A Simplified Crescent Visibility Criterion

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Abstract: **Crescent visibility has been a concern for determining the start of any lunar month. Various criteria have been offered by the astronomers since the Babylonians. The indigenous criterion proposed in this paper uses the two reliable parameters, altitude and crescent width, and makes it possible to forecast the visibility for any phase of the Moon, not just limited to crescents. Though very simple, the algorithm presented here produces rather consistent results. In addition is introduced a tool for demonstration.**

Keywords: ***crescent visibility; Moon illuminance; sky brightness; crescent width; Sun-Moon elongation***

1. INTRODUCTION

The first visibility of the waxing crescent has always been a matter of interest for many societies. The word "month" has the same root as the word "Moon" and, in a lunar calendar, a month is defined as the time slice between two maiden appearances of the youngest crescent. A month for example in the Islamic calendar begins on the day following the first evening during which the waxing crescent becomes visible. Thus, for the preparation of a lunar calendar in advance, it is necessary to constitute valid formulae for the computational determination when a crescent may become visible. Astronomers therefore have strived to express various lunar visibility criteria since the Babylonian age.

This paper will introduce an alternative criterion for the naked-eye visibility of the lunar crescent. To aid the comprehension of the case, the physical perception mechanisms for the Moon's visibility will be presented first. Historical background about visibility criteria will then be explained briefly. After expressing the methodology of the new simplified criterion, the application developed for the demonstration of this criterion will be explained. Consecutively, the results obtained by this tool and their comparisons with other criteria will be summarized.

We start out by elaborating the generic lunar visibility problem.

2. PERCEPTION OF THE CRESCENT

For any object with sufficient size to be visible in the sky, there must be sufficient contrast between the object and the surrounding background [1]. Contrast is defined as the ratio of the object's (Moon's in this case) illumination to the sky's brightness [2]. So the brightness of the Moon must be a certain level higher than the sky brightness at that azimuth and elevation.

The angle between the Sun-Moon and Earth-Moon lines is called elongation. The Moon phase angle is defined as the projection of this angle onto the ecliptic plane, i.e. the difference between the celestial longitudes of the Earth and the Moon. At the time of conjunction when the celestial longitudes are the same, the Moon phase angle will be zero and elongation becomes a minimum. This minimum elongation will be an angle (Moon declination angle at conjunction) bearing a value between +5.15° and -5.15°, since Moon's orbital plane is tilted at 5.15° with respect to the ecliptic (see Figure 1). A solar eclipse occurs if this elongation is smaller than the Moon parallax, approx. 1°.

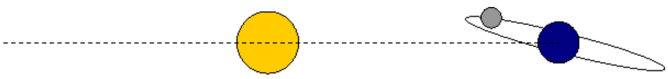

Figure 1 – Moon Declination

While the Moon in its gibbous phase is also visible during daytime, a thin crescent can only be seen after sunset, since the sky is so bright before the sunset that a new crescent is impossible to detect. As the Sun depresses further below the horizon, the sky brightness uniformly decreases. The perceived brightness of the illuminated portion of the Moon (crescent) depends upon his elongation; the sky brightness, on the other hand, is mainly related to the position of the Sun. This fact implies that the lower the Sun moves, the more will be the contrast between the thin crescent and the twilit sky. Nevertheless, the waxing crescent will also set soon after the Sun. The time lag between sunset and moonset depends on the Sun-Moon elongation and the latitude, as depicted in Figure 2[1]. When the Moon approaches the horizon, adverse effects like atmospheric refraction as well as clouds, fog, dust or pollution will diminish the brightness of the Moon and deteriorate the contrast [3].

[1] This picture is a screenshot produced by another tool of the author, namely the FELEK screensaver available at http://vakitmatik.tripod.com/felek.

