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FREQUENCY ANALYSIS OF HYDROLOGIC DATA WITH SPECIAL APPLICATION TO RAINFALL INTENSITIES

Ven Te Chow

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NOMENCLATURE

A, B = coefficients in the theoretical equation of best-fit; also, variables in a hypergeometric series

C = variable in a hypergeometric series

a,b,c = statistical parameters in the distribution function of hydrologic event

d = accumulated depth of rainfall in in.

$F(A,B,C,1)$ = hypergeometric series defined as

$$F(A,B,C,1) = 1 + \frac{A}{1}\frac{B}{C} + \frac{A(A-1)}{2!}\frac{B(B-1)}{C(C-1)} + \dots$$

$F(N)$ = function of N for the computation of statistical control curves

$F(T_M)$ = function of T_M for the computation of statistical control curves

$F(y)$ = limiting form of distribution function

$f_o(y)$ = initial distribution function of original hydrologic data of magnitude y

$f_x(A,B,C,1)$ = the $x + 1$st member of a hypergeometric series

h = arbitrary variable in Stirling formula

K = Chow's frequency factor

k = order of factorial moment

m = average number of hydrologic events per year; also the rank of statistical events arranged in an order of descending magnitude

N = total number of years of observation; also number of future trials in the theory of the number of exceedances

n = number of past observations in the theory of the number of exceedances

P = probability of recurrence of an event equal to or greater than a given magnitude, say y

P_E = probability of an annual maximum value being equal to or greater than a given magnitude, say y

P_M = probability of an annual exceedance value being equal to or greater than a given magnitude, say y

$P(n,m,N,x)$ = cumulative probability of distribution function of the number of exceedances, x, over the m-th largest value among n observations in N future trials

$P(y)$ = limiting form of $p_o(y)$

p = geometric mean probability of probabilities, p_1, p_2, p_3, . . . p_r; also, arbitrary variable in Stirling formula

$p_o(y)$ = probability of a value equal to or less than a given magnitude, y

p_r = probability of occurrence of a hydrologic event due to one of causative factors, where r is the designated number of a causative factor

T = recurrence interval in years, which is defined as the average interval of time within which the magnitude of a hydrologic event will be equaled or exceeded once on the average

T_E = recurrence interval for annual exceedance value in years

T_E' = converted value for T_E

T_M = recurrence interval for annual maximum value in years

T_M' = converted value for T_M

T_0 = recurrence interval in years for non-recurrence of an event in a designated future period

T_1 = recurrence interval in years for an event recurring at least once in a designated future period

t = duration of rainfall in minutes

x = recurrence interval in transformed scale; also number of exceedances

$\bar{x}$ = mean of x-values, or $(\Sigma x)/N$; also mean number of exceedances

$\overline{xy}$ = mean of the products of x and y, equal to $(\Sigma xy)/N$

y = magnitude of a hydrologic event

y_E = magnitude of annual exceedance values

y_M = magnitude of annual maximum values

y_E' = theoretical value of y_E

y_M' = theoretical value of y_M

y_0 = value of y which it would need to have if it were to lie exactly on the line of best-fit

$\bar{y}$ = mean of y-values, or $(\Sigma y)/N$

$\overline{y^2}$ = mean of the squares of y, or $(\Sigma y^2)/N$

γ = Euler's Constant, or 0.5772157 . . .

Δy = half-height of the confidence belt for statistical control curves

σ = standard deviation

σ_y = standard deviation for y, defined as $\sqrt{[N/(N-1)]\,(\overline{y^2} - \bar{y}^2)}$

ϕ_x = function representing $\dfrac{x!}{(x-k)!} f_x\,(A,B,C,1)$

I. INTRODUCTION

1. Significance and Purpose of the Study

The greatest need for information relating to hydrologic frequencies is apparent in various economic studies and in efficient designs of cofferdams, waterway openings in bridges, highway and railway culverts, urban storm sewers, farm terraces, airfield drainage, stream-control works, hydroelectric power installations, water-supply facilities and many other hydraulic structures and projects which are designed in consideration of the frequency of certain hydrologic events. In securing the necessary information from hydrologic data, hydraulic engineers are therefore required to possess a working knowledge of the hydrologic frequency analysis with respect to its principles and procedures.

The existing methods of hydrologic frequency analysis are numerous, and the view points and theories expressed thereupon are diverse and confusing. It is very desirable to review the manifold methods available in this field and to apply them to a certain specific problem. From the results thus obtained, it is possible to develop sound principles and practical procedures for the use of engineers. The purpose of this report is to attempt the presentation of such principles and procedures of frequency analyses for hydrologic data.

2. Development of the Study

Engineers of the Illinois Division of Highways and the Bureau of Public Roads concerned with the design of express highways were not satisfied with existing methods of utilizing records of rainfall for the design of storm drains. They recommended that an intensive mathematical study of hydrologic frequency analyses be undertaken as one phase of the Cooperative Highway Drainage Investigation at the University of Illinois. The study was started in the fall of 1948, and continued over a period of two academic years on a half-time working basis.

In the beginning, a comprehensive survey was made of available literature in the field under consideration. It was followed by a review of various statistical methods of hydrologic analysis as well as an investigation on the suitability of data to be used in the analysis. A number of selected methods were then applied to the precipitation data for Chicago. These data were used because the immediate project attention was directed to the Congress Street Express-Highway in Chicago with which the Illinois Division of Highways was primarily concerned. From the knowledge and experience gained through the first stage of research, an attempt was made to develop new and improved procedures of analysis. Finally, for testing their practical applicability, the procedures were applied again to the Chicago data, and in addition, to the data of Seattle, Washington, and Los Angeles, California.

As a result of the intensive study, thirteen preliminary reports were produced and submitted to the members of the Technical Advisory Committee of the Cooperative Highway Drainage Investigation. After reviewing these reports, the Committee considered that these studies would be of value to students and engineers working in the hydrologic field. At its meeting dated May 13, 1952, the Committee approved the publication of these studies as a final report of the study presented in a reassembled and revised form.

3. Scope of the Study

This report consists of four main parts: Part I, Introduction, in which a general account of the hydrologic study — its significance, purpose, and development — is presented; Part II, Principle and Theory of Analysis, in which the selection of data, interpretation of theory, formulas for plotting positions, fitting of theoretical curves, statistical controls, and other features are described; Part III, Procedure and Application of Analysis, in which improved methods are developed and applied to the Chicago data which are taken as an illustrative example; and Part IV, Preparation of Hydrologic Data for Analysis, in which sources and kinds of data, deficiencies of data and methods of adjustment are discussed.

4. Acknowledgments

This study was undertaken as a part of one project of the Highway Planning Survey of the Illinois Division of Highways in cooperation with the Bureau of Public Roads. The work was done under a memorandum agreement for Cooperative Investigation of Highway Drainage entered into by the University of Illinois, Engineering Experiment Station, of which Dean W. L. Everitt is the Director, and the State of Illinois, Division of Highways, of which Mr. F. N. Barker is the Chief Highway Engineer.

On the behalf of the University, the work was carried out under the administrative direction of Professor W. C. Huntington, Head of the Department of Civil Engineering, and Professor Ellis Danner, Director of the Illinois Cooperative Highway Research Program, and under the technical supervision of Professor J. J. Doland of Hydraulic Engineering.

On the part of the State of Illinois, Division of Highways, the work was under the administrative direction of Mr. H. E. Surman, Assistant Engineer of Design, Mr. W. L. Esmond, Engineer of Research and Planning, and Mr. W. E. Chastain, Engineer of Physical Research. On the part of the Bureau of Public Roads, the work was under the direction of Mr. C. F. Izzard, Chief of Hydraulics Branch, and Mr. F. L. Anthony, Materials Engineer of the Illinois District.

The Technical Advisory Committee of this research project is formed by six members: Professors J. J. Doland and H. L. Langhaar of the University of Illinois, Mr. H. E. Surman of the Illinois Division of Highways, Professor W. E. Howland of Purdue University on behalf of the Illinois Division of Highways, Mr. C. F. Izzard of the Bureau of Public Roads and Professor Wallis Hamilton of Northwestern University on behalf of the Bureau of Public Roads.

Part IV of this report has been reviewed by the U. S. Weather Bureau, Washington office. All corrections and suggestions by the Bureau have been incorporated. The author is indebted to Mr. F. W. Reichelderfer, Chief of the Bureau, and his staff for their contributions.

Special acknowledgment is gratefully made to Mr. J. C. Guillou, Research Assistant Professor of Hydraulic Engineering, University of Illinois, for his constant advice during the progress of the study, and to Mr. W. D. Potter, Head, Hydrology Section, Hydraulics Branch, the U. S. Bureau of Public Roads, for his kind review of the preliminary reports and valuable suggestions to the preparation of this final report. Acknowledgment is also made to Miss Joyce Vandiver for typing the manuscript.

II. PRINCIPLE AND THEORY OF ANALYSIS

5. General

Hydrologic data are random in nature depending on their magnitude and time of occurrence. When the data are arranged in the order of magnitude, a series of data which can be subjected to mathematical analysis is formed. The distribution of this series may be studied by the method of statistics. It is assumed at the outset that within the period of time under consideration the distribution of hydrologic data possesses a definite pattern which may be derived by mathematical theories and verified by observed data. The theoretical distribution indicates the case which is closely approached only when the data is a truly representative selection, covering the whole period of time under consideration. As this is rarely the case in the actual application of the theory, the approximation of the observed pattern to the theoretical distribution would depend upon the quality of data and the length of record. Generally speaking, hydrologic data taken from a record extending over a period of 20 yrs, using the statistical method which employs two parameters, should produce a fair approximation for practical purposes.

6. Selection of Data

The available hydrologic data are generally arranged in a chronological order. Figure 1a exhibits a hypothetical set of such data for a certain period of observation, say 20 yr as shown in the figure. The magnitude of data is expressed in an arbitrary unit. Experience has shown that many of the original data have practically no significant value in the analysis because the hydrologic design of a project is usually governed by a few of the extreme conditions only. In order to save labor and time in anlysis the data of insignificant magnitude should be excluded. For this purpose, two types of data, the annual maxima, or annual maximum values, and the annual exceedances, or annual exceedance values, are proposed for use in the analysis.

7. The Annual Maxima

The annual maximum value is the largest of all observations taken in a year. For example, the annual maximum daily flood is the largest of the 365 observations of daily discharges; the annual maximum peak

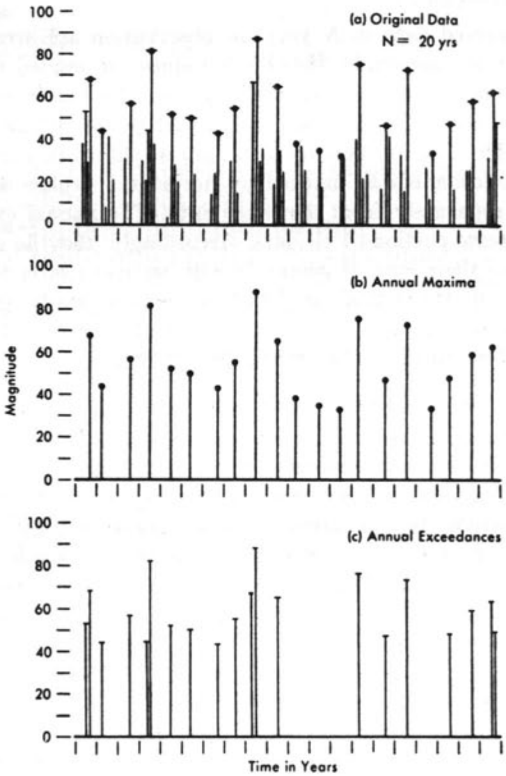

Fig. 1. Hydrologic Data Arranged in the Order of Occurrence

flood is the largest of all flood peaks observed in a year; and the annual maximum rainfall intensity for a certain duration is the largest of all observed values in a year. As there is only one value of annual maximum recorded in a year, the number of annual maxima in a period of observation is equal to the number of years of observation. It will be shown later that when the number of hydrologic data in their initial distribution becomes large, the annual maxima of the data approximate a definite pattern of distribution. In studying this pattern of distribution, only annual maxima are required in the analysis, while other data can be disregarded. In Fig. 1a, the annual maxima of the hypothetical set of data are marked with dots, and they are also shown separately from other data in Fig. 1b.

8. The Annual Exceedances

When all observed data in N years of observation are arranged in a descending order of magnitude, the top N values are named the annual exceedances. As in the case of annual maxima, the number of annual exceedances in a period of observation is equal to the number of years of observation. Thus, in the hypothetical data as shown in Fig. 1a, there are 20 annual exceedances as marked by horizontal strokes. In Fig. 1c, they are shown separately from the other data. The annual exceedances are only the extreme portion of all data. Accordingly, they do not form a complete series of their own. However, it will be shown later that as the number of data in the initial distribution becomes large, the annual exceedances would converge to an asymptotic pattern of distribution which can be subjected to mathematical manipulations.

9. Comparison of Two Approaches in Selecting Data

The annual maxima and the annual exceedances of the hypothetical data in Fig. 1a are arranged graphically in Fig. 2 in the order of magnitude. The figure shows that many annual exceedances exceed the annual maxima in magnitude. In this particular illustration, only five of twenty values are equal to the annual maxima. This is also demonstrated in Fig. 1a, in which the second largest value in a given year outranks, in magnitude, many annual maxima. Consequently, when only the annual maxima are selected, these second largest values would be omitted, resulting in the neglect of their effect in the analysis. On the other hand, when the annual exceedances alone are selected, an objection commonly recognized is that the selected data may not be fully independent events; that is, one event could affect another which follows closely after, such as one flood sets the stage for the next close flood and one storm disturbs the meteorological condition for the subsequent ones.

Logically speaking, the selection of data should be judged by the nature of the designed structure. The annual exceedances should be used if the second largest values in the year would affect the design. For instance, the damage caused by flooding sometimes results from the repetition of flood recurrence rather than from a single peak flow. Consider also the design of a culvert in which damage or destruction may be rapidly and economically repaired and then soon again exposed to damage. The case is similar for highway drainage in which the loss due to traffic interruption as a result of flooding will be weighed by the number of flood peaks and the extent of flooding which may be caused largely by associated peak flows. In other cases where the design is controlled by the most critical condition, such as the design of a spillway, the annual maxima should be used.

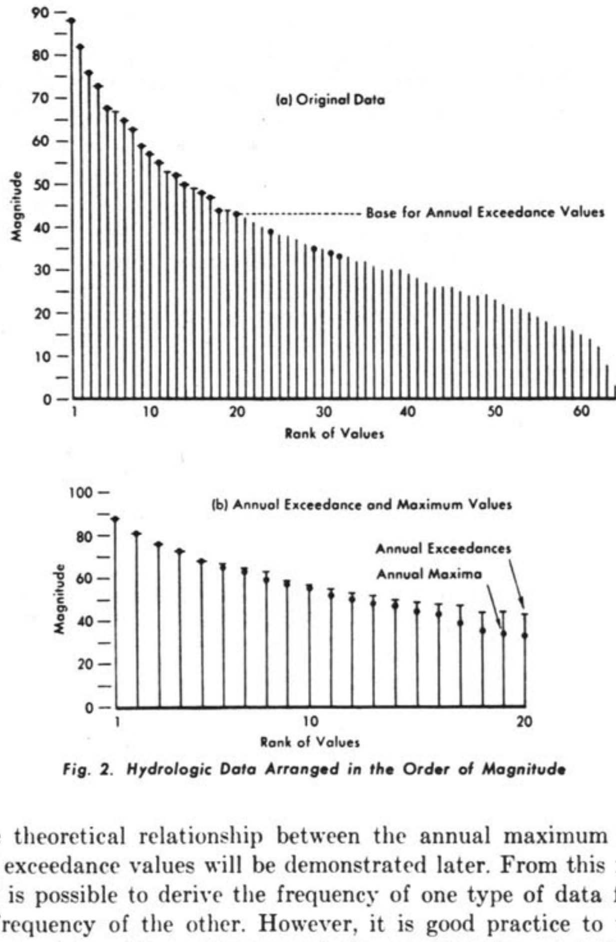

Fig. 2. Hydrologic Data Arranged in the Order of Magnitude

The theoretical relationship between the annual maximum and the annual exceedance values will be demonstrated later. From this relationship, it is possible to derive the frequency of one type of data from the given frequency of the other. However, it is good practice to work up hydrologic data with a view toward the consideration of both annual maxima and annual exceedances.

10. Recurrence Interval

The primary object of the frequency analysis of hydrologic data is to determine the recurrence interval of the hydrologic event of a given magnitude, say y. The so-called recurrence interval, denoted hereafter by T in years, is defined as the average interval of time within which the magnitude of the event y will be equaled or exceeded once on the average. For instance, if we say the 100-yr flood of a river at a certain gaging station is 17,600 cfs, the recurrence interval is 100 yrs. It means that the magnitude, 17,600 cfs, of the flood has been equaled or exceeded, as deduced from the data, on an average period of 100 yrs and it will be continued so by assuming a definite pattern of frequency distribution under the period of consideration.

The term frequency is often used interchangeably with the recurrence interval. However, it should not be construed to mean a regular or stated interval of occurrence or recurrence; this meaning is accepted in certain branches of sciences. Sometimes, the frequency may also mean the number of occurrences.

For both approaches, annual maxima and annual exceedances, the total number of events is equal to the total number of years of record. The difference lies only in the fact that the annual maximum value occurs exactly once a year, while the annual exceedance value occurs once a year on the average. Let P be the probability of recurrence of an event equal to or greater than magnitude y, then the recurrence interval T years is the reciprocal of this probability or

$$T = 1/P \tag{1}$$

For the sake of clarity, it may also be reasoned that if an event equal to or greater than y occurs once in T years, the chance of occurrence or the probability P is equal to 1 in T cases, or $P = 1/T$, hence $T = 1/P$.

There is a term known as percentage frequency often used in connection with the frequency study. It corresponds to the probability, expressed in percentage, and may be defined as the percent of observed events that were equal to, or larger than, a given event within the period of records under observation.

11. Relationship between Recurrence Intervals, T_M and T_E

Let P_E be the probability of an annual exceedance value being equal to or greater than magnitude y, and m be the average number of events per year or mN be the total number of events in N years of record. Then,

$P_E/m =$ the probability of any event being of magnitude y or greater

and

$1 - P_E/m =$ the probability of any event equal to or less than magnitude y

Accordingly,

$(1 - P_E/m)^m =$ the probability of an event of magnitude y being a maximum of the m events in a year

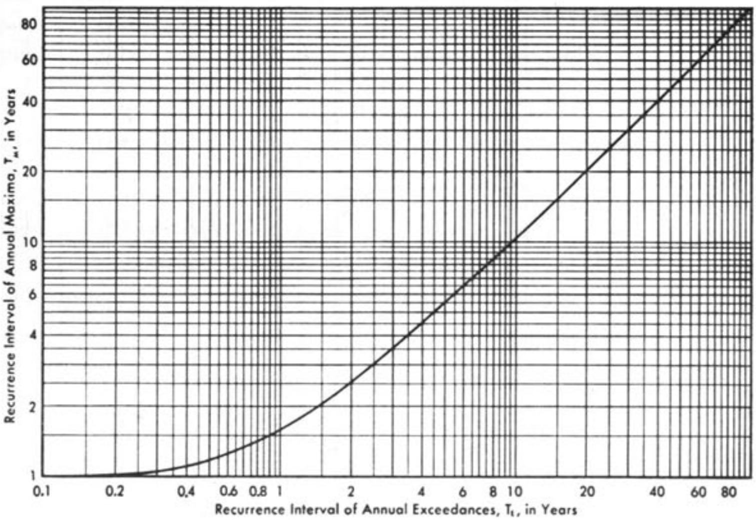

*Fig. 3. **Relationship between Recurrence Intervals, T_M and T_E***

This probability $(1-P_E/m)^m$ approaches e^{-P_E}, provided P_E is small compared with m. Consequently, the probability, P_M, of an annual maximum of magnitude y being equaled or exceeded is equal to

$$P_M = 1 - e^{-P_E} \tag{2}$$

If T_M and T_E are the recurrence intervals for annual maximum and annual exceedance values respectively, then, by Eq. 1, $P_M = 1/T_M$ and $P_E = 1/T_E$. Substituting these values of P_M and P_E into Eq. 2 and simplifying, the relationship between the two recurrence intervals is expressed by

$$T_E = \frac{1}{\log_e T_M - \log_e (T_M - 1)} \tag{3}$$

The relationship between T_M and T_E of Eq. 3 is plotted as shown in Fig. 3. This relationship has also been investigated by W. B. Langbein.* He compared it with a few actual cases and found that the ratio of T_E to T_M is slightly less than that of the theoretical values, but the plotted points are very close to their theoretical positions. As the theory of probability postulates the independence and randomness of events, it is believed that the discrepancy between the theoretical and the actual positions is due to the deviations from these two qualities in actual data.

* W. B. Langbein, "Annual Floods and the Partial-Duration Flood Series," Trans. Amer. Geophys. Union, Vol. 30, No. 6, December, 1949, pp. 879-881.

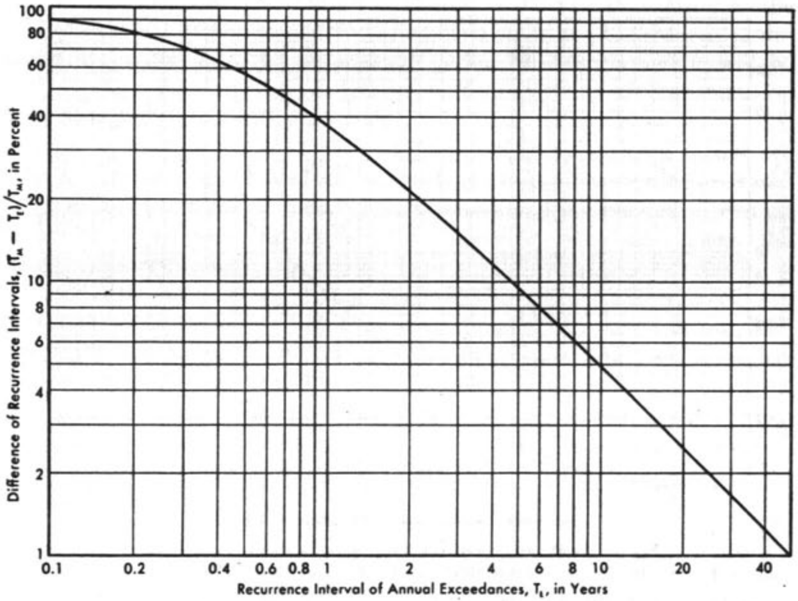

Fig. 4. Relationship of $(T_M-T_E)/T_M$ to T_E

Figure 3 also indicates the important fact that the two recurrence intervals approach numerical equality for events of large recurrence intervals. The difference between these two intervals with respect to T_M has been computed and is plotted against T_E as shown in Fig. 4.* It can be seen that a difference of about 10 percent corresponds to a 5-yr recurrence interval, T_E, and a difference of about 5 percent corresponds to a 10-yr recurrence interval. In ordinary engineering practice, a 5 percent difference is tolerable in such cases. In other words, these two approaches give essentially identical results for recurrence intervals greater than about 10 yrs. Figure 4 also shows that the difference is always positive; that is, for a given event, T_M is always greater than T_E.