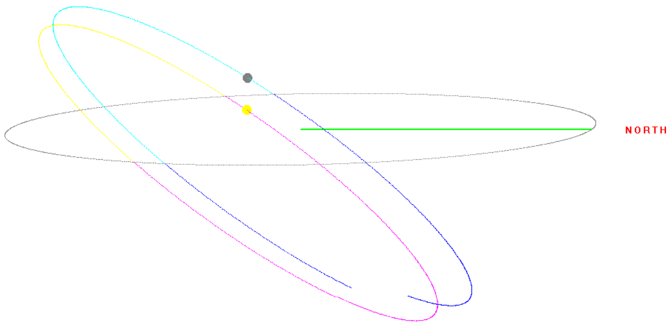

Figure 2 – Sun/Moon Trajectory

A thick crescent, lagging a sufficient amount behind the Sun, can be distinguished above the horizon during a certain period after sunset until it vanishes within the last few degrees of elevation. The younger the crescent is, the later can it be detected and the earlier it will disappear. There will be a limiting condition, where a crescent can just be identified for a very few minutes. This boundary is called the first (earliest) visibility of a crescent. Such a crescent is also perceived as shorter than full 180°, as discovered by Danjon, because the thinner edges will fall below the physiological visibility threshold [12]. The most favorable instant for the visibility is denoted as the "best time" and the least elongation for a crescent to become visible is expressed by the "Danjon limit". There will also be a unique place on Earth for each lunation, where the crescent can be first observable globally. The Sun/Moon trajectory and position at the best time and place, calculated according to the novel criteria proposed in this paper, are displayed in Figure 2. Methods for the determination of the best time and the coordinates of the best place will be presented later.

3. PREVIOUS WORK

As stated in the previous section, the crescent must be brighter than the sky in order to be visible by the observer. This implies that any visibility criterion has to manifest at least two parameters; one for the crescent illumination, the other for the sky brightness [4]. Nevertheless, in some cases (especially in the ancient times), also single parameter approaches have been practiced.

We shall summarize the basic parameters found in the literature, as follows:

3.1 Lag

Lag, which is expressed as the time delay in minutes between the sunset and the moonset, is one of the oldest parameters, used since the Babylonian era. As a fact, the more the lag, the bigger will be the elongation. A greater elongation in turn leads to a thicker crescent, implying higher illumination. On the other hand, a bigger lag means that the Sun goes deeper below horizon before the Moon vanishes, resulting in a darker sky. In general, the required contrast will depend upon the lag. However, the contrast cannot be determined by the lag only; the illumination as well as the sky brightness is related to other parameters also, especially the latitude. In high latitudes, the Sun & Moon trajectories become more decumbent, which means that the lag increases for the same elongation.

3.2 Age

Age, the other simple parameter, is defined as the time in hours passed since the conjunction. Age is only a moderate indication of the crescent illumination, since it considers neither the speed / distance of the Moon, nor the declination angle. It hardly ever gives any information about sky brightness.

3.3 Altitude

The altitude difference between the Sun and the Moon is a more recent parameter. It is also known as Arc of Vision (ARCV). For a specific point on the sky (azimuth/elevation), the brightness gradually decreases as the Sun goes down. There is also a brightness gradient on the sky in vertical direction, i.e. as one looks upwards from the horizon, for a specific time. Hence we can deduce that the brightness of the sky at the elevation of the Moon is a direct function of the ARCV. Therefore the altitude is a very good parameter to represent the sky brightness. In Figure 3, the (horizontal) western sky brightness is depicted as a function of the solar depression angle [5]. Only atmospheric conditions such as humidity or aerosols will alter the brightness.

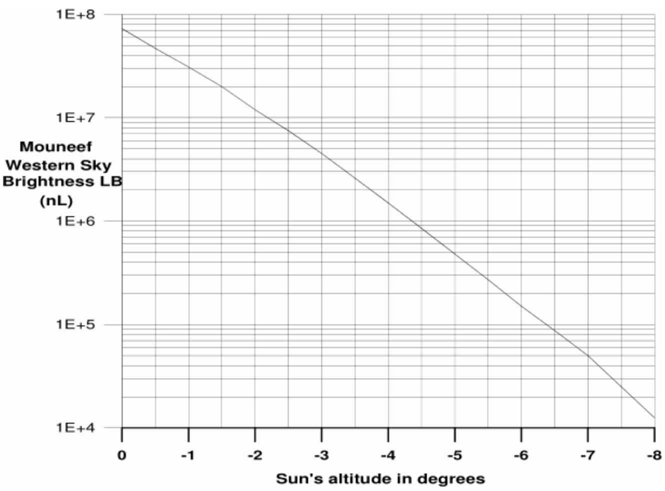

Figure 3 – Sky Brightness vs. Sun Altitude

Alternative parameters representing the altitude can be:

- Moon altitude at sunset
- Moon altitude when the Sun is 4° below horizon, which is regarded as nearly the best time [8].
- Apparent altitude of the crescent's lower limb

3.4 Azimuth

Sun-Moon azimuth difference is a common parameter generally used together with the altitude. This represents the crescent illumination. It is commonly abbreviated as DAZ (Delta Azimuth). In fact, DAZ and ARCV constitute the two orthogonal angles (see Figure 4) and, using the spherical trigonometry, one can write [16]:

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cos(ARCL) = cos(ARCV) * cos(DAZ)
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ARCL stands for Arc of Light, which is anonymous to Sun-Moon separation, or elongation.

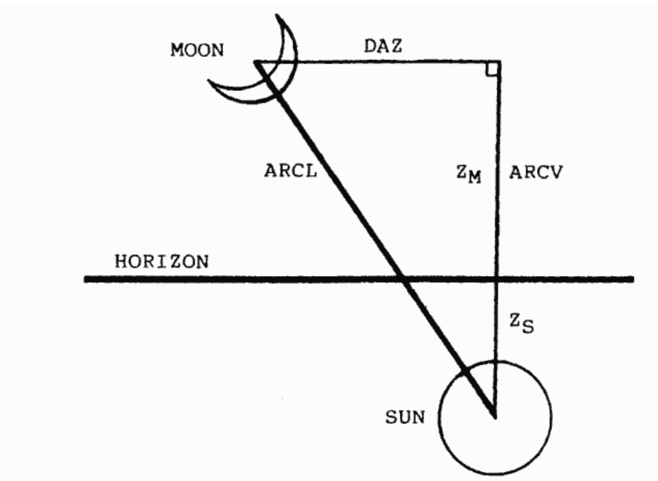

Figure 4 – Relation between ARCV, DAZ and ARCL

Azimuth and altitude together are used as two common parameters by most recent astronomers. The visibility criteria are generally shown in a graph (Figure 5). Using the above formula, we should note that for a given ARCL (elongation), ARCV (altitude) will increase as DAZ decreases. When DAZ = 0, ARCL will be maximum and equal to ARCV. The maximal Moon altitude for a specific elongation corresponds to minimal sky brightness for a given Moon illumination, maximizing the contrast. Hence the latitude where the azimuth becomes zero (the Moon is directly above the Sun) will be the unique place with the best visibility (see Figure 2).

The y-axis of the graph in Figure 5 (where DAZ = 0 and ARCL = ARCV) shows the minimum possible elongation for naked-eye visibility. Except for Fotheringham, this angle has a value of 10-11° based on statistical observation data.

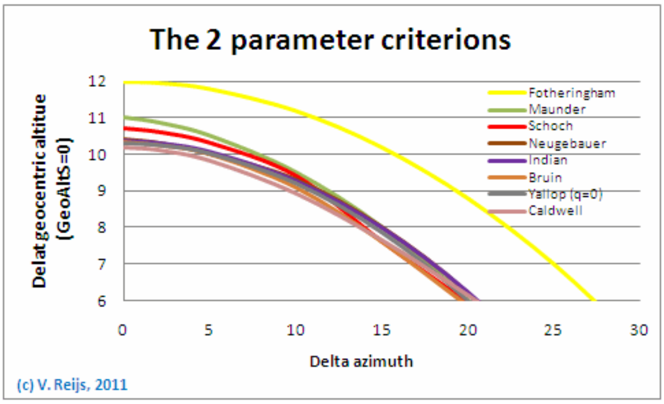

Figure 5 – Azimuth/Altitude Criteria

Ilyas extended this curve [9] in 1988 for large azimuth differences (high latitudes), as shown in Figure 6, which is known as Ilyas (C) criterion.