12. Frequency Distribution

The frequency distribution is an arrangement of numerical data according to size or magnitude. For practical purposes, it may be represented graphically in various ways.

* Ven Te Chow, Discussion of "Annual Floods and the Partial-Duration Flood Series," Trans. Amer. Geophys. Union, Vol. 31, No. 6, December, 1950, pp. 939-941.

Consider the hypothetical hydrologic data in Fig. 1a which consist of 64 items recorded in a period of 20 yrs. Using the range of the data as a guide, the data may be divided into a number of convenient sized groups as shown in Columns 1 and 2 of Table 1.

The range of magnitude may be plotted against the number of items in the form of a bar diagram as shown in Fig. 5a. A smooth curve is drawn in to fit the bar diagram. This curve is taken as the theoretical frequency curve which defines the pattern of distribution of the hydrologic data. It extends further in one direction from its peak than in the other, and thereby indicates a lack of symmetry. This unsymmetrical pattern of distribution is characteristic of most hydrologic data.

Table 1
Study of Frequency Distribution

Range of Magnitude (1)	No. of Items (2)	No. of Items Greater than the Range (3)	Percentage Greater than the Range (4)
1–5	1	63	98.4
6–10	1	62	96.9
11–15	3	59	92.2
16–20	6	53	82.8
21–25	8	45	70.3
26–30	9	36	56.3
31–35	8	28	43.7
36–40	6	22	34.4
41–45	5	17	26.6
46–50	4	13	20.3
51–55	3	10	15.6
56–60	2	8	12.5
61–65	2	6	9.4
66–70	2	4	6.3
71–75	1	3	4.7
76–80	1	2	3.1
81–85	1	1	1.6
86–90	1	0	0.0
	64		

If the magnitude is plotted against the percentage of items greater than the indicated range, as computed in Column 4 of Table 1, then a cumulative frequency curve or probability curve of the hydrologic data is obtained as shown in Fig. 5b. This probability P represented by the ordinate may be taken as the probability of recurrence of an event equal to or greater than magnitude y represented by the abscissa.

It is convenient for practical purposes to plot the data in a straight line. This can be done by transforming the scale of the ordinate in such a way, as shown in Fig. 5c, that the curve would appear as a straight line. Whenever a theoretical frequency distribution is proposed, a special probability paper with transformed scales for straight-line plotting may be designed. (See Section 24.) Any plotting of data appearing in a straight

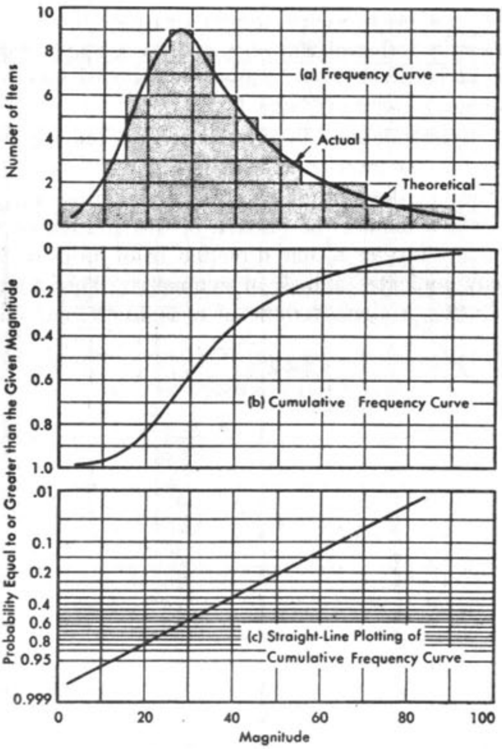

Fig. 5. Graphical Representation of Frequency Distribution of Hypothetical Hydrologic Data

line on this paper should mean that the frequency distribution of data follows a proposed law of non-symmetrical probability. If the distribution pattern of the data shows a symmetrical shape the scale of ordinates would be different from that shown in Fig. 5c.

By the principle of statistics, a theoretical law of frequency distribution may be defined by a number of parameters. J. J. Slade, Jr.* has pointed out after making a mathematical study of hydrologic frequencies that because of errors of sampling inherent in the ordinary hydrologic data, it is meaningless to compute any statistical parameters higher than the second order. Therefore, it is suggested that two statistical parameters be used to define the law for frequency distribution of hydrologic data.

* J. J. Slade, Jr., "The Reliability of Statistical Methods in the Determination of Flood Frequencies," U. S. Geological Survey, Water Supply Paper 771, 1936, pp. 421-432.

13. Distribution of Annual Maxima

The theory of extreme values which was introduced by Fisher and Tippett in 1928 is used to interpret the theoretical distribution of annual maxima.* They found that the distribution of the N largest (or the N smallest) values, each of which values is selected from one of m values contained in each of N samples, approaches a limiting form as m is increased indefinitely. The application of this theory to the frequency distribution of hydrologic data was first made by E. J. Gumbel in 1941.† In principle, it is assumed that annual maximum values of N years of record approaches a definite pattern of frequency distribution when the number of observations in each year becomes large. The theoretical treatment of this distribution is described below.

Let $f_o(y)$ be the function of the initial frequency distribution of original hydrologic data and $p_o(y)$ be the probability of a value equal to or less than magnitude y. In effect, the probability $p_o(y)$ is the cumulated frequency distribution function of $f_o(y)$; or inversely, $f_o(y)$ is the first derivative of $p_o(y)$, i.e.,

$$f_o(y) = dp_o(y)/dy \tag{4}$$

The probability that all of m observations will be equal to or less than y is

$$p(y) = [p_o(y)]^m \tag{5}$$

It should be noted that this is also the probability that y will be the largest among m observations. Therefore, for an initial distribution of $f_o(y)$, the frequency distribution function of the largest value is the first derivative of its probability $p(y)$, or

$$f(y) = dp(y)/dy$$

By Eq. 5

$$f(y) = d\,[p_o(y)]^m/dy$$
$$= m\,[p_o(y)]^{m-1}\,dp_o(y)/dy$$

By Eq. 4

$$f(y) = m\,[p_o(y)]^{m-1}f_o(y) \tag{6}$$

It is thus shown that the largest value is a statistical variable with a distribution $f(y)$ of its own expressed by Eq. 6, which distribution, in general, is different from the initial distribution $f_o(y)$, from which the largest values are selected.

* R. A. Fisher and L. H. C. Tippett, "Limiting Forms of the Frequency Distribution of the Smallest and Largest Member of a Sample," Proceedings of Cambridge Philosophical Society, Vol. 24, 1928, pp. 180-190.

† E. J. Gumbel, "The Return Period of Flood Flows," Annals Mathematical Statistics, Vol. XII, No. 2, June, 1941, pp. 163-190.

Equation 6, however, depends on the distribution $f_o(y)$. It cannot be evaluated unless $f_o(y)$ becomes known. By the theory of extreme values, it is possible to demonstrate that when m becomes large and indefinite, a limiting form of the distribution of largest values independent of the initial distribution will then be approached. This limiting form is derived as follows:

When y is an unlimited variable, the probability $p(y)$ of Eq. 5 converges toward:

$$P(y) = e^{-e^{-(a+y)/c}} \tag{7}$$

in which a and c are statistical parameters. This limiting form for the probability of largest values has been proved by Fisher and Tippett and was later affirmed by Gumbel. Accordingly, the limiting form for the frequency distribution of largest values is the first derivative of $P(y)$, or it is:

$$F(y) = \frac{1}{c} e^{-(a+y)/c - e^{-(a+y)/c}} \tag{8}$$

Fisher and Tippett also evaluated the parameters a and c by the method of moments as follows:

$$a = \gamma c - \bar{y} \tag{9}$$

and

$$c = (\sqrt{6}/\pi)\,\sigma \tag{10}$$

where $\gamma = 0.5772157\ldots$ a so-called Euler's constant

$\bar{y}$ = the mean

If N is the total number of the observed data, then the mean is the average of their magnitude y computed by the formula

$$\bar{y} = \Sigma y/N \tag{11}$$

and σ = the standard deviation which is defined by

$$\sigma = \sqrt{[N/(N-1)]\,(\overline{y^2} - \bar{y}^2)} \tag{12}$$

in which $\overline{y^2}$ is the mean of the squares of y, or

$$\overline{y^2} = \Sigma y^2/N \tag{13}$$

N and y are the same as those in Eq. 11.

Gumbel has used Eqs. 9 to 13 inclusive to compute the parameters a and c. Thus the theoretical distribution $F(y)$ can be determined and a theoretical curve represented by Eq. 8 may be drawn to fit the observed data. However, experience has shown that this method of moments does not always give as good a fit as the method of least squares. The use of the latter method which will be described later is therefore recommended.

14. Theoretical Recurrence Interval for Annual Maxima

The probability of recurrence P_M of an annual maximum value equal to or greater than magnitude y is obviously the complementary probability of $P(y)$ represented by Eq. 7, or

$$\begin{aligned} P_M &= 1 - P(y) \\ &= 1 - e^{-e^{-(a+y)/c}} \end{aligned} \tag{14}$$

The recurrence interval T_M of annual maxima may be therefore found from Eqs. 1 and 14, or

$$T_M = \frac{1}{1 - e^{-e^{-(a+y)/c}}} \tag{15}$$

Transposing and simplifying, Eq. 15 becomes

$$y = -a - c \log_e [\log_e T_M - \log_e (T_M - 1)] \tag{16}$$

Substituting Eqs. 9 and 10 for a and c, Eq. 16 becomes

$$y = \sigma K + \bar{y} \tag{17}$$

where K is the so-called frequency factor, first defined by V. T. Chow* as follows:

$$K = -\frac{\sqrt{6}}{\pi} \{\gamma + \log_e [\log_e T_M - \log_e (T_M - 1)]\} \tag{18}$$

Equation 18 is used to compute the relationship between the frequency factor and the recurrence interval. A K-T_M curve showing this relationship is given in Fig. 6.

When y is plotted against K in linear scales, Eq. 17 indicates that a straight line should be produced. On the linear scale of K, a transformed scale of T_M can be calibrated by the use of Eq. 18 or Fig. 6. As demonstrated in Fig. 5c, the data plotted on this transformed scale should also appear as a straight line if they follow the theoretical law. A special probability paper used for straight line plotting of annual maxima is shown in Fig. 10. A paper of this kind was first suggested by R. W. Powell in 1943.†

15. Distribution of Annual Exceedances

It is generally understood that a complete probability study of all observations is possible only when the entire distribution of the events is

* Ven Te Chow, "A General Formula for Hydrologic Frequency Analysis," Trans. Amer. Geophys. Union, Vol. 32, No. 2, April, 1951, pp. 231-237.

† R. W. Powell, "A Simple Method of Estimating Flood Frequencies," Civil Engineering, Vol. 13, February, 1943, pp. 105-106.

known. Since annual exceedances are the top portion of a series, they do not form a complete distribution. This is shown in Fig. 7 where ABCD indicates a hypothetical distribution curve of original data, and AB is the portion for annual exceedances only. However, as the complete data are unavailable, any law which applies to the portion AB, such as ABC′, would be good also for annual exceedances. That is to say, when any curve ABC′ of Fig. 7 is chosen to fit the distribution of annual exceedances, and since this curve fits the portion AB adequately, it is not essential that the other portion BC′ should also fit the actual distribution of BCD which is unknown. With this understanding in mind, the theoretical distribution of annual exceedances may be demonstrated:

Consider that the occurrence of a hydrologic event of certain magnitude y is a result of the joint action of many causative meteorological and geographical factors whose probabilities of occurrence are $p_1, p_2, p_3, \ldots p_r$, all being functions of y. By the theorem of multiple probabilities, the probability of the combined action is

$$p_o(y) = p_1 p_2 p_3 \ldots p_r \tag{19}$$

where r is the number of factors. Let p be the geometric mean probability of all causative factors, then

$$p_o(y) = p^r \tag{20}$$

Since r is infinitely large, $p_o(y)$ converges to a limiting form as below

$$P(y) = e^{-e^{-\frac{\log_e 10}{a}(y-b)}} \tag{21}$$

This is due to the same reasoning which has been applied to the derivation of Eq. 7.

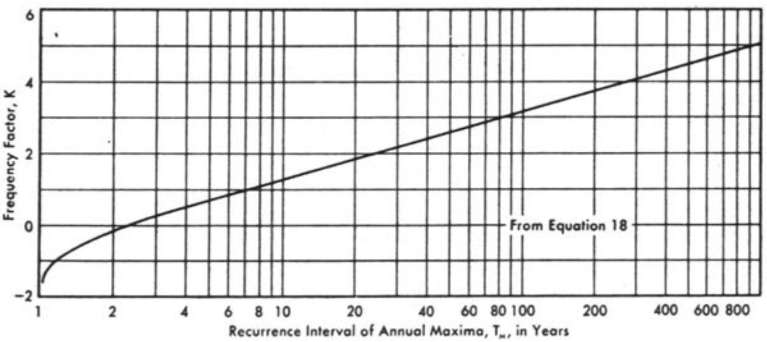

Fig. 6. K-T_M Curve

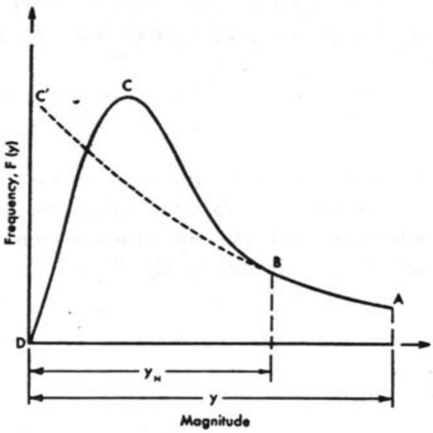

Fig. 7. Fitting Distribution Curve for Annual Exceedances

For annual exceedance values, y is of high magnitude, hence the probability of Eq. 21 may further converge to

$$P(y) = 1 - e^{-\frac{\log_e 10}{a}(y-b)} \tag{22}$$

This is the probability that the annual exceedance y will not be exceeded. The distribution function of the annual exceedances is the first derivative of Eq. 22, or

$$F(y) = \frac{\log_e 10}{a} e^{-\frac{\log_e 10}{a}(y-b)} \tag{23}$$

which gives the exponential curve represented by ABC′ in Fig. 7.

16. Theoretical Recurrence Interval for Annual Exceedances

The probability of recurrence P_E of an annual exceedance value equal to or greater than magnitude y is the complementary probability of $P(y)$ represented by Eq. 22, or

$$P_E = 1 - P(y) = e^{-\frac{\log_e 10}{a}(y-b)} \tag{24}$$

The recurrence interval T_E of annual exceedances may be found from Eqs. 1 and 24, or

$$T_E = e^{\frac{\log_e 10}{a}(y-b)} \tag{25}$$

Converting Eq. 25 into logarithmic form,

$$y = a \log_{10} T_E + b \tag{26}$$

which will give a straight-line relationship between y and T_E when plotted on a semi-logarithmic graph paper with T_E represented by a logarithmic scale.

It should be interesting to see through the mathematical relationship between T_E and T_M at this stage of demonstration. As proved previously, T_M is related to T_E by Eq. 3. It can be seen that if Eq. 3 is substituted into Eq. 16, an equation practically the same as Eq.. 26 results. Therefore, if the use of the theory of extreme values for annual maxima is mathematically adequate, then the conclusion shown by the fact that the equation obtained by substituting Eq. 3 into Eq. 16 is practically the same as Eq. 26, should explain the legitimate use of the semi-logarithmic plotting for annual exceedances.

17. Plotting Positions

In treating hydrologic data for the study of their frequency of recurrence, two phases of study are generally met. One is the frequency of recurrence for the observed distributions, and the other is the corresponding frequency of recurrence for the theoretical distributions of best-fit. The former is usually required for the purpose of plotting observed data and hence is called the "plotting positions," a term used by H. A. Foster.* The latter is treated by the mathematical theory of probability, such as that described in previous articles, and generally serves as a theoretical basis for interpreting the observed phenomena.

A correct determination of plotting positions has been a moot question and has caused a great deal of discussion. Many methods for computing plotting positions have been proposed, but few of them deserve theoretical explanation. An attempt is made here to give a rational solution to the derivation of plotting position formulas for T_M and T_E.

Assume that n past observations were taken from a certain unknown distribution of events. These n values can be arranged in an order of descending magnitude in which the rank, designated by m, of the largest value is equal to one. According to Gumbel and von Schelling,† it is possible to compute the probability that an observed value of any rank m which will be taken from the same distribution will be equaled or exceeded x times in N future trials. It can also be proved that the mean number of exceedances $\bar{x}$ for this m-th largest value to be equaled or exceeded in N future trials is

$$\bar{x} = N \frac{m}{n+1} \tag{27}‡$$

* H. A. Foster, "Duration Curves," Trans. Amer. Soc. Civ. Engrs., Vol. 99, 1934, pp. 1213-1235.

† E. J. Gumbel and H. von Schelling, "The Distribution of the Number of Exceedances," The Annals of Mathematical Statistics, Vol. XXI, No. 2, June, 1950, pp. 247-262.

‡ The derivation of this formula is referred to Appendix I.

For annual maximum values, the recurrence interval may be defined as the time in years for N future trials that the m-th largest value in annual maxima will be equaled or exceeded once on the average. In other words, the recurrence interval T_M is equal to N when the mean number of exceedance $\bar{x}$ is equal to one. From Eq. 27, when $\bar{x} = 1$,

$$1 = N \frac{m}{n+1}$$

and for $T_M = N$,

$$T_M = \frac{n+1}{m} \tag{28}$$

This indicates that the recurrence interval of an annual maximum value is equal to the number of years of record plus one and divided by the rank of the value. Equation **28** is recommended for computing the plotting positions of annual maxima.

In case of annual exceedance values, n and N are numbers of events respectively in the past years of observation and in future years of observations. Generally, they are very large values. Thus, the fraction $N/(n+1)$ approaches N/n and the former can be replaced by the latter in Eq. 27; i.e.,

$$\bar{x} = Nm/n \tag{29}$$

Furthermore, it may be reasonably assumed that the number of events is proportional to the number of years in the period under consideration. In other words, N may be taken as the recurrence interval T_E and n as the number of years of record. As the recurrence interval of annual exceedance value may be defined as the time in N future years that the m-th largest value of observed annual exceedances will be equaled or exceeded once on the average, the plotting position formula for annual exceedances is derived from Eq. **29** with $\bar{x} = 1$ and $N = T_E$ as follows:

$$T_E = n/m \tag{30}$$

which indicates that the recurrence interval of an annual exceedance value is equal to the number of years of record divided by the rank of the value.

18. Fitting Theoretical Curves

When annual maximum and annual exceedance values respectively are plotted on their corresponding special probability papers, straight lines are expected to be produced if the data follow the proposed theoretical law of distribution. Experience has shown that hydrologic data do exhibit such straight line trends. However, since the data are rarely perfect, the observed values will not coincide with the theoretical values, and hence, a scatter or variation about a straight line occurs. After the data are plotted, it is generally required that they be fitted to a straight line which would represent a correct trend.

There are two analytical methods of curve fitting: the method of moments and the method of least squares. By the first method, the theoretical probability curve is defined by a number of statistical parameters, the values of which can be evaluated by taking moments about an arbitrary value as an origin. Detailed procedures will not be described herein as the reader may find them in any standard textbook of statistics. Fisher and Tippett have used this method to evaluate the parameters for the theoretical distribution of extreme values as given in Eqs. 9 to 13 inclusive, and Gumbel has utilized them in actual application. A simplified alternative method to achieve the same purpose is to define the straight line by Eq. 17 in which the parameters y and σ are computed by Eqs. 11 and 12 respectively. Equation 17 represents a straight line on the special probability paper in which T_M is plotted on a transformed scale. The relation between T_M and K is given by Eq. 18 or by Fig. 6.

In the second method the principle of least squares is employed in determining the line that best describes the trend of the data. The principle states that a line of best-fit to a series of values is a line the sum of the squares of the deviations about which will be a minimum; the deviations are the differences between the line and the actual values. In the probability paper, the ordinate represents magnitude y and the abscissa represents, in transformed scale, the recurrence interval T_M or T_E. It is assumed that the errors which cause the scatter of plotted points exist only in the y's inasmuch as the recurrence interval is computed on a theoretical basis and is considered as an independent variable. Therefore, the deviations are the differences between the observed y's and the theoretical y_o's, where y_o is the value y would need to have if it were to lie exactly on the line of best-fit.

According to the principle stated above,

$$\Sigma(y - y_o)^2 = \text{a minimum} \tag{31}$$

Let the line of best-fit be represented by a linear equation as follows:

$$y_o = Ax + B \tag{32}$$

in which A is the slope of the line, B is the intercept of the line on y_o-axis and x is the recurrence interval in transformed scale. Since the purpose is to determine the values of A and B, it is required to differentiate the expression on the left side of Eq. 31 with respect to A and B respectively, and set the derivatives equal to zero as follows:

$$\frac{d\Sigma(y - y_o)^2}{dA} = 0 \tag{33}$$

and

$$\frac{d\Sigma(y - y_o)^2}{dB} = 0 \tag{34}$$

Substitute y_o of Eqs. 32 into Eqs. 33 and 34, and simplify the expressions, noting $\Sigma B = BN$ where N is the number of plotted values or also equal to the number of years of record, then,

$$A\Sigma x^2 + B\Sigma x = \Sigma xy \tag{35}$$

$$A\Sigma x + BN = \Sigma y \tag{36}$$

Solve Eqs. 35 and 36 simultaneously for A and B, and let $(\Sigma x)/N = \overline{x}$, $(\Sigma y)/N = \overline{y}$, $(\Sigma x^2)/N = \overline{x^2}$ and $(\Sigma xy)/N = \overline{xy}$, then,

$$A = \frac{\overline{x}\,\overline{y} - \overline{xy}}{\overline{x}^2 - \overline{x^2}} \tag{37}$$

and

$$B = \overline{y} - A\overline{x}$$

$$= \frac{\overline{x}\,\overline{xy} - \overline{x^2}\,\overline{y}}{\overline{x}^2 - \overline{x^2}} \tag{38}$$

Comparing Eq. 32 with Eq. 17 and 26, it can be seen that for curve fitting of annual maxima, x corresponds to K, and for annual exceedances, x corresponds to $\log_{10} T_E$.