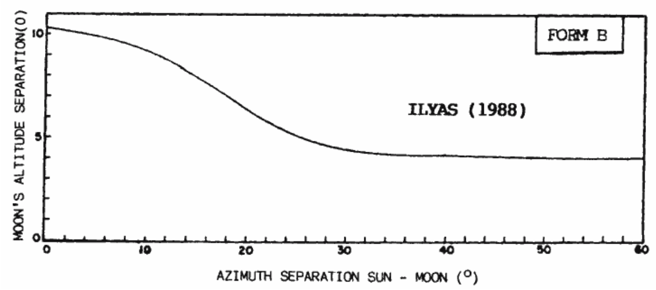

Figure 6 – Composite Extended Criterion of Ilyas

3.5 Elongation

Elongation has a strong relationship with the Moon illumination: Mathematically, the illuminated portion of the disc is given by the formula:

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Illumination = ½ * [1 – cos(Elongation)]
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For small angles, the illumination is proportional to the square of the elongation. As can be seen from the following equation [15], the elongation takes the declination angle into consideration:

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cos(Elongation) = cos(Phase) * cos(Declination)
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Ilyas, in 1984, plotted a curve with Moon's altitude vs. elongation [10], denoted as Ilyas (A). The Royal Greenwich Observatory (RGO) also uses altitude & elongation. Caldwell recently presented a paper explaining the dependence of crescent visibility on lag & elongation [1]. In Indonesia, lag, elongation, altitude and azimuth are used in combination [17].

3.6 Crescent Width

Although elongation is a direct representation for the illumination of a disc, it lacks the size. Since the distance of the Moon to the Earth is not constant due to the eccentricity of the Moon's orbit, its apparent diameter changes continually. The central width of the crescent is directly proportional to the illuminated area observed and therefore should be incorporated to optimally represent the illumination criterion. The crescent width subtends an angle small enough to write the following approximation:

$$\text{Width} \approx 11950 * \text{Illumination} / \text{Moon Distance}$$

In this formula, the crescent width is in arc-minutes and the Earth-Moon distance is in thousands of kilometers.

Bruin developed a theoretical graph in 1977, plotting the Sun altitude / ARCV vs. Moon altitude using different values of crescent width, as shown in Figure 7. For the example with a crescent width of 0.25' and ARCV of 10°, the crescent will remain visible as long as the Sun is between 2~8° of depression (points A and B). Schaefer adopted this model incorporating atmospheric correction factors [13].

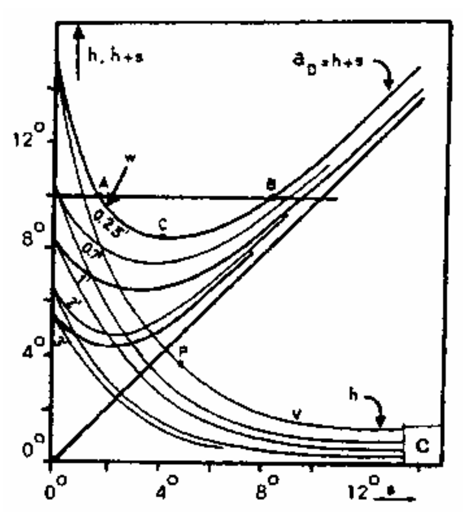

Figure 7 – Bruin's Criterion

Another research scientist, Khalid Shaukat, developed a criterion based on 900 observations, dictating Moon altitude and crescent width as parameters. According to Shaukat, (M/12.7) + (W/1.2) must be > 1 at sunset for the crescent to be visible. M denotes here the Moon altitude in degrees and W the crescent width in arc-min. The minimum Moon altitude is limited to 3.4°.

Yallop also expressed the visibility by using ARCV and W. Besides he defined the "best time" of visibility as 4/9 of the time between sunset and moonset, by interpreting the Bruin's graph. This corresponds to a topocentric Moon altitude of nearly 5.5° for ARCV=10.5° and decreases with declining ARCV.

3.7 General

Criteria with two parameters investigated so far, namely one for crescent illumination and the other for the sky brightness, has been formulated by Hoffman by combining them into an ease-of-visibility parameter v, where d represents the sky darkness and b the Moon's illumination:

$$v = d + k * b$$

He then defined a normalized ease-of-visibility parameter q. Values < 0 will mean "crescent impossible to see" and values > 1 will have a meaning of "certainly visible" [2]. Variables v_0 and v_1 are the ease-of-visibility values for the lower and upper limit, respectively:

$$q = (v - v_0) / (v_1 - v_0)$$

He next asserted that the Moon will be visible if the following condition is met:

$$h_s < x * q + h_{s0}$$

The parameter h_s is the instantaneous Sun altitude, h_{s0} the Sun altitude at the lower visibility limit and x an empirically fitted constant.