19. Statistical Control of Annual Maxima

The fact that the observed data exhibit a straight-line trend in plotting on special probability papers but do not follow exactly the path represented by the theoretical line leads to the belief that singular events cannot be forecasted with perfect confidence by the theory of probability and statistics. Consequently, it becomes essential to know the confidence in results obtained by the frequency analysis, that is, to know how well the individual event agrees with the theoretical line derived from the observed data. Gumbel has developed the technique which establishes so-called "confidence limits" for annual maximum values.* The technique is based on the principle that the theoretical value of rank m situated on the straight line and corresponding to a given recurrence interval is

* E. J. Gumbel, "Statistical Control-Curves for Flood-Discharges," Trans. Amer. Geophys. Union, Pt. II, 1942, pp. 489-500, and "The Statistical Forecast of Floods," The Ohio Water Resources Board, Columbus, Ohio, December, 1948, pp. 4-7.

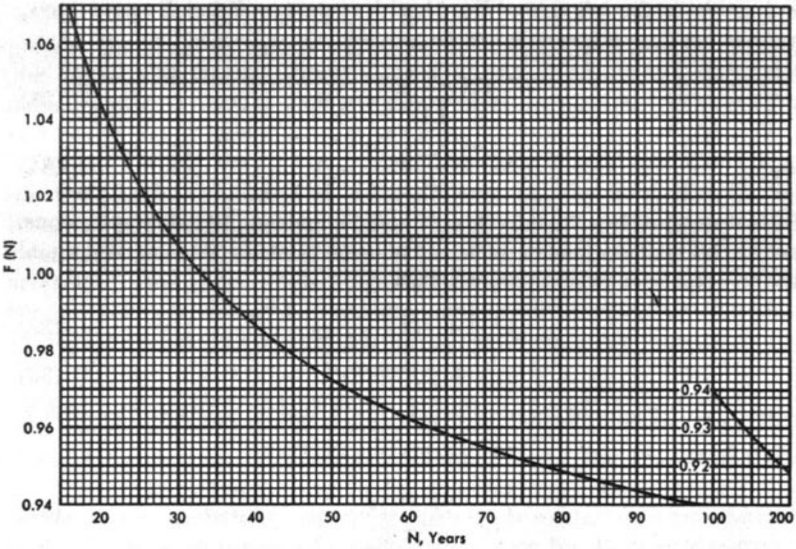

Fig. 8. Relation between N and F(N)

the approximation to the most probable m-th value. With a probability equal to 68.269%,* the mth observation is contained in a confidence belt which is bound by two control curves. The two control curves are constructed respectively above and below the theoretical straight line with a vertical distance of Δy measured from the line. In other words, the m-th observation is contained in the confidence belt defined by

$$y - \Delta y < y < y + \Delta y \tag{39}$$

It is expected that 68.269% of all observed values will fall within the confidence belt.

For practical purposes, the control curves may be constructed by a simplified method with which the half height, Δy, of the confidence belt may be computed by the following approximate rules:

(1) For the largest value $(m = 1)$,

$$\Delta y_1 = \sigma_y \cdot F\ (N) \tag{40}$$

in which σ_y is the standard deviation of the observed magnitude y, and $F\ (N)$ is a function of years of observation, N. The value of $F\ (N)$ has been computed and plotted against N as shown in Fig. 8.

* The deviations from the predicted values are usually measured by multiples of standard deviation, σ. This probability of 68.269% corresponds to the probability for a deviation of $\pm\sigma$ from the predicted value in a normal probability scale.

(2) For the second largest value ($m = 2$),

$$\Delta y_2 = \frac{0.661\,(N+1)}{N-1}\,\Delta y_1 \tag{41}$$

(3) For intermediate values,

$$\Delta y = \frac{0.877}{\sqrt{N}}\,\Delta y_1 F(T_M) \tag{42}$$

where $F(T_M)$, a function of T_M, may be found from Fig. 9 for given values of T_M. When T_M is greater than 10 yrs, $F(T_M)$ may be computed by the following formula:

$$F(T_M) = T_M^{0.5} \tag{43}$$

(4) As the smallest values are usually of no interest, their control curves are not necessary.

(5) For extrapolation beyond the largest value, the control curves are two parallels to the extrapolated straight line; the half height of the belt bound by the two parallels is equal to Δy_1 of the largest value.

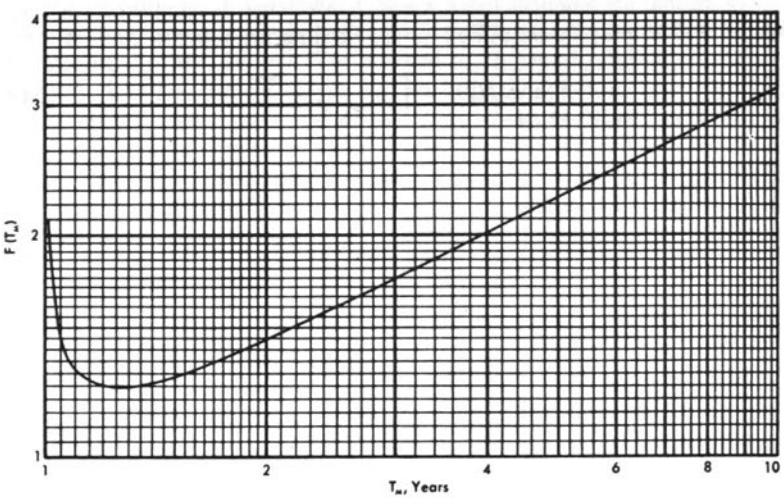

Fig. 9. Relation between T_M and $F(T_M)$

20. Statistical Control of Annual Exceedances

The control curves for annual maxima can be easily converted for the use of annual exceedances. For the largest and the second largest values and for extrapolation, the width of the confidence belt is practically identical. For intermediate values, Eq. 42 may be used, but the recurrence interval T_M in the equation should be converted from the given T_E by means of Eq. 3 or Fig. 3 which gives the theoretical relationship between T_E and T_M.

There is another method which can be employed to show the confidence in annual exceedance values. By this method, the probability that the m-th largest among n past observations will be equaled or exceeded x times in N future trials is considered. This probability is described in Appendix I, and Eq. A6 is derived for its computation.

When the event is not expected to happen at all in future trials, then $x = 0$ and Eq. A6 becomes

$$P(n,m,N,0) = \frac{n!\,(N + n - m)!}{(n - m)!\,(N + n)!} \tag{44}$$

When the event is expected to happen at least once in future trials, the probability is

$$1 - P(n,m,N,0) = 1 - \frac{n!\,(N + n - m)!}{(n - m)!\,(N + n)!} \tag{45}$$

For annual exceedances, both n and N are large as compared with m, Eqs. 44 and 45 may be simplified by the Stirling formula* as follows:

$$\underset{n,N \to \text{large}}{P(n,m,N,0)} = \left[\frac{1}{1 + N/n}\right]^m \tag{46}$$

and

$$\underset{n,N \to \text{large}}{1 - P(n,m,N,0)} = 1 - \left[\frac{1}{1 + N/n}\right]^m \tag{47}$$

Consider a probability of 68.269 percent for Eqs. 46 and 47, then from Eq. 46,

$$N/n = 1.465^{1/m} - 1 \tag{48}$$

and from Eq. 47,

$$N/n = 3.150^{1/m} - 1 \tag{49}$$

As assumed previously, the number of events for annual exceedances may be proportioned to the number of years. Therefore, the ratio N/n multiplied by the number of years of observation would give the recurrence interval T_E on control curves. Equation 48 gives the control curve for the event which will not occur in the indicated recurrence interval with a probability of 68.269 percent. Equation 49 gives the control curve for the event which will occur at least once in the indicated recurrence interval with the same probability, 68.269 percent.

* Stirling formula gives $(p+h)!/p! = p^h$ or $p!/(p-h)! = p^h$ when p becomes infinite.

III. PROCEDURE AND APPLICATION OF ANALYSIS

21. Procedure of Analysis

The theory described in Part II is applied to the frequency analysis of hydrologic data according to the following steps:

(1) Selection of data
(2) Computation of plotting positions
(3) Plotting of data
(4) Fitting of theoretical curve
(5) Construction of control curves

The analysis of rainfall intensity data at Chicago, Illinois, is taken as an example for the interpretation of each step.

22. Selection of Data

In the example, the original rainfall intensity data of various durations are the excessive precipitations recorded during the period from 1913 to 1947 at Chicago, Illinois. The method of choice of original data from Weather Bureau's official record will be fully described in Part IV. The maximum excessive precipitation data are listed in Appendix IV from which the annual maximum values and the annual exceedance values of 10-min duration rainfall depth data are selected as shown in Column 2 of Table 2 and Table 3 respectively. The values are arranged in order of decreasing magnitude with the rank m listed in Column 1 of the tables.

23. Computation of Plotting Positions

Regarding the computation of plotting positions, Eq. 28

$$T_M = \frac{n+1}{m} \tag{28}$$

is used for annual maximum values, and Eq. 30

$$T_E = n/m \tag{30}$$

is used for annual exceedance values. Column 4 in Table 2 gives the T_M values and Column 4 in Table 3 gives the T_E values.

24. Plotting of Data

For annual maximum values, the magnitude y of Column 2 in Table 2 is plotted against the recurrence interval T_M of Column 4. For annual

Table 2
Frequency Analysis of Annual Maximum Values
10-Min Duration Rainfall Depth at Chicago, Illinois

m (1)	y (2)	y^2 (3)	T_M (4)	$x=K$ (5)	x^2 (6)	xy (7)	Δy (8)
1	1.11	1.2321	36.000	2.332	5.4382	2.5885	0.177
2	0.96	0.9216	18.000	1.783	3.1791	1.7117	0.124
3	0.94	0.8836	12.000	1.455	2.1170	1.3677	0.081
4	0.92	0.8464	9.000	1.218	1.4835	1.1206	0.079
5	0.88	0.7744	7.200	1.033	1.0671	0.9090	0.070
6	0.80	0.6400	6.000	0.878	0.7709	0.7024	0.064
7	0.80	0.6400	5.143	0.745	0.5550	0.5960	0.059
8	0.76	0.5776	4.500	0.627	0.3931	0.4765	0.056
9	0.74	0.5476	4.000	0.522	0.2725	0.3863	0.052
10	0.71	0.5041	3.600	0.425	0.1806	0.3018	0.050
11	0.70	0.4900	3.272	0.337	0.1136	0.2359	0.047
12	0.68	0.4624	3.000	0.255	0.0650	0.1734	0.045
13	0.68	0.4624	2.769	0.177	0.0313	0.1204	0.044
14	0.66	0.4356	2.571	0.102	0.0104	0.0673	0.042
15	0.66	0.4356	2.400	0.032	0.0010	0.0211	0.041
16	0.66	0.4356	2.250	−0.035	0.0012	−0.0231	0.039
17	0.65	0.4225	2.118	−0.100	0.0100	−0.0650	0.038
18	0.64	0.4096	2.000	−0.164	0.0269	−0.1050	0.037
19	0.64	0.4096	1.895	−0.225	0.0506	−0.1440	0.037
20	0.63	0.3969	1.800	−0.286	0.0818	−0.1802	0.036
21	0.62	0.3844	1.715	−0.346	0.1197	−0.2145	0.035
22	0.61	0.3721	1.636	−0.405	0.1640	−0.2471	0.035
23	0.60	0.3600	1.565	−0.464	0.2153	−0.2784	0.034
24	0.58	0.3364	1.500	−0.523	0.2735	−0.3033	0.034
25	0.57	0.3249	1.440	−0.582	0.3387	−0.3317	0.033
26	0.57	0.3249	1.385	−0.643	0.4134	−0.3665	0.033
27	0.53	0.2809	1.333	−0.704	0.4956	−0.3731	0.033
28	0.52	0.2704	1.285	−0.768	0.5898	−0.3994	0.033
29	0.49	0.1401	1.242	−0.834	0.6956	−0.4087	0.033
30	0.49	0.2401	1.200	−0.904	0.8172	−0.4430	0.033
31	0.47	0.2209	1.162	−0.980	0.9604	−0.4606	0.033
32	0.41	0.1681	1.125	−1.064	1.1321	−0.4362	0.034
33	0.36	0.1296	1.092	−1.159	1.3433	−0.4172	.....
34	0.34	0.1156	1.058	−1.277	1.6307	−0.4342	.....
35	0.33	0.1089	1.029	−1.445	2.0880	−0.4769	.....
	$\Sigma=22.71$	15.8049		−0.987	27.1261	4.6705	
	$\bar{y}=0.6489$	$\overline{y^2}=0.4516$		$\bar{x}=-0.0282$	$\overline{x^2}=0.7750$	$xy=0.1334$	
	$A=0.1960$	$B=0.6544$		$\sigma_y=0.1775$			

Line of best-fit: $y=0.1960K+0.6544$

exceedance values, the magnitude y of Column 2 in Table 3 is plotted against the corresponding recurrence interval T_E of Column 4.

The probability paper for annual maxima is specially prepared with a transformed scale of T_M; while for annual exceedances, standard semi-logarithmic paper is employed. In case such paper is not available, an ordinary plotting paper of rectangular coordinates may also be employed. To plot annual maximum values on rectangular coordinate paper the magnitude y is plotted against the frequency factor K. The K-values may be computed by Eq. 18 or obtained from Fig. 6 for given plotting positions of T_M. For practical purposes, the following simplified form of Eq. 18 should be used in computation:

$$K = -\left(1.1 + 1.795 \log_{10} \log_{10} \frac{N+1}{N+1-m}\right) \tag{50}$$

Table 3
Frequency Analysis of Annual Exceedance Values
10-Min Duration Rainfall Depth at Chicago, Illinois

m (1)	y (2)	y^2 (3)	T_E (4)	$x = \log_{10} T_E$ (5)	x^2 (6)	xy (7)	T_0 (8)	T_1 (9)
1	1.11	1.2321	35.00	1.5441	2.3842	1.7140	16.30	75.20
2	0.96	0.9216	17.50	1.2430	1.5450	1.1933	7.35	27.20
3	0.94	0.8836	11.67	1.0671	1.1387	1.0031	4.76	16.30
4	0.92	0.8464	8.75	0.9420	0.8874	0.8666	3.61	11.68
5	0.88	0.7744	7.00	0.8451	0.7141	0.7437	2.78	9.04
6	0.80	0.6400	5.833	0.7659	0.5866	0.6127	2.30	7.38
7	0.80	0.6400	5.000	0.6990	0.4886	0.5592	1.96	6.24
8	0.76	0.5776	4.375	0.6410	0.4109	0.4872	1.71	5.38
9	0.74	0.5476	3.889	0.5899	0.3480	0.4365	1.52	4.75
10	0.74	0.5476	3.500	0.5441	0.2960	0.4026	1.36	4.30
11	0.71	0.5041	3.182	0.5027	0.2527	0.3569	1.24	3.85
12	0.70	0.4900	2.917	0.4649	0.2161	0.3254	1.13	3.52
13	0.68	0.4624	2.692	0.4301	0.1850	0.2925	1.04	3.23
14	0.68	0.4624	2.500	0.3979	0.1583	0.2706	.97	2.98
15	0.68	0.4624	2.333	0.3679	0.1354	0.2502	.90	2.78
16	0.67	0.4489	2.188	0.3400	0.1156	0.2278	.85	2.61
17	0.66	0.4356	2.059	0.3137	0.0984	0.2070	.80	2.45
18	0.66	0.4356	1.944	0.2887	0.0833	0.1905	.75	2.32
19	0.66	0.4356	1.842	0.2653	0.0704	0.1751	.71	2.18
20	0.65	0.4225	1.750	0.2430	0.0590	0.1580	.68	2.07
21	0.64	0.4096	1.667	0.2219	0.0492	0.1420	.64	1.97
22	0.64	0.4096	1.591	0.2017	0.0407	0.1291	.61	1.88
23	0.63	0.3969	1.522	0.1824	0.0333	0.1149	.59	1.80
24	0.62	0.3844	1.458	0.1638	0.0268	0.1016	.56	1.72
25	0.62	0.3844	1.400	0.1461	0.0213	0.0906	.54	1.64
26	0.61	0.3721	1.346	0.1290	0.0166	0.0787	.52	1.58
27	0.60	0.3600	1.296	0.1126	0.0127	0.0676	.50	1.52
28	0.60	0.3600	1.250	0.0969	0.0094	0.0581	.48	1.47
29	0.59	0.3481	1.207	0.0817	0.0067	0.0482	.47	1.41
30	0.59	0.3481	1.167	0.0671	0.0045	0.0396	.45	1.37
31	0.58	0.3364	1.129	0.0527	0.0028	0.0306	.43	1.32
32	0.58	0.3364	1.094	0.0390	0.0015	0.0226	.42	1.28
33	0.57	0.3249	1.061	0.0257	0.0007	0.0146	.41	1.24
34	0.57	0.3249	1.029	0.0124	0.0002	0.0071	.40	1.20
35	0.57	0.3249	1.000	0.0000	0.0000	0.0000	.38	1.17
	$\Sigma = 24.41$	17.5911		14.0284	10.4002	11.4182		
	$\bar{y} = 0.6974$	$\overline{y^2} = 0.5026$		$\bar{x} = 0.4008$	$\overline{x^2} = 0.2971$	$\overline{xy} = 0.3262$		
	$A = 0.3421$	$B = 0.5603$						

Line of best-fit: $y = 0.3421 \log_{10} T_E + 0.5603$

in which m is the rank and N is the total number of years in record. According to the theory, the plotted points should exhibit a straight-line trend. If desirable, a transformed scale of recurrence interval T_M may be constructed beside the linear scale of frequency factor K. Thus, the plotting positions can be plotted directly on the T_M-scale. This indicates the procedure by which the special probability paper is constructed.

The probability paper used for annual exceedances is the semi-logarithmic paper on which the plotting positions of T_E are plotted in logarithmic scale against the magnitude y. An ordinary rectangular paper may also serve the same purpose, on which values of $\log_{10} T_E$, instead of T_E, are plotted against the magnitude y.

25. Fitting of Curves

The theoretical curve is fitted to observed data by the method of least squares which is described in Section 18. As the equations derived by this method apply only to data plotted in rectangular coordinates, the linear function of recurrence interval T_M or T_E should be employed for deriving the equation of a theoretical straight line of best-fit. In other words, for annual maxima, the theoretical straight line is constructed to fit the values of y and K instead of values of y and T_M; and for annual exceedances, the line is to fit y and $\log_{10} T_E$ instead of y and T_E. In the two cases, the abscissas for x are K and $\log_{10} T_E$ respectively.

To use Eqs. 37 and 38, values of $\bar{y}$, $\bar{x}$, $\overline{x^2}$ and $\overline{xy}$ are computed in Tables 2 and 3. The two constants in the straight line Eq. 32 are therefore computed by Eqs. 37 and 38. In Table 2, the constants for annual maxima are $A = 0.1960$ and $B = 0.6544$. The equation of the theoretical line of best-fit is

$$y = Ax + B \tag{51}^*$$

or

$$y = 0.1960x + 0.6544 \tag{52}$$

By means of Eq. 52, the rainfall depth of 10-min duration for a given recurrence interval can be estimated.

In Table 3, the constants of the straight line for annual exceedances are computed as $A = 0.3421$ and $B = 0.5603$. The theoretical equation is

$$y = A \log_{10} T_E + B \tag{53}$$

or

$$y = 0.3421 \log_{10} T_E + 0.5603 \tag{54}$$

which can be used to estimate the theoretical value of y for a given T_E.

26. Construction of Control Curves

The confidence belt for the theoretical probability curve is constructed by the methods described in Sections 19 and 20.

(1) For Annual Maxima. The half-height of the confidence belt, Δy, for the largest value is computed by Eq. 40 in which $\sigma_y = 0.1775$ and $F_1(N) = 0.996$ as obtained from Fig. 8 with $N = 35$. The computed $\Delta y_1 = 0.177$.

The value of Δy for the second largest value, or Δy_2, is computed by Eq. 41 in which $N = 35$ and $\Delta y_1 = 0.177$ as obtained above. The computed $\Delta y_2 = 0.124$. For intermediate values, Δy is computed by Eq. 42.

* A comparison between Eq. 51 and Eq. 17 might lead to a misunderstanding that values of A and B in Eq. 51 would correspond to values of σ and $\bar{y}$ in Eq. 17. However, a big difference should be noticed; Eq. 17 represents the theoretical straight line which is obtained by the method of moments, while Eq. 51 represents the line obtained by the method of least squares. In the given example, $\bar{y} = 0.6489$ and $\sigma = 0.1775$, and the line obtained by the method of moments, or by Eq. 17, is $y = 0.1775x + 0.6489$ which is different from Eq. 52, obtained by the method of least squares.

All required values of Δy are listed in Column 8 of Table 2. The control curves are constructed as shown in Fig. 10.

(2) For Annual Exceedances. By means of the theoretical relationship between the recurrence intervals of annual maxima and annual exceedances, the control curves for annual exceedances may be constructed

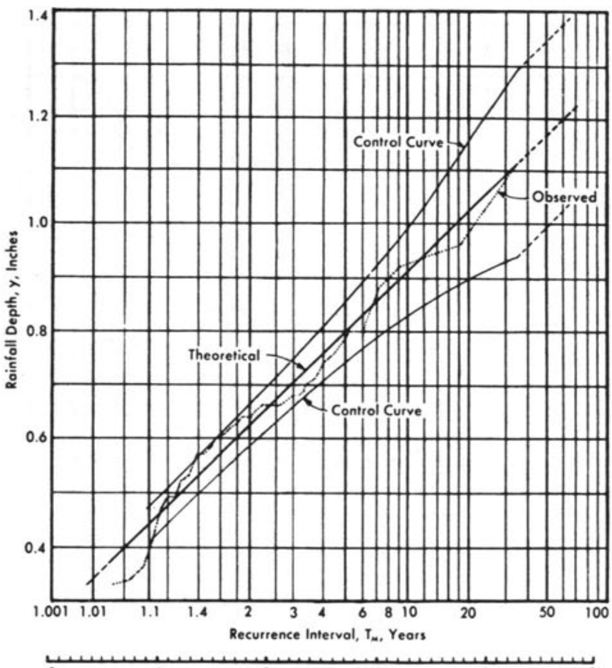

Fig. 10. Probability Curve of Annual Maxima

in the following manner: Convert the recurrence interval T_E to T_M by the aid of Fig. 3, and find the value of Δy, corresponding to this converted T_M, from the control curves of annual maxima or by interpolation of the computed values of Δy for annual maxima. For instance, with $m = 3$ in Table 3, the recurrence interval T_E is 11.67 yrs. From Fig. 3, the converted T_M is 12.20 yrs. Then, for $T_M = 12.20$, the value of Δy for this third largest value $y = 0.94$ of all annual exceedances is found from

Fig. 10 to be 0.092. The control curves of the annual exceedances are shown in Fig. 11.

The computation of the control curve for non-recurrence of the event in the designated future period with a probability of 68.3 percent and the control curve for recurrence of the event at least once in the designated

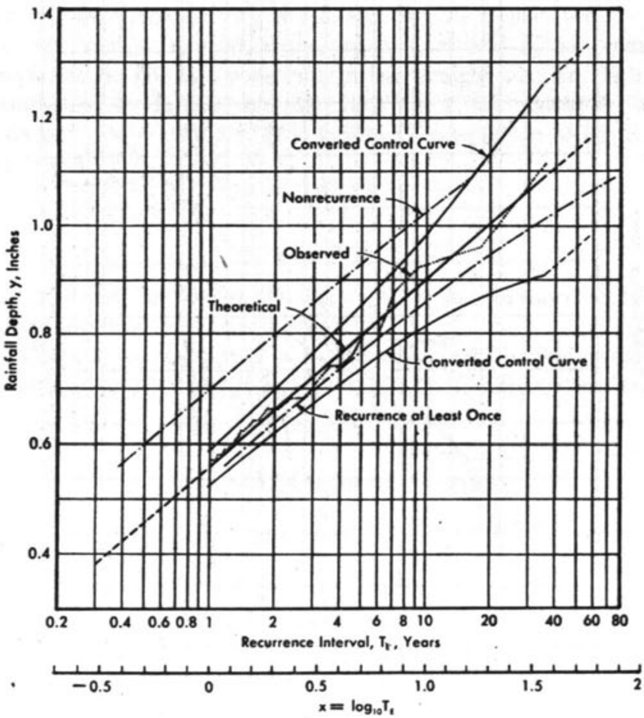

Fig. 11. Probability Curve of Annual Exceedances

future period with a probability of 68.3 percent involves the use of Eqs. 48 and 49 respectively. These values of T_0 and T_1 are computed and listed in Columns 8 and 9 of Table 3. For instance, the largest observed value, $y = 1.11$, in Table 3 has a plotted recurrence interval of 35.00 yrs. The theoretical value of y corresponding to this recurrence interval, as shown by the theoretical line in Fig. 11, is $y = 1.09$. Accordingly, the control curves are plotted at $y = 1.09$, $T_0 = 16.30$ yrs and

$y = 1.09$, $T_1 = 75.00$ yrs. The control curves for non-recurrence and for recurrences at least once are as shown in Fig. 11.

27. Off-Control Data

Occasionally, it may be observed from the plotting of a frequency curve that the plotted point for the largest value or values deviate markedly from the general path indicated by the curve of best-fit. The deviation may be so great that the plotted points fall far out of the area bound by the control curves. It is believed by some that such an outstanding event or events follow some other law which applies to a series of events that might occur at very long intervals. In other words, an event has occurred within the period of record which is entirely true but is incompatible, from a mathematical viewpoint, with those events with which it is associated in the given sample. Therefore, a more logical practice is to recognize that the largest value or values in the available hydrologic data may have an actual recurrence interval which is several times greater than the length of record. Whether or not the outstanding event or events follow the usual laws or some other law than that represented by the rest of the data, it would seem obvious that they cannot be assumed to have a recurrence interval equal to the value computed by the plotting position formulas in which the available length of the record is used. Consequently, the use of such off-control data as the largest value or values in the computation of a theoretical frequency curve would produce a different result from that which the sample should indicate. Since the off-control data would not be homogeneous with the rest of the sample, they should be excluded from the computations for the theoretical curve. In other words, when the theoretical curve first computed indicates that one or more observations included in the data are of an off-control nature, the theoretical curve is recomputed by omitting the off-control data. An estimate of the recurrence interval of the off-control data can therefore be made by extending the recomputed curve until it crosses the magnitude of the off-control points.

28. Extrapolation of Frequency Curves

In order to estimate the frequency of hydrologic events having a recurrence interval considerably greater than the length of record, the extrapolation of frequency plotting of existing hydrologic data becomes necessary. Due to the great uncertainty involved in such procedure, it is generally recognized that extrapolations should be condemned for the determination of the probable frequency of hydrologic events for which certain major structures should be designed. These structures include: large dams and their spillways, reservoirs, high sea walls, levees or flood walls protecting cities, etc., the failure of which would result in a calamity involving loss of human lives and other catastrophic situations. However, in modern practice, the frequency analysis of hydrologic data has been of important use in determining the economic justification of many water-control projects. In such cases, the process of extrapolation within a certain practical limit, say 3 or 4 times the period of record depending on circumstances, may be employed to produce reasonably satisfactory results. This application should be considered justifiable where there is no other method of analysis available.

There are several methods of extrapolation. The simplest one is to extend the frequency curve to a desired recurrence interval by eye. A better method is to extend the straight line of the theoretical curve so that the control curves for the extrapolated portion may be constructed by the procedure given in Section 19.

Fragmentary data which were observed prior to or subsequent to a period of continuous records may be used in connection with the continuous records to obtain more accurate extrapolated frequency data. Such fragmentary data are generally of prominent magnitude only and do not cover a complete series of records compatible with the continuous record. The method proposed is as follows:

The plotting positions are determined by combining the data available in the fragmentary information with the data contained in the continuous record. The recurrence interval is determined from the length in years of the fragmentary period plus the length of the period in years of the continuous record. An extrapolated frequency curve can then be plotted from which values outside of the range of the continuous period may be reasonably estimated. The extrapolated frequency curve should be smoothed out and should match the general trend indicated by the curve plotted from the continuous record.

Sometimes, rainfall and runoff data may be correlated for extrapolation purposes. When one kind of the two has a longer period of record, then the data of the extra period may be converted to another kind by means of unit-graph method, and are used to plot the extrapolated curve of the latter kind.

29. Conversion between Annual Maxima and Annual Exceedances

It may happen that the data available for frequency analysis are only one type of the two; that is, either annual maxima or annual exceedances.

However, it is possible to estimate the other unknown kind of the data by the use of the theoretical relationship between the two kinds of data as developed previously in Section 11.

Take the data of 10-min duration rainfall depth at Chicago, Illinois, as an example. The conversion is to be made in both ways. The computation is given in Table 4. In the table, Column 1 lists the rank m. Columns 2 and 3 list the observed depth y_M and plotting postion T_M for

Table 4
Conversion between Annual Maxima and Annual Exceedances

	Annual Maxima				Annual Exceedances			
Rank	*Observed*		*Converted*	*Theoretical*	*Observed*		*Converted*	*Theoretical*
m	y_M	T_M	T_E'	y_M'	y_E	T_E	T_M'	y_E'
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1	1.11	36.00	35.5	1.111	1.11	35.00	35.50	1.088
2	0.96	18.00	17.5	1.003	.96	17.50	18.00	.985
3	0.94	12.00	11.5	.939	.94	11.67	12.15	.924
4	0.92	9.00	8.5	.892	.92	8.75	9.30	.882
5	0.88	7.20	6.7	.857	.88	7.00	7.52	.859
6	0.80	6.00	5.5	.826	.80	5.833	6.36	.822
7	0.80	5.143	4.6	.800	.80	5.000	5.55	.799
8	0.76	4.500	4.0	.777	.76	4.375	4.90	.779
9	0.74	4.000	3.45	.756	.74	3.889	4.44	.762
10	0.71	3.600	3.05	.737	.74	3.500	4.03	.746
11	0.70	3.272	2.74	.720	.71	3.182	3.74	.732
12	0.68	3.000	2.46	.704	.70	2.917	3.43	.719
13	0.68	2.769	2.21	.689	.68	2.692	3.23	.707
14	0.66	2.571	2.02	.674	.68	2.500	3.04	.696
15	0.66	2.400	1.87	.660	.68	2.333	2.86	.686
16	0.66	2.250	1.70	.647	.67	2.188	2.72	.676
17	0.65	2.118	1.57	.634	.66	2.059	2.60	.667
18	0.64	2.000	1.46	.622	.66	1.944	2.48	.659
19	0.64	1.895	1.34	.610	.66	1.842	2.37	.651
20	0.63	1.800	1.25	.598	.65	1.750	2.30	.643
21	0.62	1.715	1.16	.586	.64	1.667	2.20	.636
22	0.61	1.636	1.06	.575	.64	1.591	2.12	.629
23	0.60	1.565	.97	.563	.63	1.522	2.05	.623
24	0.58	1.500	.90	.551	.62	1.458	2.00	.616
25	0.57	1.440	.83	.540	.62	1.400	1.95	.610
26	0.57	1.385	.77	.528	.61	1.346	1.89	.604
27	0.53	1.333	.70	.516	.60	1.296	1.84	.598
28	0.52	1.285	.66	.503	.60	1.250	1.80	.593
29	0.49	1.242	.59	.491	.59	1.207	1.76	.588
30	0.49	1.200	.54	.477	.59	1.167	1.72	.583
31	0.47	1.162	.48	.466	.58	1.129	1.70	.578
32	0.41	1.125	.42	.446	.58	1.094	1.67	.573
33	0.36	1.092	.38	.427	.57	1.061	1.64	.569
34	0.34	1.058	.30	.402	.57	1.029	1.61	.564
35	0.33	1.029	.24	.371	.57	1.000	1.58	.560

annual maxima. Columns 6 and 7 list the corresponding values, y_E and T_E, for annual exceedances. Corresponding to values of T_M in Column 3, the converted values of T_E', as shown in Column 4, are found from Fig. 3 which gives the theoretical relationship between the two kinds of recurrence intervals. Similarly, the converted values T_M' for annual exceedances are found as shown in Column 8. Column 5 gives the theoretical values of annual maxima; the values are computed from Eq. 52 which is obtained by the method of least squares. For instance, with T_M in

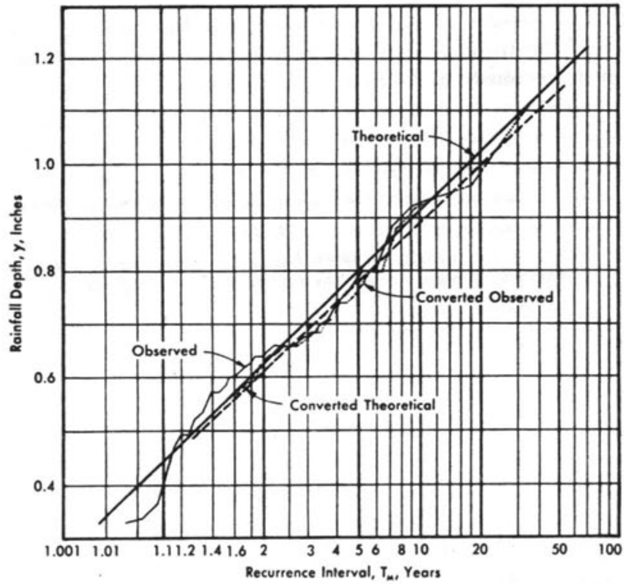

Fig. 12. Conversion of Annual Exceedances to Annual Maxima

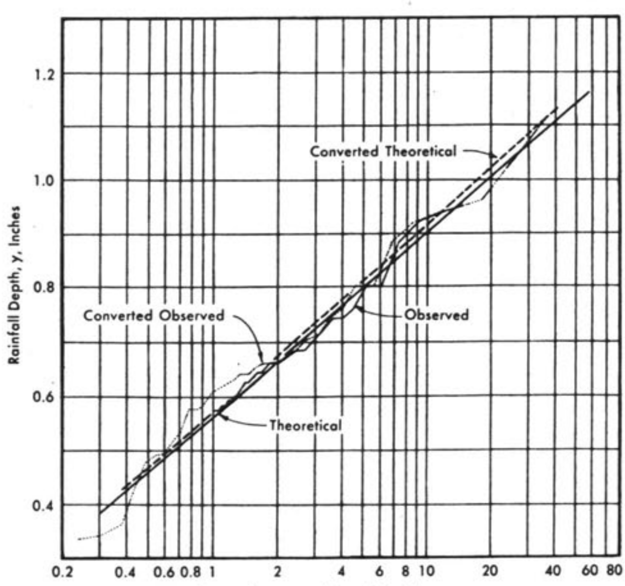

Fig. 13. Conversion of Annual Maxima to Annual Exceedances

Column 3 equal to 18.00 yrs, the frequency factor K or x is 1.783 (see Table 2) and the theoretical $y_M = 0.196 \times 1.783 + 0.654 = 1.003$. Similarly, the theoretical values of y_E' are given in Column 9 for which Eq. 54 is used in computing the values.

Figure 12 gives a comparison between the actual observed and theoretical plottings of annual maxima and the converted plottings. The converted observed curve is obtained by plotting values of y_E in Column 6 of Table 4 against the converted T_M' in Column 8. The converted theoretical curve is obtained by plotting values of y_E' in Column 9 against T_M' in Column 8. Similarly, Fig. 13 gives a comparison between the actual plottings and the converted plottings for annual exceedances, in which the converted observed curve is obtained by plotting y_M in Column 3 against T_E' in Column 4 and the converted theoretical curve is obtained by plotting y_M' against T_E'.

A close examination of Figs. 12 and 13 reveals the practicability of the technique of conversion from one type of data to another, as the discrepancy between the actual plotting and the converted plotting is very small (max. 2 to 3 percent) and, hence, it is tolerable for practical purposes. There are several other interesting points as observed from this example. The plottings indicate that the observed annual exceedances give a narrower band of spread than those of annual maxima for the corresponding recurrence intervals. Furthermore, the annual maxima, either the actual or the converted, demonstrate a higher magnitude than do the annual exceedances.

30. Rainfall Intensity-Duration-Frequency Curve

For the engineering use of rainfall data, it is required to know the relationship between four fundamental rainfall characteristics: namely, intensity, duration, frequency and area of distribution. The knowledge about the area of distribution may be obtained by a regional study of the data. However, such study is beyond the scope of the present paper. Therefore, only the relationship between intensity, duration and frequency is considered herewith.

In the previous example, maximum rainfall depths of 10-min duration are analyzed. For other durations, computations for both annual maxima and annual exceedances are given in Tables 5 and 6. Observed and theoretical curves are plotted in Figs. 14 and 15, in which the control curves are not shown.

The intensity of rainfall for a given duration is equal to the depth divided by the duration. The value of intensity is expressed in in. per hr. Depths of rainfall for given durations and recurrence intervals are obtained from the theoretical probability curves of Figs. 14 and 15. The

Table 5
Computation for Annual Maximum Values
Rainfall Depths at Chicago, Illinois

			y for Duration (min)													
m	T_M	$x = K$	5	10	15	20	25	30	35	40	45	50	60	80	100	120
1	36.000	2.332	0.61	1.11	1.29	1.45	1.64	1.81	1.90	1.98	2.17	2.38	2.81	2.96	3.36	3.36
2	18.000	1.783	0.58	0.96	1.22	1.39	1.58	1.70	1.86	1.97	2.04	2.08	2.30	2.35	2.40	2.43
3	12.000	1.455	0.55	0.94	1.16	1.39	1.49	1.61	1.77	1.92	2.04	2.06	2.15	2.30	2.32	2.37
4	9.000	1.218	0.53	0.92	1.16	1.38	1.45	1.57	1.76	1.85	2.03	2.03	2.15	2.30	2.30	2.30
5	7.200	1.033	0.51	0.88	1.15	1.35	1.45	1.55	1.71	1.81	1.85	1.95	2.03	2.28	2.30	2.30
6	6.000	0.878	0.50	0.80	1.12	1.32	1.39	1.49	1.61	1.77	1.80	1.85	2.02	2.20	2.26	2.30
7	5.143	0.745	0.50	0.80	1.08	1.16	1.34	1.48	1.59	1.75	1.76	1.85	1.92	2.03	2.03	2.03
8	4.500	0.627	0.50	0.76	1.00	1.16	1.29	1.44	1.53	1.61	1.75	1.75	1.85	1.85	1.85	1.85
9	4.000	0.522	0.48	0.74	0.93	1.13	1.25	1.43	1.53	1.60	1.66	1.74	1.75	1.75	1.75	1.75
10	3.600	0.425	0.45	0.71	0.87	1.10	1.21	1.39	1.49	1.55	1.61	1.61	1.61	1.61	1.61	1.74
11	3.272	0.337	0.42	0.70	0.87	1.05	1.21	1.38	1.43	1.53	1.53	1.53	1.53	1.55	1.55	1.61
12	3.000	0.255	0.41	0.68	0.86	1.05	1.18	1.31	1.43	1.49	1.49	1.49	1.53	1.53	1.53	1.57
13	2.769	0.177	0.41	0.68	0.85	1.00	1.16	1.29	1.39	1.39	1.39	1.40	1.49	1.49	1.49	1.55
14	2.571	0.102	0.41	0.66	0.85	0.97	1.12	1.26	1.38	1.38	1.38	1.38	1.40	1.40	1.40	1.53
15	2.400	0.032	0.41	0.66	0.84	0.97	1.10	1.20	1.26	1.31	1.34	1.35	1.38	1.38	1.40	1.49
16	2.250	−0.035	0.40	0.66	0.83	0.96	1.10	1.19	1.21	1.23	1.26	1.30	1.36	1.37	1.38	1.46
17	2.118	−0.100	0.39	0.65	0.82	0.95	1.07	1.16	1.16	1.22	1.23	1.23	1.35	1.35	1.36	1.40
18	2.000	−0.164	0.39	0.64	0.80	0.94	1.05	1.12	1.16	1.16	1.22	1.23	1.28	1.34	1.36	1.38
19	1.895	−0.225	0.38	0.64	0.78	0.88	1.05	1.11	1.11	1.16	1.17	1.22	1.23	1.25	1.35	1.38
20	1.800	−0.286	0.38	0.63	0.77	0.86	1.03	1.05	1.11	1.14	1.16	1.20	1.22	1.23	1.23	1.35
21	1.715	−0.346	0.38	0.62	0.76	0.84	0.90	1.05	1.05	1.11	1.15	1.19	1.20	1.23	1.23	1.23
22	1.636	−0.405	0.37	0.61	0.76	0.84	0.89	0.93	1.05	1.10	1.14	1.16	1.16	1.22	1.23	1.23
23	1.565	−0.464	0.37	0.60	0.72	0.83	0.87	0.91	1.04	1.08	1.11	1.11	1.11	1.16	1.22	1.22
24	1.500	−0.523	0.36	0.58	0.71	0.82	0.85	0.91	0.94	1.06	1.08	1.08	1.08	1.11	1.16	1.21
25	1.440	−0.582	0.35	0.57	0.69	0.78	0.83	0.88	0.93	1.05	1.06	1.06	1.06	1.08	1.11	1.16
26	1.385	−0.643	0.35	0.57	0.69	0.78	0.83	0.88	0.92	0.92	0.98	0.98	0.98	1.06	1.08	1.11
27	1.333	−0.704	0.34	0.53	0.67	0.77	0.82	0.84	0.89	0.92	0.97	0.97	0.98	1.05	1.06	1.08
28	1.285	−0.768	0.31	0.52	0.63	0.74	0.78	0.84	0.87	0.92	0.92	0.93	0.94	1.00	1.00	1.06
29	1.242	−0.834	0.31	0.49	0.63	0.67	0.72	0.82	0.84	0.88	0.88	0.88	0.94	0.98	0.98	1.02
30	1.200	−0.904	0.30	0.49	0.57	0.65	0.68	0.69	0.75	0.81	0.86	0.88	0.91	0.96	0.96	1.01
31	1.162	−0.980	0.29	0.47	0.57	0.57	0.67	0.67	0.74	0.79	0.79	0.79	0.82	0.94	0.94	0.98
32	1.125	−1.064	0.26	0.41	0.53	0.56	0.57	0.57	0.58	0.63	0.67	0.72	0.79	0.93	0.93	0.96
33	1.092	−1.159	0.24	0.36	0.46	0.53	0.54	0.56	0.58	0.59	0.60	0.63	0.68	0.75	0.90	0.93
34	1.058	−1.277	0.24	0.34	0.42	0.50	0.53	0.55	0.57	0.57	0.57	0.57	0.57	0.57	0.57	0.57
35	1.029	−1.445	0.20	0.33	0.40	0.41	0.43	0.53	0.53	0.53	0.53	0.53	0.53	0.53	0.53	0.53
		$\bar{x} = -0.0282$	$\bar{y} = 0.3966$	0.6489	0.8131	0.9357	1.0306	1.1191	1.1906	1.2509	1.2911	1.3174	1.3746	1.4311	1.4609	1.4986
		$\overline{x^2} = 0.7750$	$\overline{y^2} = 0.1667$	0.4516	0.7117	0.9503	1.1591	1.3668	1.5687	1.7377	1.8647	1.9520	2.1618	2.3505	2.4734	2.5820
			$\overline{xy} = 0.0698$	0.1334	0.1712	0.2089	0.2391	0.2692	0.3001	0.3217	0.3482	0.3679	0.4189	0.4374	0.4539	0.4584
			$A = 0.105$	0.196	0.251	0.304	0.346	0.389	0.431	0.461	0.497	0.523	0.591	0.617	0.640	0.647
			$B = 0.400$	0.654	0.820	0.944	1.040	1.130	1.202	1.264	1.305	1.332	1.391	1.449	1.479	1.517
			$\sigma_y = 0.0985$	0.178	0.228	0.278	0.314	0.343	0.394	0.422	0.454	0.473	0.530	0.558	0.591	0.588

Table 6
Computation for Annual Exceedance Values
Rainfall Depths at Chicago, Illinois