Note that while the criteria mentioned before describe the conditions for the earliest visibility of a crescent, Hoffman's rule will be valid for any phase of the Moon.

4. PROPOSED CRITERION

After elaborating the parameters and their pro-cons, we deduced that the most two suitable parameters for the visibility (or contrast) would be the crescent width (for illumination) and the altitude (for sky brightness).

Our proposal will include both of them; however the altitude is separated into two independent variables; namely the Sun altitude and the Moon altitude. We also shall distinguish several cases for the visibility:

- Moon is not visible when the apparent upper limb is under the horizon. This implies that the Moon altitude should roughly be greater than -0.5°.
- The Moon is visible, independent of the Sun position, if its illumination is greater than the daylight sky brightness. The limiting corresponding crescent width has been observed as roughly 2.5~3 arc-min.

- When the Sun altitude is less than 4° and it becomes yellowish, the sky brightness will no more be uniform; it will decrease as ARCV increases, due to refraction. This is also valid after sunset; the twilight brightness is inversely proportional to the elevation. For a given Sun altitude under 4°, the limiting Moon altitude will decrease (more sky brightness) as the crescent width increases (more illumination), in order the contrast to remain the same.
- For a given Moon altitude, the limiting Sun altitude will increase (more sky brightness) as the crescent width increases (more illumination), in order the contrast to remain the same. However, the change rate of the Sun altitude will be smaller because a small depression of the Sun leads to a high decrease of the brightness (see Figure 3).
- When the Moon altitude is such low that the atmospheric extinction dominates and the contrast diminishes, ARCV should be increased for compensation. Theoretically, after astronomical twilight (Sun below 18°) when the sky brightness is practically zero, a very thin crescent (width ≈ 0) could also be detectable. In 1966, a crescent vestige was photographed from space, when the elongation was only 2° [11].

Ilyas, in 1994, cited that the increased atmospheric extinction when the Moon is closer to the horizon should be compensated by a shift in altitude. Furthermore, he categorized all the existing criteria with two parameters as 4th order, and stated that the (modern) 5th order criterion should include extinction [9].

In this proposal, we shall accordingly modify the Hoffman equation and also handle two different cases, the one when the Moon is higher than 4° (practically no extinction) and the other when it is lower (extinction considered):

$$h_{s1} < a_1 * d' + k * b' + x * q + h'_{s01}$$
$$h_{s2} < a_2 * d' + k * b' + x * q + h'_{s02}$$

h_{s1}	: Sun altitude (S) in degrees, when M > 4°
h_{s2}	: Sun altitude (S) in degrees, when M < 4°
d'	: Topocentric Moon altitude (M) in degrees
b'	: Topocentric Crescent width (W) in arc-min
k	= 3.33
x	= 2° (see *Probability of Visibility*)
a_1	= 0.33 (M > 4°)
a_2	= 3.33 (M < 4°)
h'_{s01}	= -7.67° (M > 4°)
h'_{s02}	: -19.67° (M < 4°)

If we take all these conditions into consideration, the correspondent stipulation we offer for the visibility will be:

$$(S < S_{max} \text{ OR } S_{max} > 4) \text{ AND } M > -0.5$$

S denotes Sun altitude in degrees, M stands for topocentric Moon altitude in degrees and W represents the crescent width in arc-minutes. S_{max} is calculated as follows:

$$S_{max} = (M - 20 + 10 * W) / 3 \quad \text{if: } M \geq 4°$$
$$S_{max} = (10 * M - 56 + 10 * W) / 3 \quad \text{if: } M < 4°$$

If the Sun elevation is greater than 4°, the refraction will become negligible and the sky brightness will be uniform, i.e. the visibility will be independent of the Moon altitude so M is set to a constant value (4°).

4.1 Effect of Height

With increasing height above sea level, air density continually declines. This in turn causes the refraction to be smaller, diminishing the sky brightness. Since the Moon illumination is not affected, the contrast will be higher, favoring the visibility. It is a common practice to climb a nearby mountain in order to witness the first emergence of the thinnest crescent. Note that in March 2002 at a sight of 2,200 meter height, the crescent with ARCL = 7.6° has been already distinguished [14]. Similarly, calculation by using a photometric model results in a minimum ARCL of 7.5° for 2,000 m height [7].