			Duration (min)													
m	T_E	$x = \log T_E$	5	10	15	20	25	30	35	40	45	50	60	80	100	120
1	35.0	1.5441	0.61	1.11	1.29	1.45	1.64	1.81	1.90	1.98	2.17	2.38	2.81	2.96	3.36	3.36
2	17.50	1.2430	0.58	0.96	1.22	1.39	1.58	1.70	1.86	1.97	2.04	2.08	2.30	2.35	2.40	2.43
3	11.67	1.0671	0.55	0.94	1.16	1.39	1.49	1.61	1.77	1.92	2.04	2.06	2.15	2.30	2.32	2.37
4	8.75	0.9420	0.53	0.92	1.16	1.38	1.45	1.57	1.76	1.85	2.03	2.03	2.15	2.30	2.30	2.30
5	7.00	0.8451	0.51	0.88	1.15	1.35	1.45	1.55	1.71	1.81	1.85	1.95	2.03	2.28	2.30	2.30
6	5.833	0.7659	0.50	0.80	1.12	1.32	1.39	1.49	1.61	1.77	1.80	1.85	2.02	2.20	2.26	2.30
7	5.000	0.6990	0.50	0.80	1.08	1.21	1.34	1.48	1.59	1.75	1.76	1.85	1.92	2.03	2.03	2.03
8	4.375	0.6410	0.50	0.76	1.00	1.16	1.29	1.44	1.53	1.61	1.75	1.75	1.85	1.85	1.85	1.91
9	3.889	0.5899	0.48	0.74	0.96	1.16	1.27	1.43	1.53	1.60	1.66	1.61	1.75	1.75	1.75	1.85
10	3.500	0.5441	0.45	0.74	0.93	1.13	1.25	1.39	1.49	1.55	1.61	1.61	1.61	1.61	1.75	1.75
11	3.182	0.5027	0.42	0.71	0.87	1.10	1.21	1.38	1.43	1.53	1.51	1.53	1.53	1.55	1.61	1.74
12	2.917	0.4649	0.42	0.70	0.87	1.05	1.21	1.31	1.43	1.49	1.49	1.49	1.53	1.54	1.55	1.61
13	2.692	0.4301	0.41	0.68	0.86	1.05	1.18	1.29	1.39	1.39	1.39	1.40	1.49	1.53	1.54	1.57
14	2.500	0.3979	0.41	0.68	0.86	1.00	1.16	1.27	1.38	1.38	1.38	1.39	1.48	1.49	1.53	1.55
15	2.333	0.3679	0.41	0.68	0.85	0.97	1.16	1.26	1.30	1.31	1.34	1.38	1.40	1.45	1.49	1.54
16	2.188	0.3400	0.41	0.67	0.84	0.97	1.12	1.20	1.27	1.30	1.31	1.36	1.39	1.40	1.45	1.53
17	2.059	0.3137	0.40	0.66	0.83	0.96	1.10	1.20	1.26	1.27	1.30	1.30	1.38	1.39	1.40	1.53
18	1.944	0.2887	0.39	0.66	0.82	0.96	1.10	1.19	1.21	1.24	1.27	1.30	1.36	1.38	1.40	1.49
19	1.842	0.2653	0.39	0.66	0.82	0.95	1.07	1.16	1.17	1.23	1.26	1.27	1.35	1.37	1.39	1.46
20	1.750	0.2430	0.39	0.65	0.80	0.95	1.05	1.16	1.16	1.22	1.25	1.26	1.30	1.35	1.39	1.45
21	1.667	0.2219	0.38	0.64	0.80	0.94	1.05	1.12	1.16	1.16	1.23	1.25	1.28	1.34	1.38	1.40
22	1.591	0.2017	0.38	0.64	0.79	0.94	1.04	1.11	1.16	1.16	1.22	1.23	1.27	1.34	1.38	1.39
23	1.522	0.1824	0.38	0.63	0.78	0.91	1.03	1.06	1.11	1.16	1.19	1.23	1.27	1.30	1.36	1.38
24	1.458	0.1638	0.37	0.62	0.77	0.91	1.02	1.05	1.11	1.15	1.17	1.22	1.25	1.27	1.36	1.38
25	1.400	0.1461	0.37	0.62	0.76	0.88	0.98	1.05	1.10	1.14	1.16	1.20	1.25	1.27	1.36	1.36
26	1.346	0.1290	0.37	0.61	0.76	0.87	0.98	1.04	1.09	1.13	1.16	1.19	1.23	1.25	1.35	1.35
27	1.296	0.1126	0.37	0.60	0.76	0.86	0.96	1.04	1.05	1.11	1.15	1.19	1.22	1.25	1.30	1.30
28	1.250	0.0969	0.37	0.60	0.76	0.86	0.95	1.03	1.05	1.10	1.14	1.17	1.22	1.23	1.27	1.27
29	1.207	0.0817	0.37	0.59	0.73	0.84	0.93	1.02	1.04	1.09	1.14	1.16	1.21	1.23	1.27	1.27
30	1.167	0.0671	0.36	0.59	0.73	0.84	0.91	0.96	1.04	1.08	1.12	1.16	1.20	1.22	1.25	1.25
31	1.129	0.0527	0.36	0.58	0.72	0.83	0.90	0.96	1.02	1.08	1.11	1.15	1.16	1.22	1.23	1.24
32	1.094	0.0390	0.36	0.58	0.72	0.83	0.89	0.96	1.02	1.06	1.09	1.12	1.16	1.21	1.23	1.23
33	1.061	0.0257	0.35	0.57	0.72	0.82	0.88	0.93	1.00	1.05	1.08	1.11	1.13	1.20	1.22	1.22
34	1.029	0.0124	0.35	0.57	0.72	0.82	0.88	0.91	0.99	1.04	1.08	1.09	1.12	1.16	1.21	1.21
35	1.000	0.0000	0.35	0.57	0.71	0.79	0.87	0.91	0.96	1.03	1.06	1.08	1.12	1.16	1.20	1.21
		$\overline{x} = 0.4008$	$\overline{y} = 0.4214$	0.6974	0.8777	1.0240	1.1366	1.2297	1.3043	1.3631	1.4089	1.4437	1.5111	1.5637	1.6126	1.6437
		$\overline{x^2} = 0.2971$	$\overline{xy} = 0.1944$	0.3262	0.4107	0.4799	0.5315	0.5792	0.6211	0.6535	0.6820	0.7032	0.7519	0.7851	0.8147	0.8288
			$a = 0.1868$	0.3421	0.4315	0.5092	0.5568	0.6322	0.7201	0.7853	0.8593	0.9128	1.0718	1.1604	1.2337	1.2454
			$b = 0.3465$	0.5603	0.7048	0.8199	0.9134	0.9763	1.0157	1.0484	1.0645	1.0778	1.0815	1.0986	1.1181	1.1445

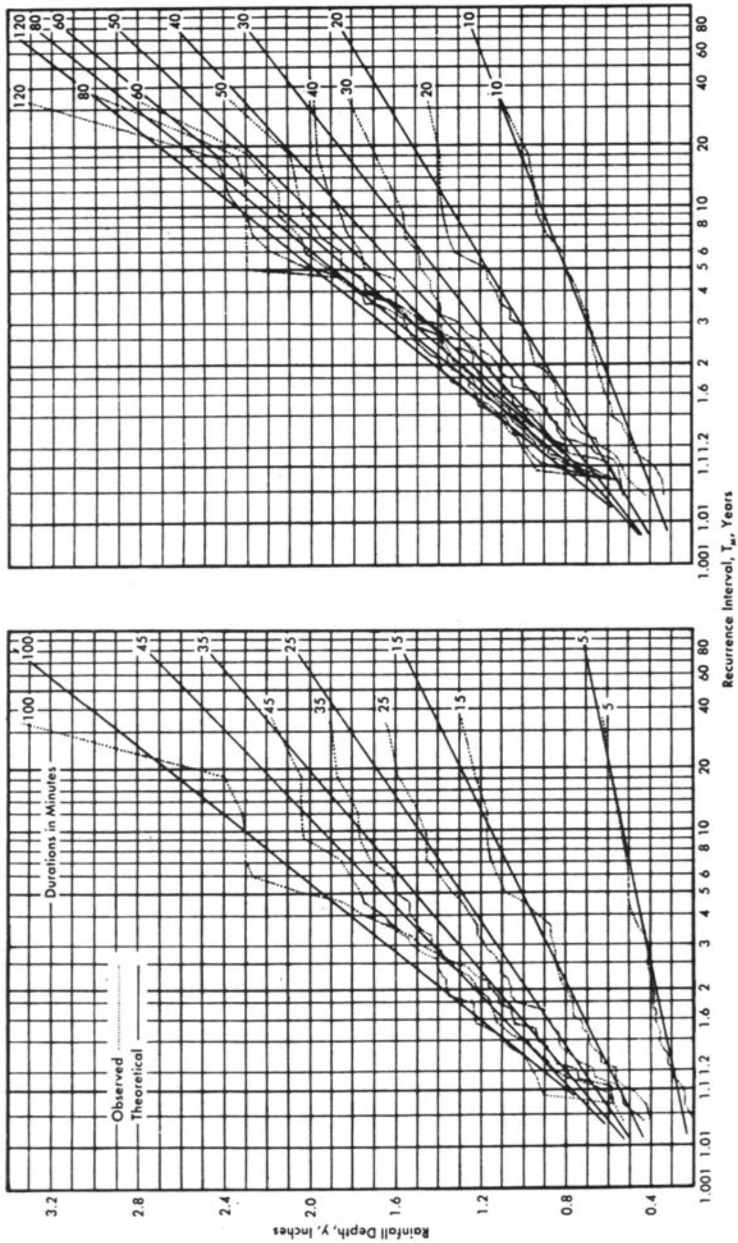

Fig. 14. Frequency Curves for Rainfall Intensities at Chicago, Illinois, by the Method of Annual Maxima

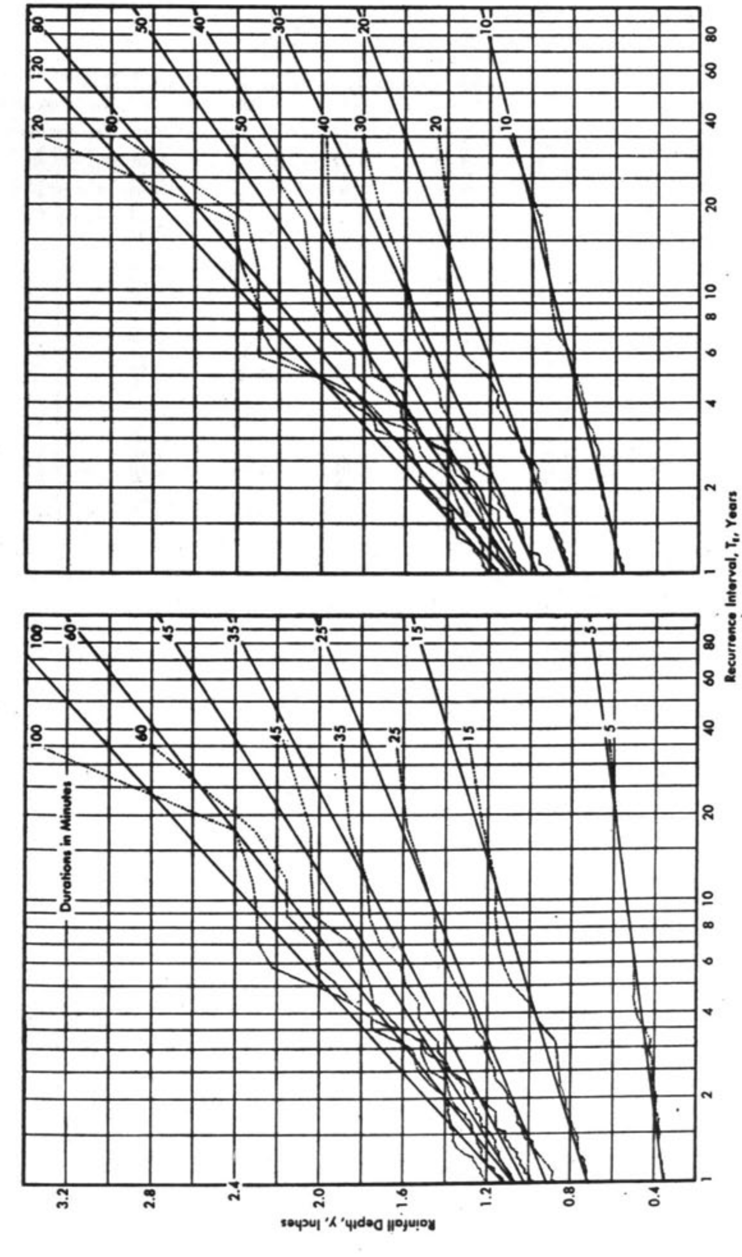

Fig. 15. Frequency Curves for Rainfall Intensities at Chicago, Illinois, by the Method of Annual Exceedances

Table 7
Rainfall Intensities at Chicago, Illinois

A: Using annual maximum values
B: Using annual exceedance values

Duration (min)		Rainfall Intensities (in. per hr) for recurrence interval (yr) 2	5	10	25	50	100
5	A	4.56	5.64	6.36	7.32	7.92	8.64
	B	4.92	5.76	6.48	7.32	8.04	8.65
10	A	3.78	4.80	5.52	6.36	7.03	7.63
	B	4.02	4.80	5.40	6.24	6.84	7.44
15	A	3.12	4.04	4.60	5.36	5.92	6.44
	B	3.36	4.04	4.52	5.24	5.72	6.24
20	A	2.67	3.48	4.02	4.74	5.22	5.73
	B	2.91	3.51	3.99	4.59	5.07	5.52
25	A	2.35	3.10	3.58	4.20	4.65	5.08
	B	2.59	3.12	3.53	4.06	4.46	4.88
30	A	2.12	2.82	3.28	3.88	4.30	4.72
	B	2.34	2.84	3.22	3.70	4.10	4.48
35	A	1.92	2.59	3.02	3.57	3.98	4.43
	B	2.11	2.59	2.96	3.46	3.84	4.20
40	A	1.77	2.37	2.78	3.30	3.68	4.05
	B	1.94	2.39	2.75	3.21	3.57	3.93
45	A	1.63	2.21	2.59	3.08	3.44	3.79
	B	1.76	2.21	2.56	3.01	3.36	3.71
50	A	1.49	2.05	2.42	2.89	3.24	3.60
	B	1.62	2.05	2.39	2.82	3.15	3.48
60	A	1.29	1.81	2.15	2.60	2.92	3.26
	B	1.40	1.83	2.15	2.58	2.90	3.22
80	A	1.01	1.42	1.69	2.02	2.29	2.54
	B	1.09	1.43	1.69	2.04	2.30	2.57
100	A	0.83	1.16	1.38	1.63	1.87	2.08
	B	0.89	1.19	1.41	1.71	1.92	2.14
120	A	0.71	0.99	1.17	1.42	1.59	1.76
	B	0.76	1.00	1.20	1.44	1.63	1.82

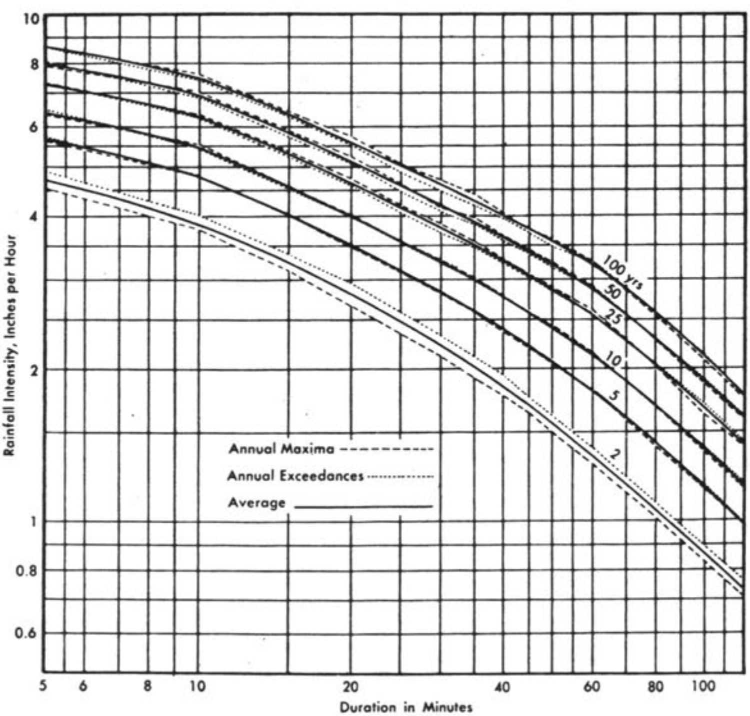

Fig. 16. Intensity-Duration-Frequency Curves of Rainfall Intensities at Chicago, Illinois

computation for rainfall intensities of Chicago, Illinois, is given in Table 7. In Fig. 16, the corresponding rainfall intensity-duration-frequency curves are constructed, in which the intensity is plotted against the duration for various recurrence intervals. Curves for both approaches of data analysis, annual maxima and annual exceedances, are shown.

A similar analysis for flood data is important in analyzing the future operation of control works. The most useful analysis is to establish the relationship between the three elements: discharge, duration, and frequency.

IV. PREPARATION OF HYDROLOGIC DATA FOR ANALYSIS

31. General

Hydrologic data required for frequency analysis may be obtained from various agencies and state, municipal, and private organizations. Published data are accessible at many public and university libraries and at Federal, state and municipal offices. The prominent Federal agencies for collecting and publishing hydrologic data in the United States are: U. S. Weather Bureau; U. S. Geological Survey; the Corps of Engineers; Tennessee Valley Authority; Public Health Service; Mississippi River Commission; and Soil Conservation Service. For detailed information reference should be made to National Resources Planning Board, Technical Paper 10, "Principal Federal Sources of Hydrologic Data," by the Special Advisory Committee on Hydrologic Data of the Water Resources Committee (1943) and "Inventory of Unpublished Hydrologic Data," U. S. Geological Survey Water — Supply Paper 837. In many regions, the Federal agencies are in cooperation with the state agencies.

It should be noted that in many cases the original hydrologic data can not be used directly for analysis because they have deficiencies of various kinds: for example, incompleteness of data which makes analysis impossible; errors and inconsistencies in recording, compiling and publishing data which would either require corrections and adjustments or produce erroneous results. In any case, it must be warned that hydrologic data should not be used without examining the record for deficiencies. In this part of the report, discussions are focused only upon rainfall intensities with reference to the data at Chicago, Illinois. The deficiencies in data are to be discussed and recommendations as to how such deficiencies may be compensated for in a frequency study will be given. However, the general principle described here should be applied as well to any other kind of hydrologic data.

32. Precipitation Data

The words "precipitation" and "rainfall" are often interchangeable, although the former includes forms of precipitated water other than rainfall, such as snow, sleet, hail, etc.

The amount of precipitation at strategic locations is recorded by gages maintained by public and private agencies; the observed records or precipitation data are published from time to time by different agencies.

Precipitation data may be classified in various ways. The "official Weather Bureau data" are those records maintained by the Bureau which were obtained by means of Bureau-approved observational techniques, with approved instruments that have approved exposures. Data are also recorded, but may or may not be published, by different agencies. The "excessive precipitation data" are those which satisfy the definition of excessive precipitation (see Section 34), and the "actual or original data" is the term which refers to all data observed and published without restrictions of any definition. "Point data" refers to the record of precipitation at a particular point where the gage has been installed for measuring and recording.

The "areal data" is a collection of point data which supplies information of precipitation distribution within the area under consideration; in the area a significant number of gages are strategically installed such that the information may be considered representative. The treatment given here is for point data only; the areal data will not be included. The "total-storm precipitation data" is the accumulated depth of precipitation during a specified time-interval of a certain rain-making storm or storms. The "maximum precipitation data" is the greatest recorded data for a certain time-interval in the period of observation without regard to any particular storm. In Weather Bureau's publications the precipitation data used for intensity-frequency analysis is furnished as the maximum excessive point data. Prior to 1935, only values of accumulated depth were given.

Observation of precipitation for the collection of data is subject to both accidental and cumulative errors. Accidental errors are often small when compared with the daily variations in precipitation and are compensating so that they can be safely ignored. Cumulative error is serious. It may be due either to faulty observation causing error of appreciable size or due to inaccurate measuring devices, improper exposure of gage, etc. Such deficiencies result in a persistent error.

In addition to the observational errors which are more or less inherent in all data, the method of preparing and presenting the data and the change of gage site may impair the uniformity of data. All kinds of errors and non-uniformity are referred to, hereafter, as deficiencies.

When the data are not sufficient in quantity for a satisfactory analysis, two remedial methods are recommended. The additional data for longer durations of storms may be supplied by the method of the extended duration principle. The data for a short period of observation may be augmented by the station-year method (see Section 38). Satisfactory results can be obtained by these methods if properly applied.

33. Source of Precipitation Data

Early fragmental weather records were kept by private individuals, U. S. Army Surgeons, officers of the General Land Office, and many educational institutions; but the first comprehensive system using uniform methods to cover the country as a whole was organized under the direction of the Smithsonian Institution about 1848. Unfortunately, many records of this service were destroyed when the building was burned in 1865.

In 1870, Congress placed the work under the direction of the Chief Signal Officer of the U. S. Army. In 1891, the work was reorganized as the Weather Bureau and placed under the administration of the U. S. Department of Agriculture. In 1940, it was transferred to the Department of Commerce.

At the present time (1952) there are approximately 350 first-order Weather Bureau stations with paid observers and complete equipment of standard instruments. This network is supplemented by more than 150 second-order Weather Bureau stations with paid observers, about 250 Civil Aeronautics Administration stations, more than 250 Supplementary Airway Weather Recording stations operated by airlines personnel, and more than 9,000 climatological substations manned by voluntary observers using instruments supplied by the Weather Bureau.

For excessive precipitation of short duration, data can be found in the publications of the Weather Bureau: Annual Climatological (or Meteorological) Summaries and Meteorological Yearbook.* The publication of the latter is usually somewhat later than the Summaries due to the time involved in compiling the data. Excessive precipitation data for years 1896 to 1934, inclusive, have been presented in the appropriate annual reports of the Chief of the Weather Bureau, and for the years 1935 to 1942 in appropriate issues of the United States Meteorological Yearbook. The published data prior to 1896 consist of a record of maximum amount of rainfall in 5- and 10-min periods, and in 1- and 24-hr periods. The annual report for 1895 to 1896 contains a summary of the records which up to the latter time had been observed at the principal stations supplied with automatic gages. From 1897 to 1935 the accumulated depth of precipitation for each 5 to 120 min is given. After 1935, the maximum amounts for durations up to 180 min are published. Publication of the Meteorological Yearbook as an annual was discontinued with the 1942 issue. A final issue consolidated for 1943-1949 has just been issued. Excessive short duration rainfall data beginning with 1950 appear in the annual issues of Climatological Data, National Summary.

Microfilm copies of rainfall recording charts together with other climatological information such as sunshine, temperature, etc., may be obtained from the office of U. S. Weather Bureau. As the interpretation of

* Before 1935, it was entitled "Report of the Chief."

these microfilms requires some experience and skill, it is wise to use the published data of the Bureau whenever available.

In general, precipitation data in important cities of the United States cover periods sufficiently long to produce records of 30 to 50 yr, which is considered as an acceptable length of time for statistical analysis.

34. Excessive Precipitation

The "excessive precipitations" are those equal to or greater than certain limits or specified limiting values of the fall of precipitation.

For years, the U. S. Weather Bureau adopted rules relating to the tabulation of excessive precipitation. The present method (1952), adopted with data for the calendar year 1936, gives the maximum fall of excessive precipitation for the periods 5 to 180 min; the maximum amounts are taken for the periods in which the fall is greatest for the given time, and are tabulated to show maximum amounts for 5, 10, 15, 20, 30, 45, 60, 80, 100, 120, 150, and 180 min, even if the fall does not equal the excessive rate for some of the periods. For all Weather Bureau stations the following table shows limits at and above which precipitation is considered as excessive.

Duration (*Min*)	Limits for Excessive Precipitation (*Accumulated depth in in.*)
5	0.25
10	0.30
15	0.35
20	0.40
25	0.45
30	0.50
35	0.55
40	0.60
45	0.65
50	0.70
60	0.80
80	1.00
100	1.20
120	1.40
150	1.70
180	2.00

This table is made up from the formula:

$$d = 0.01t + 0.20 \tag{55}$$

in which "d" is the accumulated depth in in. and "t" is the duration in min.

It is essential to review the history of the changes in regulations for the method of tabulating excessive precipitation data by the U. S. Weather Bureau. Published excessive precipitation data prior to 1896 consist of maximum amounts of rainfall in 5- and 10-min periods, also in 1 and 24 hr periods.

Excessive precipitation data for the years 1896 to 1935 inclusive generally present the accumulated amounts of precipitation for each 5-, 10-, or 20-min interval during storms in which the rate of fall equaled or exceeded 0.25 in. in any 5-min period, 0.30 in any 10-min period, or 0.35 in any 15-min period, etc., the tabulation beginning with the 5-min period where the rate of 0.05 in. in 5 min began and continuing by 10- or 20-min intervals up to 120 min. For convenience of reference, the original statement which appeared in "Instructions for Preparing Meteorological Forms," No. 1015 and 1017, Art. 200, 1935, Division of Climate and Crop Weather, U. S. Weather Bureau, is quoted as follows:

> 200. In tabulating excessive precipitation data . . . for storms in which the rate of fall equals or exceeds the limits . . . , the accumulated amounts for each excessive interval will be entered in the spaces provided for that purpose. . . . Under head 'Excessive Rate' will be entered the time precipitation began to fall at an excessive rate and the time the precipitation fell below that rate. This, however, must not be construed to mean that short periods during the progress of a storm where the rate fell temporarily below the excessive rate shall be excluded. Under the heading 'Accumulated depths of precipitation', etc., will be entered the accumulated amounts for the respective periods, all periods beginning at the same point of time, which should be identical with that given under 'Excessive rate began'. If the excessive duration is less than 120 minutes, the tabulation should be continued for 120 minutes notwithstanding that the depth after the ending of the excessive rate are below the limits in the table.* When the excessive duration extends beyond 120 minutes, the tabulation will be made for each 50 minutes separately so long as the excessive rate continues.

The sentence marked by an asterisk was added to the rule in March, 1934. The data published under this rule which is based upon the so-called extended duration principle are more complete and permit a more accurate analysis inasmuch as the precipitation for excessive storms is given for intervals as long as 120 min, although the excessive rate may not have continued for that length of time. Apparently, this rule was not followed in records previous to 1933, and any precipitation occurring within the 120-min period, but subsequent to the expiration of the excessive rate, is not shown.

In the year 1936, the method of tabulating excessive precipitation was changed primarily to meet the needs of many sewerage engineers. The general principle is not much different from the present method as described at the beginning of this Bulletin. However, in tabulating the excessive precipitations the 15-min amount was not computed for 1936-1943 and the 150-min amount was not computed for 1944 through 1948.

For the years 1936 through 1948 stations in Southern States, including North Carolina, South Carolina, Georgia, Florida, Alabama, Mississippi, Tennessee, Arkansas, Louisiana, Texas, Oklahoma, and San Juan, P.R., where heavy storms are comparatively frequent, the following table of the excessive precipitation limits was used:

Durations (*Min*)	Limits for Excessive Precipitation (*Accumulated depth in in.*)
5	0.40
10	0.50
15	0.60
20	0.70
25	0.80
30	0.90
35	1.00
40	1.10
45	1.20
50	1.30
60	1.50
80	1.90
100	2.30
120	2.70
150	3.30
180	3.90

This table is made up from a formula similar to Eq. 55:

$$d = 0.02t + 0.30 \tag{56}$$

Its use, however, was discontinued at the end of 1948 and Eq. 55 is used by all sections for 1949 and the following years.

Concerning the publication of data, a summary of maximum precipitation data for the years prior to 1896 is published in the annual report of the Chief of the Weather Bureau for 1895-1896. Data for the years 1896 through 1934 have been published in the appropriate annual reports of the Chief of the Weather Bureau. For the years 1935 through 1942 these data are published in the appropriate issue of the United States Meteorological Yearbook. Data for 1943 through 1949 will be presented in the final issue of the United States Meteorological Yearbook. For 1950 and each succeeding year excessive precipitation will be presented in annual issues of Climatological Data, National Summary.

As the climatic conditions vary from place to place, the Weather Bureau rule for defining excessive precipitation is too general to be applied to specific cases and would not even be applicable to some particular regions where the climatic conditions are so odd that data obtained by the rule would be extremely scanty or extremely abundant. It seems that the rule should be modified in these particular cases. In this respect, Rowe* has proposed that "the base for excessive precipitation

* R. Robinson Rowe, "Hydrologic Data for Highway Design," Trans. Amer. Geophys. Union, Vol. 28, No. 5, 1947, pp. 739-741.

should be changed so that the volume of data to be compiled and published will not be much greater than actually needed for computation of frequencies. The present base gives too many data for 15-30 minute bursts and too little for 5-minute bursts. It gives fewer data for North Coast stations than for the rest of the State of California. Hence, the suggestion implies several non-linear definitions of excessive precipitation appropriate to the several climatic regions of the State. Definitions for mountain and desert areas may be distinctly different." In developing new definitions, data over the entire country should be comprehensively reviewed and studied. It certainly involves a tremendous amount of labor to handle such a job. Until new definitions are made available, the old rules are in current usage.

35. Precipitation Data at Chicago, Illinois

Appendices II and III respectively give the excessive precipitation data and the maximum precipitation data at Chicago, Illinois. The maximum precipitation data from 1913 to 1935 in Appendix III were derived from the data of the same period in Appendix II. These original data are the official precipitation data at the City of Chicago as collected by the Chicago Weather Bureau office and published in Chicago's "Annual Meteorological Summary" from 1913 to 1945, and in "Annual Climatological Summary" from 1946 to 1947. The data cover a whole period of 35 years, from 1913 to 1947. As the design of highway culverts is usually predicated upon short duration, high intensity precipitation with a maximum time of about 2 hr, the tables cover only data with a duration up to 120 min.

36. Inaccuracy of Data

The accuracy of hydrologic data is often questionable due to errors of various kinds. The most important kinds of error encountered in rainfall data are as follows:

(1) Failure to Register Maximum Precipitations. Most of the Weather Bureau's early data were derived from records obtained from a tipping-bucket type of rain gage. The tabulations of the time for amount of rainfall were made at 5-min intervals. True maximum intensities can be obtained from such tabulations only when changes in intensities occur at the beginning or end of a 5-min interval. When the maximum intensities begin or end near the middle of a 5-min interval, the tabulated maximum intensity will be less than the actual intensity. Further discussion of this subject will be given in Section 38.

(2) Error due to Personal Factor. Failure to read, record, transcribe, or tabulate correctly is the error due to personal factor. It is inevitable, particularly as the greatest number of observers are carrying on the work voluntarily; it is not probable that the close attention is given to accuracy which is to be expected from paid observers. The experience of the Weather Bureau and those who are interested in weather data reveals the fact that occasional inconsistencies in records are found, indicating errors of this kind. It is difficult sometimes to decide whether or not to alter or discard certain data. It is advisable to check the data carefully and correct or reject them on justifiable grounds. However, as excessive precipitation of the type used in the present study is always worked up by paid, usually commissioned observers, the least amount of errors in these data should be expected.

(3) Failure to Catch the Precipitation Correctly due to Instrumental Defects. The instrumental defects may be caused by lack of calibration, faulty maintenance, or failure in mechanism. For the tipping-bucket type gage which was for the most of the period of record the standard recording gage of the Weather Bureau, the inaccuracies arose from the fact that the funnel could not discharge very heavy falls fast enough. In addition, the inaccuracy may also be due to corrosion, dirt and faulty leveling. The unavoidable friction which exists at the pivot of the bucket prevents the instrument from making a correct record of high intensities at short durations. Correction may be made by distributing in the data an excess as shown by stick measurement over the automatic record. In fact, all the Bureau's tipping-bucket charts are corrected to agree with 6-hr stick measurements. For the weighing type of gage which has in recent years come into popular use with the Weather Bureau, Soil Conservation Service and other agencies, the error due to mechanical friction is almost eliminated; however, it has the disadvantage of difficulties with reversing mechanism for the travel of pen, effects of temperature on the spring balance, and shrinkage and expansion of the chart paper due to changes in humidity. It is generally assumed that this type of error has been corrected by observers in preparing data for publication.

(4) Failure to Catch the Precipitation Extensively due to Limitation in Number of Gages. A single rain-gage gives a record of the precipitation at one point only, with no indication of the variation in any direction. It merely takes a sample, within the scope of the collector ring, out of a storm that may have an area of 20 sq miles. Not uncommonly the record fails to show the critical data for a heavy storm. For example, all available excessive precipitation data of the Weather Bureau are point

data which refer only to a single point, the point of measurement where the rain gage was installed. Extensive frontal type storms in the plain regions may, even within the storm center, produce uniform average depth over several sq miles of area. But thunderstorms are known to deposit heavy amounts of rain over areas so small as to be measured in city blocks. Therefore, the distribution of rainfall is not always uniform, and the point-rainfall is not always a true representative data for an area. For a region which is known to be meteorologically non-homogeneous, a regional frequency study should be made. In such study, data collected from a network of gages strategically installed within and close to the region must be available for analysis. The results of frequency study of point data at all gage stations should be tied together for an over-all regional consideration. However, there is an argument about the significance of frequency analysis using point data for a small area or an area which is more or less known as meteorologically homogeneous. It is believed that hydrologic events entering such a region can center over any point with equal probability. Therefore, based on the data of a sufficiently long record, the point data analysis should give the same result for other places of the area.

37. Non-Uniformity in Quality of Data

The quality of data is usually impaired due to the following causes:

(1) Change of Precision in Taking Observations. As the technique of measuring hydrologic data is being improved from time to time, the data collected in early times are no doubt less precise and reliable than those obtained during recent years. For this reason, fragmental data in the early years should be carefully examined and discarded if necessary.

(2) Change of Gage Site. In recent years many weather stations in cities and towns have either been moved to nearby airports or supplemented by the airport stations in order to secure better exposure. Thus the complete record consists of a series of data collected at a number of sites; the quality of data should not be considered as uniform throughout the whole period of observation, even though the distance to which the station was moved from one site to another is not great. This does not mean that the non-uniform data of this kind as collected from different gage sites should not be combined and processed in statistical analysis. In homogeneous areas and with no great difference in exposure of gage, considering the data from two sites is no worse than using the station-year method. (See Section 38.)

In the case of Chicago's precipitation data, the official weather station in the City of Chicago has been moved two times. From January 1912 to January 1926, the station was at the U. S. Court House, Federal Building, 219 Clark Street; then the station was moved to the University of

Chicago, 58th Street and University Avenue. In January 1942, the station was moved again to the Municipal Airport, 6200 South Cicero Avenue. Fig. 17 shows the general location of these stations.

(3) Change in Method of Tabulation. In Chicago's Annual Summaries, the data published before 1935 are accumulated depth of precipitation; after 1935, the data are published as the maximum rate of precipitation, and the recorded duration was extended to 180 min even if the fall does not equal the excessive rate for some of the periods. Therefore, in order to make the data consistent and uniform in quality, the

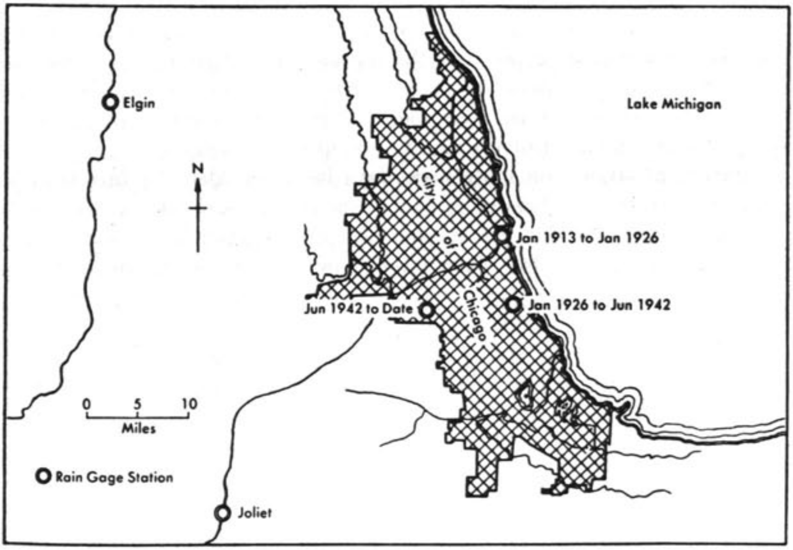

Fig. 17. Location of Weather Bureau's Gage Stations at Chicago and its Vicinity

accumulated depths before 1935 should be converted to maximum values, and the duration should be extended to 180 min to complete the data at higher durations. If the accumulation took more than 180 min and the data were available, then the duration should be extended enough longer to make the data complete for the purpose of study. The conversion procedure may be either graphical or analytical. However, the analytical procedure, as shown later in Section 38, was found to be more practical for present purposes. The extension of data to 180-min duration can be done either by referring back to the original record or by the method of extended duration principle to be described in Section 38.

(4) Trend of Precipitation. Climatologists have found that there is a tendency for the average yearly precipitation to increase or decrease

over a given period. Such tendency is known as a trend. However, the trend generally should not be appreciable in the comparatively short period of observation in which the data were taken. For a longer period of observation, where the trend is noticeable, corrections should be made for trend before the data are analyzed. The question of cyclic changes in climatic condition is still an hypothesis yet to be proved.* Hazen has maintained that these changes are pulses which must be expected and accepted when they come, but there is not reason to expect that any one pulse, or any change that occurs in the life time of one generation, will change the condition so radically that intelligent estimates based on past experience will be no longer valuable.†

(5) Short Period of Record. When the period of record is so short that the data cannot be taken as the representative sample of the hydrologic event, the data should be subjected to a normalcy test to determine how good a sample it is. Necessary adjustment is therefore made to keep the data in uniformity with the precipitation normal.

Statistical studies on hydrologic data have revealed the fact that to insure statistical significance in the coefficient of skewness, a statistical parameter of third order, the number of years required for its determination should be at least 140. Data have shown that for a significant coefficient of variation, the data should be taken from more than 30 yr of record, and for a reasonably reliable mean, data based on 20 yr of record is necessary. Generally speaking, hydrologic data taken from a record over 20 yr using the statistical method employing two parameters should produce a good approximation for practical purposes.

38. Adjustment of Data

Some of the significant deficiencies inherent in the original precipitation data may be adjusted before the data are analyzed. The following are several methods of adjustment for this purpose:

(1) Normalcy Test. When the period of record is comparatively short and the quality of data is questionable, the normalcy test is used to determine how well a sample expresses the normal precipitation defined by long period of record. In principle, the test is usually made by comparing the results of frequency analysis between the short-period data and other long-period data in the vicinity. When no suitable comparative data in the vicinity of the short-period station is available, data from Yarnell's charts or Chow's charts‡ may be used roughly for this purpose. For detailed procedure, reference should be made to "Normalcy Tests of

* Ven Te Chow, "Do Climatic Variations Follow Definite Cycles?" Civil Engineering, Vol. 20, No. 7, July, 1950, p. 470.

† Allen Hazen, "Flood Flows," John Wiley and Sons, 1930, p. 169.

‡ D. L. Yarnell, "Rainfall Intensity-Frequency Data," U. S. Department of Agriculture, Miscellaneous Publication 204, 1936, and Ven Te Chow, "Design Charts for Finding Rainfall Intensity Frequency," Water and Sewage Works, Vol. 99, No. 2, February, 1952, pp. 86-88, or Concrete Pipe News, Vol. 4, No. 6, June, 1952, pp. 8-10.

Precipitation and Frequency Studies of Runoff on Small Watersheds," by W. D. Potter, Technical Bulletin No. 985, Soil Conservation Service, June, 1949.

(2) Consistency Test by Double Mass Plotting. The consistency of precipitation records is affected by various factors, such as a change in location of a station within the record period, faulty gage exposures, and other indeterminate sources of instrumental and observational errors of cumulative type. The extent to which the consistency is impaired by these factors may be tested and adjusted by a double mass plotting method. It has been found, by accumulating the annual amounts in reverse chronological order and determining the mean accumulated precipitation for all stations, that the mean accumulated precipitation for a large group of stations is not significantly affected by the changes in individual stations. Under the assumption based on this observed fact, a mass curve constructed by plotting accumulated amounts at one station against that of the group of stations, or of any station in the group known to be consistent, should result in a practically unbroken straight line. This is a line of consistent slope, provided the entire record for the single station was observed at the same site and under the same condition. If the records are inconsistent, it will be noted that points of the mass curve do not plot along a straight line, but follow one slope for a series of years and then abruptly change to a different slope. The point (year) where the slope changes indicates an alteration in the observational regime at the station being plotted. This, as mentioned above, may be caused by a change in station site or gage exposure or by variations in techniques of observation. The former means the variability due to sampling different sets of terrain parameters, each at a satisfactorily exposed site within a small area; and the latter refers to variability due to the difference between "good" and "poor" exposure at a place. In this study, it may be safely assumed that the gages at Chicago have no great difference in the quality of exposure. As the station histories are usually incomplete, especially in the earlier years, it is possible that a number of other breaks appearing on the plotting may be caused by the station having been moved a significant distance without its identification having been changed. The entire record may then be adjusted to present conditions by multiplying the observed precipitation data by the ratio of the slope of mass curve for the most recent period of observation to the slope prevailing at the time that the earlier data were obtained. The slope established by the latest period is the control for the adjustment.

As mentioned previously in Section 35, the official weather station at Chicago, Illinois, has been moved twice throughout the period of observation. To test the consistency of data, double mass curves were constructed to correlate the total annual precipitation data observed at

Chicago to those at Joliet and Elgin which are about 30 miles west and southwest respectively of Chicago. (See Fig. 17.) Slight breaks were observed on the curves at 1924 and 1942, indicating changes in station site at these years, but the slight discrepancies shown on the plottings do not justify an adjustment of the data.

(3) Corrections for Tabulated Maximum Precipitation. As mentioned in Section 36, the tabulated maximum precipitation data may fail to show the true maximum value which occurs in 5-min intervals as this interval is the smallest unit used for tabulation. However, corrections applied to these tabulated maximum precipitation data to determine the actual maximum values can be made only approximately. According to W. D. Potter,* the 5-, 10- and 15-min maximum intensities computed from Weather Bureau records should be increased 8 percent to approximate the true maximum intensities, the 30-min intensities should be increased 7 percent, and the 60-min intensities 4 percent.

Schafmeyer and Grant have made an examination of all the original rain-gage charts of the excessive rainfall records observed at the Federal Building in Chicago for the 10-yr period, 1919-1928, and found the relation of maximum precipitation for selected 5-, 10- and 15-min periods to the tabulated precipitation as published by Weather Bureau.† The result of this study shows that the average percentage excess of maximum rainfall over tabulated rainfall is 9.9 percent for 5-min intensities, 3.3 percent for 10-min intensities, and 2.3 percent for 15-min intensities. It seems that the percentage of increase of the true maximum intensities over the recorded values varies greatly. Hence, at the present time, no definite rule could be developed for general application. If required, it is recommended that the true values may be obtained by increasing the value at a uniform varying rate from 10 percent for 5-min intensities to 0 percent for 60-min intensities. Fortunately, the tabulations of maximum precipitation published by the Weather Bureau do not need to be adjusted because they are as nearly true maxima as can be measured.

(4) Conversion of Accumulated Depths of Precipitation to Maximum Values. Data of excessive precipitation published by the U. S. Weather Bureau are expressed in maximum values after 1935; while the data for 1935 and earlier which are expressed in accumulated values should be converted into maximum values in order to be consistent with the later data. The procedure of conversion may be either graphical or analytical.

By the graphical procedure, storm mass diagrams are prepared for

* W. C. Potter, "Analytical Procedure for Determining the Effect of Land Use on Surface Runoff," Agricultural Engineering, February, 1948, p. 65.

† A. J. Schafmeyer and B. C. Grant, "Rainfall Intensities and Frequencies," Trans. American Society of Civil Engineers, Vol. 64, 1938, pp. 347-348.

each storm by plotting the given values of excessive precipitation for that particular storm against the consecutive periods of time. The maximum amount of precipitation corresponding to a given interval of time can be easily obtained by inspection from the mass diagram and then be plotted as an envelope curve for each storm. From the envelope curves for the storms occuring in each year, a maximum precipitation depth or intensity can be selected for any duration of time.

By the analytical procedure, the differences in accumulated depth for different durations are computed. The maximum difference for each duration is obtained by inspection.

Of the two procedures, the graphical procedure is faster and more flexible. Higher rate of precipitation can be estimated from the slope of the mass curve without much difficulty, but the personal factor involved in this procedure is appreciable and affects the accuracy of the result to a great extent. The analytical procedure gives a definite result and hence is used in the example. The numerical procedure of conversion is illustrated as follows:

The excessive precipitation for March 31, 1929, is listed in the Annual Summary of the Chicago Weather Bureau as in Table 8.

Table 8
Excessive Precipitation on March 31, 1929, Chicago, Illinois

Date	Excessive Rate		Accumulated depth during excessive rate for consecutive periods of time (min)							
	Began	*Ended*	*5*	*10*	*15*	*20*	*25*	*30*	*35*	*40*
March 31	7:12 p.m.	8:30 p.m.	.12	.21	.23	.29	.37	.48	.51	.59*

* Continued as follows: 45 min, 0.68; 50 min, 0.79; 60 min, 0.89; 80 min, 1.23.