The apparent horizon shifts down with the height. At 1,000 meter height, this shift is roughly 1°. S_{max} will therefore be corrected for height as follows:

$$S'_{max} = S_{max} + E$$
$$E = \cos^{-1}(R / (R + h))$$

Here R denotes the Earth radius and h the height above MSL. The minimum Moon altitude of -0.5° should also be corrected for height in a similar manner.

4.2 Probability of Visibility

The q-value defined by Yallop to quantify the ease of visibility has a minimum value of 0.216 for "easily visible" and a maximum value of -0.014 for "optical aid to find the crescent" [6]. The difference is 0.23 which corresponds to 2.3 degrees. This spread reflects the uncertainty related to the observations and incorporates atmospheric conditions as well as observers' capabilities. Ilyas [9] and Schaefer [13] also investigated this effect and found a similar spread of ±1°. This spread corresponds to the coefficient x in the Hoffman's equation and therefore we take it as 2°.

Accordingly, we incorporate this into our criteria as the percent probability (P) and the final equation becomes:

$$S''_{max} = S'_{max} - P / 50 + 1$$

S''_{max} and S'_{max} will be equal for 50% probability.

The resulting visibility algorithm is sketched below.

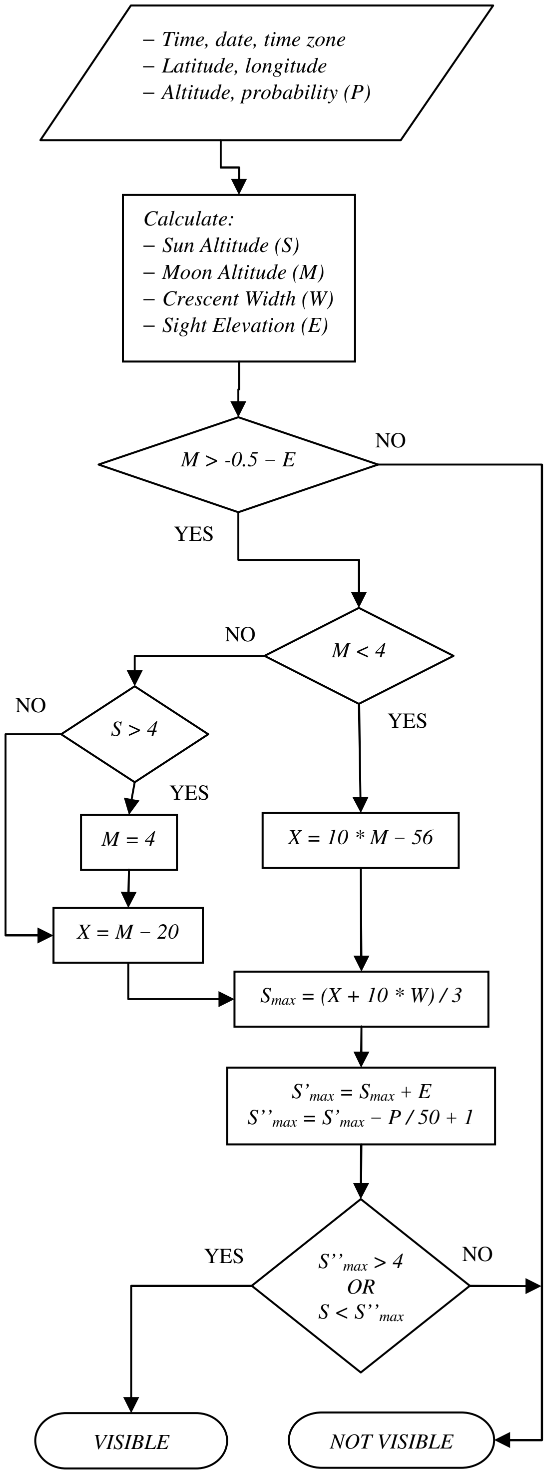

Figure 8 – Flowchart of Proposed Lunar Visibility Algorithm

The algorithm claimed can be sketched as a graph, displaying S_{max} (y-axis) vs. M (x-axis) for various crescent widths (W) in Figure 9:

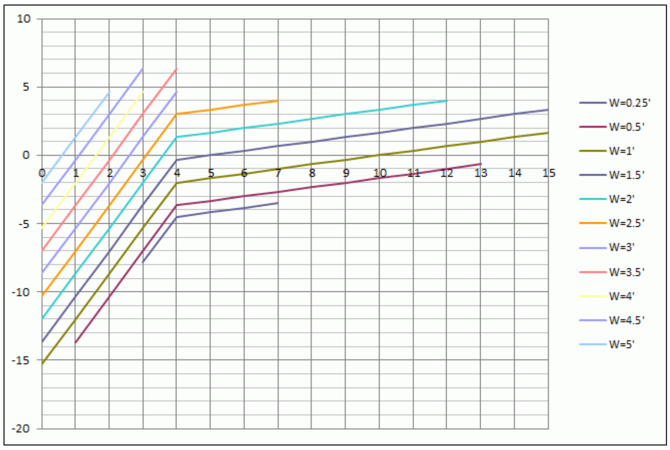

Figure 9 – Visibility Graph of Proposed Criterion

5. DEMONSTRATION TOOL

To demonstrate the performance of the criterion and compare its results with the other criteria in literature, a tiny software program[2] has been developed as a screen saver. EHILLE, this screen saver, can be easily configured to supply the necessary parameters (Figure 10).

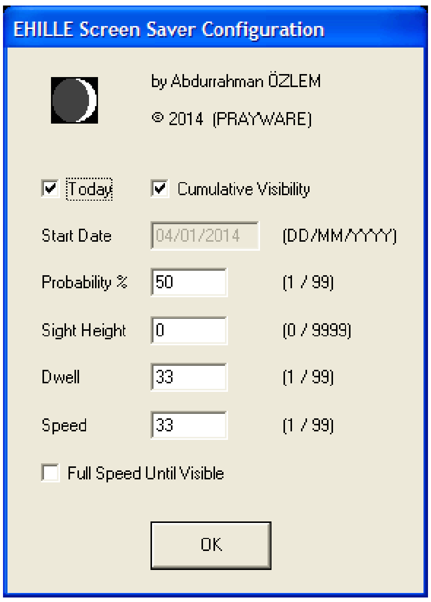

Figure 10 – Configuration Screen of EHILLE

[2] This tool is available at http://vakitmatik.tripod.com/ehille.

The area of instantaneous visibility is painted on a Mercator map in real-time and the painted areas are then combined to form the cumulative area of visibility, which has the shape of a parabola. The vertex of this parabola represents the “best place” on Earth and the area widens westward, being symmetric on a roughly horizontal line. The position of the vertex is unique for each lunation.

The software first computes the time of conjunction and shifts the Mercator map accordingly, such that the parabola lies more or less on the same place, its vertex being placed near the right border. Next is calculated the start time, which is nearly one hour before the first global visibility. Then the time is progressed with the entered speed and the visibility is checked continually.

A relatively simple approach to draw the parabola would be to compute the visibility for each pixel (corresponding to a certain latitude & longitude) by executing the novel algorithm for every tested time. However, this necessitates more than 500 million loops for a complete run, requiring a huge amount of calculation time. Therefore the software uses a smart search & track method which speeds up the process nearly 5,000-fold, as detailed below.

Following the start, the software searches the best place on the map for visibility. Beginning from the center of the right border, a vertical search (up and down) is performed as to maximize ARCV to find the latitude where the Moon is vertical to the Sun (DAZ = 0). Consecutively, a horizontal search determines the longitude where the Moon altitude is 4°, which is most favorable condition according to our algorithm. The combined search fixes the position with the highest possibility of visibility, and this position is tested & updated only once for each time using the algorithm. The time is then incremented one minute and the search is repeated.

After detection of the maiden visibility at this best place, the coordinates & local time is displayed on the screen. Now, the software checks the visibility in a vertical scan and saves the latitude limits of visibility, up and down, which forms the border points of the parabola. The connection line is then painted. This scan is repeated for neighbor longitudes, left and right, until visibility ceases. The instantaneous visibility area is formed thereby. For the next minute, the software shifts the former “best place” to left (¼°) and the horizontal scan is initiated from the saved points, considerably shrinking the computing time.

A screenshot of the tool is displayed in Figure 11 to visualize the output. The graph looks very similar to those obtained through the famous Moon Calculator program by Dr. Monzur Ahmed.