The conversion procedure is given in Table 9. Line 1 shows the original data in which the marked values are interpolated. Line 2 shows the increments for every 5 min, which are the differences between two consecutive values shown in Line 1. The italicized value *0.12* is, by inspection, the maximum value in Line 2, and therefore is the maximum precipitation for any 5-min duration. Line 3 shows the increments for every 10 min, which increments are the differences between values in Line 1 with a difference of 10 min, or simply the sum of every two adjacent values in Line 2. The maximum in this Line 3 is italicized as *0.21*. Similarly, Lines 4 to 17 show respectively the increments for every 15, 20, 25, 30, etc. up to 120 min. All italicized values are maximum precipitations; they are arranged as in Table 10.

Table 9
Sample of Conversion Procedure from Accumulated Depth to Maximum Depth

	Duration (min)																
	0	*5*	*10*	*15*	*20*	*25*	*30*	*35*	*40*	*45*	*50*	*55*	*60*	*65*	*70*	*75*	*80*
1.	0	.12	.21	.23	.29	.37	.48	.51	.59	.68	.79	.84▲	.89	.97▲	1.05▲	1.14▲	1.23
2.		*.12**	.09	.02	.06	.08	.11	.03	.08	.09	.11	.05	.05	.08	.08	.09	.09
3.		*.21*	.11	.08	.14	.19	.14	.11	.17	.20	.16	.10	.13	.16	.17	.18	
4.			.23	.17	.16	.25	.22	.22	.20	*.28*	.25	.21	.18	.21	.25	.26	
5.			.29	.25	.27	.28	.30	.31	.31	.33	.30	.29	.26	.30	*.34*		
6.				.37	.36	.30	.36	.39	*.42*	.36	.38	.38	.37	.35	.39		
7.				.48	.39	.38	.45	*.50*	.47	.41	.46	.46	.46	.44			
8.					.51	.47	.47	*.56*	.55	.52	.49	.54	.55	.55			
9.					.59	.56	.58	.61	.60	.60	.57	.63	*.64*				
10.						.68	.67	.63	.66	.68	.68	.66	*.72*				
11.						*.79*	.72	.68	.74	.76	.77	.75					
12.							.84	.77	.76	.82	.85	*.86*					
13.							.89	.85	.84	.91	*.94*						
14.								.97	.93	.93	*1.00*						
15.								*1.05*	1.02	1.02							
16.									*1.14*	1.11							
17.									*1.23*								

* Italicized values are maximum precipitations
▲ Values interpolated

The original data in Appendix II, expressed as accumulated depths for 1913 to 1935, are converted to maximum values as listed in Appendix III. In these tables, some incomplete intermediate data for several durations were supplemented by graphical interpolation. By this interpolation procedure, known data of the storm are plotted first; then the intermediate values are taken from the smooth curve passing through the plotted points. All interpolated values in the tables are marked with a signal. The italicized values in the Appendices are below the limits for excessive precipitation; hence, they are not used as excessive precipitation in analysis. However, since the data are insufficient for high durations, as 80-, 100-, and 120-min, some italicized values are used because they provide the best data available.

Table 10
Maximum Excessive Precipitation for Storm of March 31, 1929

Date	Duration (min)															
	5	*10*	*15*	*20*	*25*	*30*	*35*	*40*	*45*	*50*	*55*	*60*	*65*	*70*	*75*	*80*
Mar. 31	.12	.21	.28	.34	.42	.50	.56	.64	.72	.79	.86	.94	1.00	1.05	1.14	1.23

(5) The Station-Year Method. When the length of record at a single station is very short and the data collected at the station are scanty, it is sometimes advisable to combine the records of several stations and then to treat them as a single record whose length is equal to the sum of the individual records. This idea of combining records of different stations is the basis of the station-year method which was first used in rainfall analysis by Engineers of the Miami Conservancy District.* It was found

* "Storm Rainfall of Eastern United States," Miami Conservancy District, Engineering Staff, Technical Reports, Pt. 5, Revised ed., 1936, p. 67.

that the volume of data increased in this way would make possible more accurate estimates of the frequencies of excessive rainfall intensities. However, certain restrictions must be taken into consideration:

(a) The region from which the data are collected should be meteorologically homogeneous; that is, the expectation of excessive storms should be approximately the same for all stations. The invalidity of this method to mountainous regions is therefore apparent.

(b) A reasonable length of record must be available from each of the individual stations. It is considered that a period of 10 yr for each station of independent record is a minimum for analysis.

(c) The records obtained from individual stations should be independent; that is, no single storm should be counted more than once. Accordingly, the individual stations from which records are collected should be so spaced that one and only one station measures each storm. As the high intensity rainfalls of short duration are usually the result of thunderstorms which may cover a relatively small area, the interdependence between stations is likely to be high.

It should be noted that the requirements of homogeneity on the one hand and of independence on the other are nearly mutually exclusive; the restrictions mentioned must be tolerated to a certain extent depending on the conditions under consideration.

(6) Extended Duration Principle. By this principle, all storms in which the total precipitation was sufficient to show significant average rates for periods longer than the actual duration of the rainfall were considered as though they had continued for the longer time. It is found that this principle produces more complete data and permits more accurate analysis for the hydrologic event. The principle was first initiated by Meyer for studying rainfall frequencies.* Later, the Weather Bureau adopted this principle for expanding tabulation of excessive precipitation data for durations up to 180 min. This was done only after 1935. Before 1935, the Weather Bureau data failed to show the precipitation, if any, after the greatest excessive value was reached. The application of the extended duration principle is therefore used to extend the greatest excessive value beyond the listed durations up to 180 min. This may be noticed in the preparation of Appendix IV, in which the excessive precipitation data supplemented by this principle are shown at the right side of the heavy zigzag line.

* Adolph F. Meyer, "The Elements of Hydrology," John Wiley and Sons, Inc., 2nd ed., 1928, p. 160.

APPENDIX I
DERIVATION OF THE FORMULA FOR MEAN NUMBER OF EXCEEDANCES

Based on the work by Gumbel and von Schelling, the derivation of Eq. 27, or the formula for mean number of exceedances, proceeds in the following manner:

If p_o is taken as the probability for a value less than magnitude y, then the probability for a value equal to or greater than y is $1\text{-}p_o$. By the Bernoulli theorem, the probability that y will be equaled or exceeded x times among N future trials is equal to the $x + 1$st term in the development of a binomial series, or

$$P\,[p_o, N, x] = \binom{N}{x} (1 - p_o)^x p_o^{\,N-x} \tag{A1}$$

Take the probability p_o as a variate, then the probability of choosing a value having a probability p_o among n past observations will be

$$d\,P\,[n,m,p_o] = \binom{N}{1} dp_o \binom{n-1}{m-1} (1 - p_o)^{m-1} p_o^{\,n-m}$$

or

$$= \binom{n}{m} m p_o^{\,n-m} (1 - p_o)^{m-1}\, dp_o \tag{A2}$$

By the theorem of compound probability, the cumulative probability, or the distribution, of the number of exceedances over the m-th largest value among n observations in N future trials is equal to the integration of the product of two probabilities given respectively by Eq. A1 and Eq. A2 for the entire range of observation, or

$$\begin{aligned} P\,[n,m,N,x] &= \int_o^1 P\,[p_o, N,x]\, dP\,[n,m,p_o] \\ &= \int_o^1 \binom{N}{x} (1 - p_o)^x p_o^{\,N-x} \binom{n}{m} m p_o^{\,n-m} (1 - p_o)^{m-1} dp_o \\ &= \binom{n}{m} m \binom{N}{x} \int_o^1 p_o^{\,N-x+n-m} (1 - p_o)^{m+x-1} dp_o \end{aligned} \tag{A3}$$

The integral in Eq. A3 is a Beta function which may be evaluated in terms of Gamma functions; that is,

$$\int_o^1 p_o^{N-x+n-m}(1-p_o)^{m+x-1}dp_o$$

$$= \frac{\Gamma(N-x+n-m+1)\,\Gamma(m+x)}{\Gamma(N+n+1)} \tag{A4}$$

Developing the Gamma functions, Eq. A4 becomes

$$\int_o^1 p_o^{N-x+m-n}(1-p_o)^{m+x-1}dp_o$$

$$= \frac{(N-x+n-m)!\,(m+x-1)!}{(N+n)!}$$

$$= \frac{1}{(N+n)\binom{N+n-1}{m+x-1}} \tag{A5}$$

Substituting Eq. A5 for the value of the integral into Eq. A3, then

$$P(n,m,N,x) = \frac{\binom{n}{m} m \binom{N}{x}}{(N+n)\binom{N+n-1}{m+x-1}}$$

$$= \frac{(x+m-1)!\,n!\,N!\,(N+n-x-m)!}{(m-1)!\,x!\,(n-m)!\,(N-x)!\,(N+n)!} \tag{A6}$$

It is evident that the conditions for Eq. A6 are:

$$\left.\begin{array}{l} 1 \leqq m \leqq n; \\ 0 \leqq x \leqq N; \\ \sum_{x=0}^{x=N} P(n,m,N,x) = 1 \end{array}\right\} \tag{A7}$$

Equation A6 gives the probability that the m-th largest among n past observations will be equaled or exceeded x times in N future trials.

The probability by Eq. A6 may be represented by the $x + 1$st member in the development of

$$\frac{\binom{A+B-C}{A}}{\binom{A-C}{A}} F(A,B,C,1)$$

where $F\,(A,B,C,\,1)$ is a hypergeometric series defined as

$$F\,(A,B,C,1) = 1 + \frac{A}{1}\,\frac{B}{C} + \frac{A\,(A-1)}{2!}\,\frac{B\,(B-1)}{C\,(C-1)} + \ldots \tag{A8}$$

in which

$$\left.\begin{aligned} A &= m \\ B &= -N \\ C &= m - n - N \end{aligned}\right\} \tag{A9}$$

The $x + 1$st member of this series is

$$f_x(A,B,C,1) = \frac{A\,(A+1)\,\ldots\,(A+x-1)\,B\,(B+1)\,\ldots\,(B+x-1)}{x!\;C\,(C+1)\,\ldots\,(C+x-1)}$$

$$= \frac{(A+x-1)!\;(B+x-1)!\;(C-1)!}{x!\;(A-1)!\;(B-1)!\;(C+x-1)!} \tag{A10}$$

Therefore the probability by Eq. A6 may be expressed as

$$P(n,m,N,x) = \frac{\binom{A+B-C}{A}}{\binom{A-C}{A}}\,f_x(A,B,C,1) \tag{A11}$$

One of its conditions from Eq. A7 is

$$\sum_{x=0}^{x=N} P(n,m,N,x) = 1 \tag{A12}$$

Substituting Eq. A11 into Eq. A12,

$$\frac{\binom{A+B-C}{A}}{\binom{A-C}{A}} \sum_{x=0}^{x=N=-B} f_x(A,B,C,1) = 1$$

or

$$\sum_{x=0}^{x=-B} f_x(A,B,C,1) = \frac{\binom{A-C}{A}}{\binom{A+B-C}{A}} \tag{A13}$$

The relation expressed by Eq. A13 is used for the evaluation of the factorial moment $\bar{x}_k$ of order k. This factorial moment is defined by

$$\bar{x}_k = \sum_{x=k}^{x=N} x(x-1)\,\ldots\,(x-k+1)P(n,m,N,x)$$

$$= \sum_{x=k}^{x=N} \frac{x!}{(x-k)!}\,P(n,m,N,x) \tag{A14}$$

From Eq. A11, Eq. A14 becomes

$$\bar{x}_k = \frac{\binom{A+B-C}{A}}{\binom{A-C}{A}} \sum_{x=k}^{x=-B} \frac{x!}{(x-k)!} f_x(A,B,C,1)$$

$$= \frac{\binom{A+B-C}{A}}{\binom{A-C}{A}} \sum_{x=k}^{x=-B} \phi_x \tag{A15}$$

in which

$$\phi_x = \frac{x!}{(x-k)!} f_x\,(A,B,C,1)$$

$$= \frac{(A+x-1)!\,(B+x-1)!\,(C-1)!}{(x-k)!\,(A-1)!\,(B-1)!\,(C+x-1)!} \tag{A16}$$

When $x = k$,

$$\phi_k = \frac{(A+k-1)!\,(B+k-1)!\,(C-1)!}{0!\,(A-1)!\,(B-1)!\,(C+k-1)!} \tag{A17}$$

When $x = k + 1$,

$$\phi_{k+1} = \frac{(A+k)!\,(B+k)!\,(C-1)!}{1!\,(A-1)!\,(B-1)!\,(C+k)!} \tag{A18}$$

Hence, from Eqs. A17 and A18,

$$\phi_{k+1} = \phi_k \frac{(A+k)\,(B+k)}{1!\,(C+k)!} \tag{A19}$$

Similarly, it can be proved that

$$\phi_{k+n} = \phi_k \frac{(A+k+n-1)\,(B+k+n-1)}{n!\,(C+k+n-1)} \tag{A20}$$

Then

$$\sum_{x=k}^{x=-B} \phi_x = \phi_k \left[1 + \frac{(A+k)\,(B+k)}{1!\,(C+k)!} + \ldots\right] \tag{A21}$$

The members in the bracket form a hypergeometric series,

$$\sum_{x=k}^{x=-B} \phi_x = \phi_k \sum_{x=0}^{x=-B-k} f_x[(A+k),(B+k),(C+k),1] \tag{A22}$$

By Eq. A13,

$$\sum_{x=k}^{x=-B} \phi_x = \phi_k \frac{\binom{A-C}{A+k}}{\binom{A+B-C+k}{A+k}} \tag{A23}$$

Therefore Eq. A15 becomes

$$\bar{x}_k = \frac{\binom{A+B-C}{A}}{\binom{A-C}{A}} \phi_k \frac{\binom{A-C}{A+k}}{\binom{A+B-C+k}{A+k}}$$

$$= \frac{\binom{A+B-C}{A}(A+k-1)!\,(B+k-1)!\,(C-1)!\binom{A-C}{A+k}}{\binom{A-C}{A}0!\;(\mathrm{A}-1)!\,(\mathrm{B}-1)!\,(\mathrm{C}+\mathrm{k}-1)!\binom{A+B-C+k}{A+k}} \quad \text{(A24)}$$

Substituting Eq. A9 into Eq. A24 for A, B, and C,

$$\bar{x}_k = \frac{n!\,(N+n-m)!\,(m-1+k)!\,(-N-1+k)!\,(m-n-N-1)!}{(n+k)!\,(N+n-m-k)!\,(m-1)!\,(-N-1)!\,(m-n-N-1+k)!} \quad \text{(A25)}$$

When $k = 1$, the factorial moment $\bar{x}$ is of the first order or it is equal to the mean value of x. Therefore, the mean number of exceedance, $\bar{x}$, over the m-th largest value in N future trials is obtained readily from Eq. A25 by substituting $k = 1$; or

$$\bar{x} = N\frac{m}{n+1} \quad \text{(A26)}$$

APPENDIX II
EXCESSIVE PRECIPITATION DATA AT CHICAGO, ILLINOIS, COMPILED BY THE CHICAGO WEATHER BUREAU OFFICE, 1913-1935

Dates	Excessive Rate		Accumulated Depths during Excessive Rate for Consecutive Periods of Time (min)													
	Began	*Ended*	*5*	*10*	*15*	*20*	*25*	*30*	*35*	*40*	*45*	*50*	*60*	*80*	*100*	*120*
July 14, 1913	7:38 pm	8:18 pm	0.13	0.29	0.40	0.50	0.59	0.67	0.74	0.79						
Aug. 7, "	10:21 pm	10:28 pm	0.31	0.37												
Aug. 7-8, "	11:24 pm	11:39 pm	0.12	0.42	0.56											
Aug. 18, "	1:31 pm	1:47 pm		0.49	0.63	0.67										
Apr. 27, 1914	5:26 pm	5:31 pm	0.27													
May 27, "	9:54 pm	10:14 pm	0.18	0.33	0.41	0.49										
June 4, "	2:49 am	3:04 am	0.21	0.35	0.40											
July 16, "	1:59 pm	2:32 pm	0.27	0.51	0.70	0.82	1.15	1.48	1.61							
Aug. 9, "	2:52 pm	3:11 pm	0.08	0.35	0.70	0.91										
Aug. 13, "	11:41 am	12:06 pm	0.08	0.22	0.41	0.58	0.68									
Sept. 1, "	2:44 am	2:59 am	0.14	0.27	0.38	0.40										
May 15, 1915	6:07 pm	6:30 pm	0.08	0.25	0.31	0.40	0.48									
June 12, "	11:02 pm	11:47 pm	0.18	0.31	0.46	0.56	0.72	0.82	0.89	0.92	0.98					
July 7, "	3:41 pm	3:54 pm	0.20	0.33	0.45											
July 18, "	7:58 am	8:13 am	0.08	0.29	0.37											
Aug. 3, "	2:01 am	2:31 am	0.15	0.28	0.44	0.52	0.58									
Aug. 23, "	8:27 pm	8:39 pm	0.17	0.36	0.45											
Sept. 10, "	1:41 pm	1:54 pm	0.26	0.32	0.37											
May 14, 1916	3:48 pm	3:54 pm	0.30	0.32												
July 19, "	11:36 am	12:12 pm	0.12	0.16	0.21	0.56	0.79	0.90	1.05	1.08						
Aug. 20, "	6:37 pm	6:45 pm	0.30	0.36												
Sept. 5, "	5:48 pm	6:08 pm	0.17	0.31	0.37	0.43										
May 19, 1917	5:24 pm	5:47 pm	0.12	0.21	0.25	0.37	0.47									
June 28, "	3:57 pm	4:10 pm	0.24	0.32												
July 26, "	2:32 pm	2:47 pm	0.23	0.41	0.57											
July 24, 1918	5:25 pm	5:40 pm	0.24	0.38	0.41	0.43										
July 28, "			0.23	0.47	0.53											
June 11, 1919	4:27 pm	4:36 pm	0.29	0.42												
June 14, "	1:50 pm	2:01 pm	0.19	0.35	0.39											
June 19, "	6:50 am	7:03 am	0.21	0.33	0.40											
Sept. 10, "	4:38 am	4:51 am	0.14	0.38	0.47											
Oct. 5, "	8:13 am	8:48 am	0.09	0.28	0.61	1.02	1.19	1.38	1.53							
Oct. 9-10, "	11:37 pm	12:44 am	0.06	0.17	0.22	0.30	0.38	0.49	0.56	0.61						
June 14, 1920	2:35 pm	2:43 pm	0.22	0.41												
June 14, "	6:14 pm	6:26 pm	0.21	0.39	0.43											
June 29, "	4:52 pm	4:57 pm	0.53													
July 13, "	6:41 pm	7:08 pm	0.13	0.18	0.25	0.34	0.50	0.54								
Sept. 5, "	5:47 pm	6:32 pm	0.06	0.10	0.17	0.34	0.53	0.70	0.83	1.22						
July 7, 1921	5:34 pm	6:00 pm	0.32	0.71	1.08	1.35	1.45	1.49								
Aug. 19, "	7:12 pm	8:31 pm	0.20	0.36	0.58	0.58	0.58	0.58	0.58	0.58	0.65	0.74	1.12	1.45		
Sept. 2, "	5:10 pm	5:23 pm	0.23	0.42	0.48											
Sept. 29, "	4:25 pm	4:32 pm	0.28	0.32												
Sept. 29, "	6:34 pm	6:42 pm	0.25	0.34												
Apr. 10, 1922	3:51 pm	4:11 pm	0.11	0.26	0.34	0.40										
Apr. 10, "	10:23 pm	11:13 pm	0.06	0.09	0.13	0.24	0.33	0.48	0.63	0.68	0.73	0.78				
July 11, "	4:40 pm	5:33 pm	0.06	0.25	0.42	0.67	0.78	0.88	0.91	0.98	1.04	1.12	1.21			
July 22, "	9:07 pm	9:18 pm	0.19	0.36	0.41											
Aug. 24, "	4:06 pm	4:27 pm	0.28	0.57	0.69	0.77	0.79									
Sept. 10, "	1:59 am	3:00 am	0.08	0.31	0.39	0.43	0.59	0.74	0.87	0.88	0.88	1.09	1.53	1.55		
June 25, 1923	10:53 pm	11:07 pm	0.05	0.22	0.35											
July 6, "	9:19 pm	9:41 pm	0.08	0.15	0.28	0.42	0.45									
Aug. 7, "	3:11 pm	3:20 pm	0.10	0.30												
Aug. 11, "	1:06 am	2:19 am	0.10	0.37	0.72	1.17	1.49	1.58	1.71	1.77	1.80	1.85	2.02	2.30		
Aug. 11, "	8:49 pm	9:09 pm	0.16	0.43	0.66	0.82										
Aug. 27, "	3:27 am	3:42 am	0.10	0.31	0.40											
Aug. 27, "	6:58 am	7:12 am	0.16	0.45	0.56											
July 20, 1924	8:50 pm	9:30 pm	0.16	0.32	0.71	0.98	1.18	1.37	1.57	1.75						