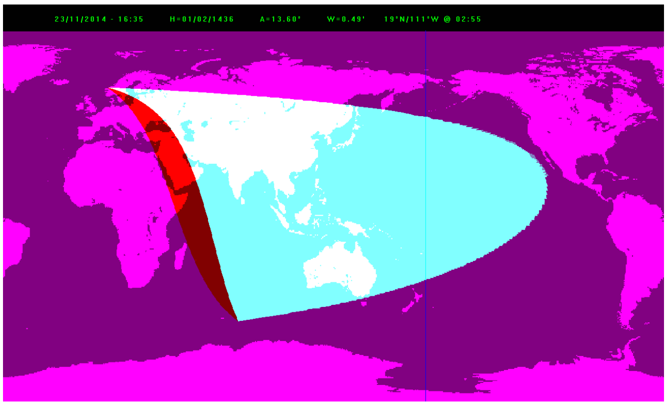

Figure 11 – Visibility Parabola Obtained by EHILLE

6. COMPARISON & CONCLUSION

Several cases will be analyzed in this section and compared with the criteria available in literature, as to evaluate the validity of the proposed method. Since our criteria uses topocentric Moon elevation, we will consider the parallax of Moon (≈ 0.95°) for the calculation of the error.

Case #1: DAZ = 0°, ARCV = 10.5°

This case is the upper-left corner of the mean of the 7 curves in Figure 5. Regarding the “best time” or 4/9 rule, the Moon altitude will be taken as 5.5°. The average crescent width for ARCL = 10.5° is 0.26’ and the necessary Sun depression is calculated as 3.97°. The result has an error of 0.08°.

Case #2: DAZ = 12°, ARCV = 9°

The Moon altitude will be taken as 4.5°. The average crescent width is 0.53’ and the necessary Sun depression is calculated as 3.42°. The error result has an error of 0.13°.

Case #3: DAZ = 20°, ARCV = 6.5°

This case is the lower-right corner of the mean of the 7 curves in Figure 5. The Moon altitude will be taken as 4°. The average crescent width is 1.03’ and the necessary Sun depression is calculated as 1.91°. The result has an error of 0.36°.

Case #4: DAZ = 35°, ARCV = 4.5°

This case is chosen to test against Ilyas (B) criterion in Figure 6. The Moon altitude will be taken as 2.5°. The average crescent width is approx. is 2.84’ and the necessary Sun depression is calculated as 0.86°. The result has an error of 0.19°.

Case #5: S = -4°, W = 0.25'

This case validates the algorithm against the Bruin criterion in Figure 7 (Point C). The Moon altitude is calculated as 4.5° and the error is almost zero.

Case #6: S = -4°, W = 0.7'

This case is 2nd validation against the Bruin criterion. The Moon altitude is calculated as 3.7° and the error is almost zero.

Case #7:

This case tests the algorithm against Shaukat's criterion: If we take S = -1° at sunset and assume a site height of 2,000 meters ($E \approx 1.5°$), our first condition can be rewritten as $(M/12.5) + (W/1.25) > 1$, quite similar to Shaukat's condition.

6.1 Discussion on Islamic Calendar

When relying on local calendar, the Visibility Separator Parabola [3] clearly defines the new lunar month; the crescent is visible on the area within the parabola and the next month begins, whereas inhabitants on the area outside the parabola should wait for another day. However, if we consider a global calendar instead, the problem is how to set the International Lunar Date Line (ILDL), or better "Lunar Month Line". An old but rational rule is test whether the crescent will be visible on the Earth before 12:00 pm (UCT), i.e. midnight at Greenwich. If yes, the following day is declared as the first day of the new Islamic month; if not, it will be the last day of the old month. This rule is regarded as reasonable because at that time Sun sets at the 90°W meridian and the crescent can only be seen west of that, where the Great Ocean resides. So if the crescent is observable after 12:00 pm (UCT), probably nobody will be able to see the crescent on that night; otherwise, at least some people on the west coasts of the American continent may see the crescent, and, since meanwhile the night still continues on a large majority of the Earth, that night and the following day can be declared as the beginning of the new lunar month. Figure 10 visualizes such a case, where the visibility starts just after the midnight at Greenwich and the crescent is first visible in Australia.

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