Dates	Excessive Rate		Accumulated Depths during Excessive Rate for Consecutive Periods of Time (min)													
	Began	*Ended*	*5*	*10*	*15*	*20*	*25*	*30*	*35*	*40*	*45*	*50*	*60*	*80*	*100*	*120*
Aug. 4, 1924	12:51 am	1:20 am	0.06	0.24	0.34	0.70	0.93	1.02								
Aug. 5, "	7:34 pm	8:49 pm	0.12	0.24	0.47	0.57	0.69	0.80	0.88	1.08	1.12	1.15	1.22	1.54		
Aug. 5, "	10:33 pm	10:51 pm	0.16	0.27	0.36	0.45										
Aug. 6, "	2:12 pm	2:24 pm	0.15	0.34	0.41											
Aug. 8, "	10:27 am	10:32 am	0.34													
Apr. 13, 1925	4:07 pm	4:14 pm	0.26	0.38												
May 16, "	1:51 pm	1:56 pm	0.25													
June 15, "	3:41 am	4:30 am	0.05	0.17	0.18	0.22	0.30	0.30	0.35	0.54	0.75	0.87				
June 17, "	4:50 am	5:03 am	0.23	0.40	0.53											
June 24, "	12:30 pm	1:15 pm	0.12	0.30	0.43	0.56	0.67	0.74	0.89	1.08	1.25					
Aug. 13, "	12:57 am	1:31 am	0.10	0.31	0.47	0.74	1.15	1.31	1.38							
Sept. 8, "	3:35 pm	4:00 pm	0.05	0.14	0.26	0.46	0.52									
May 18, 1926	4:48 pm	5:13 pm	0.09	0.22	0.52	0.90	1.04									
June 11, "	11:20 am	11:48 am	0.22	0.32	0.43	0.52	0.62	0.67								
June 11, "	4:23 pm	4:38 pm	0.07	0.37	0.54											
June 11, "	7:54 pm	8:03 pm	0.17	0.34												
June 13, "	4:44 pm	5:24 pm	0.08	0.30	0.51	0.72	1.03	1.26	1.59	1.85						
June 25, "	5:54 pm	6:28 pm	0.10	0.26	0.34	0.50	0.85	1.03	1.09							
Sept. 23, "	3:04 pm	4:04 pm	0.08	0.13	0.18	0.22	0.26	0.31	0.36	0.44	0.48	0.55	0.81			
May 9, 1927	4:27 pm	4:38 pm	0.23	0.38	0.44											
May 24, "	12:30 am	12:55 am	0.11	0.21	0.30	0.39	0.46									
July 1, "	4:13 pm	4:43 pm	0.25	0.55	0.77	0.94	1.05	1.11								
July 6, "	9:29 pm	9:39 pm	0.24	0.32												
Aug. 8, "	2:46 pm	3:11 pm	0.48	0.61	0.73	0.79	0.87									
Sept. 17, "	7:24 pm	7:53 pm	0.06	0.18	0.50	0.69	0.73	0.81								
Sept. 17, "	10:23 pm	11:23 pm	0.05	0.14	0.23	0.40	0.46	0.63	0.69	0.70	0.70	0.78	0.97			
June 20, 1928	2:14 pm	3:14 pm	0.22	0.72	1.10	1.38	1.58	1.70	1.86	1.92	2.04	2.06	2.30			
June 24, "	4:49 pm	5:56 pm	0.11	0.21	0.28	0.32	0.35	0.42	0.52	0.58	0.62	0.68	0.99			
July 2, "	8:49 am	8:59 am	0.17	0.41												
July 3, "	2:27 am	2:57 am	0.08	0.14	0.25	0.47	0.68	0.78								
Aug. 3, "	10:08 pm	10:28 pm	0.27	0.47	0.55	0.61										
Aug. 29, "	2:23 pm	2:57 pm	0.18	0.42	0.53	0.73	1.01	1.20	1.30							
Sept. 14, "	4:56 pm	5:26 pm	0.10	0.21	0.35	0.50	0.54	0.61								
Mar. 31, 1929	7:12 pm	8:30 pm	0.12	0.21	0.23	0.29	0.37	0.48	0.51	0.59	0.68	0.79	0.89	1.23		
June 11, "	2:20 pm	2:50 pm	0.16	0.40	0.52	0.63	0.72	0.77								
June 11, "	6:39 pm	7:08 pm	0.06	0.34	0.45	0.52	0.74	0.84								
June 27, "	8:44 pm	8:59 pm	0.30	0.49	0.54											
Aug. 10, "	4:38 pm	4:51 pm	0.21	0.39	0.46											
Aug. 26, "	4:59 pm	5:42 pm	0.17	0.37	0.63	0.74	0.74	0.74	0.74	0.88						
Sept. 28, "	11:25 pm	11:48 pm	0.07	0.10	0.24	0.49	0.58									
June 5, 1930	2:59 pm	3:08 pm	0.12	0.32												
July 5, "	6:04 am	7:12 am	0.19	0.33	0.40	0.41	0.43	0.55	0.58	0.63	0.67	0.72	0.82	0.96		
June 23, 1931	2:15 am	4:15 am	0.18	0.41	0.67	0.76	0.88	0.96	0.99	1.00	1.00	1.02	1.13	1.22	1.36	1.53
July 19, "	9:24 pm	10:13 pm	0.15	0.36	0.68	0.91	0.98	1.04	1.17	1.24	1.31	1.40				
July 21, "	4:29 pm	4:49 pm	0.16	0.26	0.54	0.66										
Aug. 10-11, "	11:17 pm	12:10 am	0.12	0.25	0.37	0.39	0.44	0.54	0.62	0.74	0.80	1.17	1.27			
Aug. 11, "	5:44 am	6:04 am	0.26	0.71	1.22	1.39										
Aug. 27, "	9:00 pm	9:23 pm	0.32	0.74	0.96	1.21	1.27									
Sept. 2, "	4:01 pm	4:40 pm	0.05	0.12	0.21	0.24	0.35	0.57	0.78	0.90						
Sept. 25, "	7:09 am	7:39 am	0.07	0.11	0.19	0.34	0.57	0.81								
May 7, 1932	6:29 pm	6:51 pm	0.06	0.21	0.43	0.54	0.59									
June 5, "	11:47 am	12:12 pm	0.23	0.46	0.59	0.69	0.78									
June 26, "	4:50 pm	5:05 pm	0.55	0.94	1.16											
June 30, "	11:30 pm	11:40 pm	0.24	0.43												
July 7, "	2:24 am	2:39 am	0.20	0.47	0.76											
July 7, "	5:41 am	5:53 am	0.20	0.31	0.35											
July 26, "	5:54 pm	6:32 pm	0.11	0.23	0.29	0.39	0.49	0.57	0.72	0.78						
June 29, 1933	6:34 pm	7:18 pm	0.31	0.68	0.80	0.85	1.10	1.38	1.53	1.81	2.03					
July 2, "	2:05 am	2:25 am	0.41	0.80	1.00	1.16										
June 22, 1934	9:50 am	10:10 am	0.26	0.44	0.63	0.72	0.72	0.72	0.72	0.72						
Aug. 15, "	4:19 am	4:51 am	0.12	0.24	0.37	0.63	0.99	1.19	1.21	1.23						
Aug. 23, "	11:39 pm	11:59 pm	0.05	0.18	0.40	0.45	0.48	0.50	0.50	0.51						
Sept. 29, "	7:57 pm	8:34 pm	0.08	0.19	0.37	0.51	0.72	0.78	0.85	0.88						
May 12, 1935	4:28 am	6:01 am	0.06	0.18	0.28	0.35	0.48	0.53	0.74	0.95						
June 16, "	5:50 pm	6:09 pm	0.11	0.26	0.39	0.45	0.45	0.46	0.46	0.47						
June 16, "	6:43 pm	7:07 pm	0.10	0.16	0.27	0.38	0.47	0.47	0.47	0.47						
June 18, "	8:51 pm	9:10 pm	0.08	0.37	0.75	0.94	0.95	0.96	1.02	1.03						
June 26, "	9:42 am	9:47 am	0.27	0.28	0.28	0.28	0.28	0.28	0.28	0.28						
July 5, "	12:29 pm	12:42 pm	0.26	0.41	0.46	0.48	0.49	0.50	0.50	0.50						
July 23, "	6:57 pm	7:09 pm	0.21	0.30	0.36	0.37	0.38	0.39	0.40	0.40						
July 25, "	1:13 pm	1:24 pm	0.19	0.31	0.32											
July 28, "	2:16 am	2:35 am	0.16	0.36	0.48	0.58	0.59	0.59	0.60	0.60						
Aug. 2, "	9:48 pm	10:12 pm	0.37	0.71	0.87	0.96	1.03	1.05	1.05	1.06						
Aug. 17, "	4:33 pm	4:49 pm	0.08	0.30	0.37	0.38	0.38	0.38	0.39	0.39						

APPENDIX III
MAXIMUM EXCESSIVE PRECIPITATION DATA AT CHICAGO, ILLINOIS, 1913-1947

Date		Duration (min)													
		5	*10*	*15*	*20*	*25*	*30*	*35*	*40*	*45*	*50*	*60*	*80*	*100*	*120*
July 14	1913	*0.16*	*0.29*	0.40	0.50	0.59	0.67	0.74	0.79						
Aug. 7	"	0.31	0.37												
Aug. 7-8	"	0.30	0.44	0.56											
Aug. 18	"	0.28▲	0.49	0.63	0.67										
Apr. 27	1914	0.27													
May 27	"	*0.18*	0.33	0.41	0.49										
June 4	"	*0.21*	0.35	0.40											
July 16	"	0.33	0.66	0.79	0.97	1.21	1.48	1.61							
Aug. 9	"	0.35	0.62	0.83	0.91										
Aug. 13	"	*0.19*	0.36	0.50	0.60	0.68									
Sept. 1	"	*0.14*	*0.27*	0.38	0.40										
May 15	1915	*0.17*	*0.25*	*0.32*	0.40	0.48									
June 12	"	*0.18*	0.31	0.46	0.56	0.72	0.82	0.89	0.92	0.98					
July 7	"	*0.20*	0.33	0.45											
July 18	"	*0.21*	*0.29*	0.37											
Aug. 3	"	*0.16*	*0.29*	0.44	0.52	0.58									
Aug. 23	"	*0.19*	0.36	0.45											
Sept. 10	"	0.26	0.32	0.37											
May 14	1916	0.30	0.32												
July 19	"	0.35	0.58	0.69	0.84	0.89	0.93	1.05	1.08						
Aug. 20	"	0.30	0.36												
Sept. 5	"	*0.17*	0.31	0.37	0.43										
May 19	1917	*0.12*	*0.22*	*0.26*	*0.37*	0.47									
June 28	"	*0.24*	0.32												
July 26	"	*0.23*	0.41	0.57											
July 24	1918	*0.24*	0.38	0.41	0.43										
July 28	"	*0.23*	0.47	0.53											
June 11	1919	0.29	0.42												
June 14	"	*0.19*	0.35	0.39											
June 19	"	*0.21*	0.33	0.40											
Sept. 10	"	*0.24*	0.38	0.47											
Oct. 5	"	0.41	0.74	0.93	1.10	1.29	1.44	1.53							
Oct. 9-10	"	*0.11*	*0.19*	*0.27*	*0.34*	*0.43*	0.50	0.56	0.61						
June 14	1920	*0.22*	0.41												
June 14	"	*0.21*	0.39	0.43											
June 29	"	0.53													
July 13	"	*0.16*	*0.25*	*0.32*	*0.37*	0.50	0.54								
Sept. 5	"	0.39	0.52	0.69	0.88	1.05	1.12	1.16	1.22						
July 7	1921	0.39	0.76	1.08	1.35	1.45	1.49								
Aug. 19	"	*0.22*	0.38	0.58	0.58	0.58	0.58	0.58	*0.58*	0.65	0.74	1.12	1.45		
Sept. 2	"	*0.23*	0.42	0.48											
Sept. 29	"	0.28	0.32												
Sept. 29	"	0.25	0.34												
Apr. 10	1922	*0.15*	*0.26*	*0.34*	0.40										
Apr. 10	"	*0.15*	0.30	0.39	0.50	0.55	0.60	0.65	0.69	0.73	0.78				
July 11	"	0.25	0.42	0.61	0.72	0.82	0.88	0.92	0.98	1.06	1.12	1.21			
July 22	"	*0.19*	0.36	0.41											
Aug. 24	"	0.29	0.57	0.69	0.77	0.79									
Sept. 10	"	*0.23*	0.44	0.65	0.66	0.67	0.79	0.94	1.10	1.14	1.23	1.53	1.55		
June 25	1923	*0.17*	0.30	0.35											
July 6	"	*0.14*	*0.27*	*0.34*	0.42	0.45									
Aug. 7	"	*0.20*	0.30												
Aug. 11	"	0.45	0.80	1.12	1.39	1.49	1.61	1.71	1.77	1.80	1.85	2.02	2.30		
Aug. 11	"	0.27	0.50	0.66	0.82										
Aug. 27	"	*0.21*	0.31	0.40											
Aug. 27	"	0.29	0.45	0.56											

NOTES:
(1) Criterion for excessive precipitation is $0.01T + 0.20$ in., where T is duration in minutes.
(2) Data before 1936 are computed from "Accumulated depths" given by Chicago Weather Bureau Office.
(3) Data after 1933 were published according to "Extended duration" principle.
(4) "▲" indicates the value interpolated from a curve which is plotted by using the known data of the same storm.
(5) The italicized values are not excessive precipitations, because they are below the criterion.

Date		Duration (min)													
		5	*10*	*15*	*20*	*25*	*30*	*35*	*40*	*45*	*50*	*60*	*80*	*100*	*120*
July 20	1924	0.39	0.66	0.86	1.05	1.25	1.43	1.59	1.75						
Aug. 4	"	0.36	0.59	0.69	0.87	0.96	1.02								
Aug. 5	"	*0.23*	0.35	0.47	0.57	0.69	0.80	0.88	1.08	1.12	1.15	1.22	1.54		
Aug. 5	"	*0.16*	*0.27*	0.36	0.45										
Aug. 6	"	*0.19*	0.34	0.41											
Aug. 8	"	0.34													
Apr. 13	1925	0.26	0.38												
May 16	"	0.25													
June 15	"	*0.21*	0.40	0.52	0.57	0.57	0.65	0.69	0.70	0.82	0.87				
June 17	"	*0.23*	0.40	0.53											
June 24	"	*0.19*	0.36	0.51	0.58	0.69	0.78	0.96	1.13	1.25					
Aug. 13	"	0.41	0.68	0.84	1.05	1.21	1.31	1.38							
Sept. 8	"	*0.20*	0.32	0.41	0.47	0.52									
May 18	1926	0.38	0.68	0.82	0.95	1.04									
June 11	"	*0.22*	0.32	0.43	0.52	0.62	0.67								
June 11	"	0.30	0.47	0.54											
June 11	"	*0.17*	0.34												
June 13	"	0.33	0.59	0.87	1.13	1.34	1.55	1.77	1.85						
June 25	"	0.35	0.53	0.69	0.77	0.93	1.03	1.09							
Sept. 23	"	*0.13*	*0.26*	*0.33*	*0.37*	0.45	0.50	0.55	*0.59*	*0.63*	*0.68*	0.81			
May 9	1927	*0.23*	0.38	0.44											
May 24	"	*0.11*	*0.21*	*0.30*	*0.39*	0.46									
July 1	"	0.30	0.55	0.77	0.94	1.05	1.11								
July 6	"	*0.24*	0.32												
Aug. 8	"	0.48	0.61	0.73	0.79	0.87									
Sept. 17	"	0.32	0.51	0.63	0.69	0.75	0.81								
Sept. 17	"	*0.17*	*0.26*	0.40	0.49	0.58	0.64	0.69	0.70	0.74	0.80	0.97			
June 20	1928	0.50	0.88	1.16	1.38	1.58	1.70	1.86	1.92	2.04	2.06	2.30			
June 24	"	*0.16*	0.31	0.37	0.41	0.47	0.57	0.64	0.67	0.71	0.78	0.99			
July 2	"	*0.24*	0.41												
July 3	"	*0.22*	0.43	0.54	0.64	0.70	0.78								
Aug. 3	"	0.27	0.47	0.55	0.61										
Aug. 29	"	0.28	0.48	0.59	0.83	1.02	1.20	1.30							
Sept. 14	"	*0.15*	*0.29*	0.40	0.50	0.54	0.61								
Mar. 31	1929	*0.12*	*0.21*	*0.28*	*0.34*	*0.42*	0.50	0.56	0.64	0.72	0.79	0.94	1.23		
June 11	"	*0.24*	0.40	0.52	0.63	0.72	0.77								
June 11	"	0.28	0.39	0.46	0.68	0.78	0.84								
June 27	"	0.30	0.49	0.54											
Aug. 10	"	*0.21*	0.39	0.46											
Aug. 26	"	0.26	0.46	0.63	0.74	0.74	0.74	0.74	0.88						
Sept. 28	"	0.25	0.39	0.48	0.51	0.58									
June 5	1930	*0.20*	0.32												
July 5	"	*0.19*	0.33	0.40	0.41	*0.43*	0.55	0.58	0.63	0.67	0.72	0.82	*0.96*		
June 23	1931	0.26	0.49	0.67	0.76	0.88	0.96	0.99	1.00	1.00	1.02	1.13	1.22	1.36	1.53
July 19	"	0.32	0.55	0.76	0.91	0.98	1.04	1.17	1.24	1.31	1.40				
July 21	"	0.28	0.40	0.54	0.66										
Aug. 10-11	"	0.37	0.43	0.55	0.63	0.73	0.78	0.83	0.92	1.05	1.17	1.27			
Aug. 11	"	0.51	0.96	1.22	1.39										
Aug. 27	"	0.42	0.74	0.96	1.21	1.27									
Sept. 2	"	*0.22*	0.43	0.55	0.66	0.69	0.78	0.85	0.90						
Sept. 25	"	*0.24*	0.47	0.62	0.70	0.74	0.81								
May 7	1932	*0.22*	0.37	0.48	0.54	0.59									
June 5	"	*0.23*	0.46	0.59	0.69	0.78									
June 26	"	0.55	0.94	1.16											
June 30	"	*0.24*	0.43												
July 7	"	0.29	0.56	0.76											
July 7	"	*0.20*	0.31	0.35											
July 26	"	*0.15*	*0.23*	*0.33*	*0.39*	0.49	0.57	0.72	0.78						
June 29	1933	0.37	0.68	0.80	0.96	1.18	1.38	1.53	1.81	2.03					
July 2	"	0.41	0.80	1.00	1.16										
June 22	1934	0.26	0.44	0.63	0.72	0.72	0.72	0.72	0.72						
Aug. 15	"	0.36	0.62	0.82	0.95	1.07	1.19	1.21	1.23						
Aug. 23	"	*0.22*	0.35	0.40	0.45	0.48	0.50	*0.50*	*0.51*						
Sept. 29	"	*0.21*	0.35	0.53	0.64	0.72	0.78	0.85	0.88						
May 12	1935	*0.21*	0.42	0.47	0.60	0.67	0.77	0.89	0.95						
June 16	"	*0.15*	*0.28*	0.39	0.45	0.45	*0.46*	*0.46*	*0.47*						
June 16	"	*0.11*	*0.22*	*0.31*	*0.38*	0.47	*0.47*	*0.47*	*0.47*						
June 18	"	0.38	0.67	0.86	0.94	0.95	0.96	1.02	1.03						
June 26	"	0.27	*0.28*	*0.28*	*0.28*	*0.28*	*0.28*	*0.28*	*0.28*						
July 5	"	0.26	0.41	0.46	0.48	0.49	0.50	*0.50*	*0.50*						
July 23	"	*0.21*	0.30	0.36	*0.37*	*0.38*	*0.39*	*0.40*	*0.40*						
July 25	"	*0.19*	0.31	*0.32*											
July 28	"	*0.20*	0.36	0.48	0.58	0.59	0.59	0.60	0.60						
Aug. 2	"	0.37	0.71	0.87	0.96	1.03	1.05	1.05	1.06						
Aug. 17	"	*0.22*	0.30	0.37	*0.38*	*0.38*	*0.38*	*0.39*	*0.39*						
May 1	1936	*0.21*	0.38	0.45▲	0.50	0.57▲	0.63	0.64▲	0.65▲	0.65	*0.66*▲	*0.67*	*0.69*	*0.72*	*0.72*
June 29	"	*0.18*	0.32	0.36▲	*0.38*	*0.38*▲	*0.39*	*0.39*▲	*0.39*▲	*0.39*	*0.40*▲	*0.40*	*0.40*	*0.41*	*0.41*
Aug. 16	"	0.33	0.46	0.46▲	0.46	0.46▲	*0.46*	*0.46*▲	*0.46*▲	*0.46*	*0.46*▲	*0.47*	*0.48*	*0.49*	*0.49*
Sept. 11	"	*0.24*	0.40	0.54▲	0.66	0.76▲	0.86	0.90▲	0.93▲	0.94	0.95▲	0.95	*0.98*	*1.04*	*1.09*

Date		Duration (min)													
		5	10	15	20	25	30	35	40	45	50	60	80	100	120
Sept. 13	1936	0.61	1.11	1.29▲	1.45	1.64▲	1.81	1.90▲	1.98▲	2.04	2.08▲	2.15	2.20	2.26	2.30
Sept. 15	"	0.29	0.38	0.41▲	0.43	0.44▲	0.44	0.44▲	0.45▲	0.48	0.50▲	0.57	0.80	0.84	1.02
Oct. 6	"	0.35	0.54	0.63▲	0.68	0.71▲	0.73	0.76▲	0.79▲	0.82	0.83▲	0.84	1.00	1.08	1.21
June 4	1937	0.21	0.33	0.35▲	0.37	0.40▲	0.41	0.41▲	0.42▲	0.42	0.42▲	0.42	0.42	0.42	0.42
June 13	"	0.28	0.43	0.46▲	0.47	0.48▲	0.49	0.50▲	0.51▲	0.51	0.52▲	0.53	0.54	0.55	0.55
June 13	"	0.41	0.57	0.60▲	0.62	0.62▲	0.62	0.62▲	0.63▲	0.63	0.63▲	0.63	0.63	0.63	0.63
June 21	"	0.34	0.64	0.85▲	1.00	1.10▲	1.20	1.26▲	1.31▲	1.34	1.36▲	1.36	1.37	1.38	1.38
Aug. 19	"	0.21	0.28	0.38▲	0.48	0.51▲	0.53	0.55▲	0.56▲	0.57	0.58▲	0.60	0.61	0.61	0.62
Oct. 4	"	0.22	0.37	0.48▲	0.59	0.66▲	0.72	0.80▲	0.85▲	0.91	0.93▲	0.97	0.98	1.00	1.02
June 1	1938	0.30	0.48	0.54▲	0.58	0.60▲	0.61	0.61▲	0.62▲	0.62	0.62▲	0.62	0.62	0.62	0.62
June 6	"	0.27	0.39	0.50▲	0.57	0.64▲	0.68	0.71▲	0.72▲	0.73	0.74▲	0.76	0.82	0.85	0.87
July 22	"	0.37	0.52	0.57▲	0.61	0.64▲	0.67	0.71▲	0.74▲	0.76	0.76▲	0.76	0.76	0.76	0.76
July 25	"	0.35	0.48	0.56▲	0.62	0.64▲	0.66	0.70▲	0.72▲	0.73	0.78▲	0.89	0.91	0.91	0.93
Aug. 5	"	0.27	0.42	0.47▲	0.50	0.53▲	0.54	0.54▲	0.55▲	0.55	0.58▲	0.64	0.67	0.68	0.69
Sept. 9	"	0.20	0.33	0.45▲	0.56	0.61▲	0.66	0.75▲	0.81▲	0.86	0.88▲	0.91	1.05	1.36	1.46
Sept. 11	"	0.23	0.41	0.56▲	0.65	0.68▲	0.69	0.69▲	0.70▲	0.70	0.71▲	0.73	0.77	0.78	0.79
May 27	1939	0.37	0.64	0.72▲	0.78	0.81▲	0.85	0.93▲	1.05▲	1.15	1.20▲	1.28	1.34	1.39	1.74
June 10	"	0.35	0.49	0.65▲	0.77	0.83▲	0.88	0.91▲	0.94▲	0.96	0.98▲	0.99	1.00	1.01	1.01
July 6	"	0.25	0.39	0.48▲	0.55	0.64▲	0.78	0.91▲	1.03▲	1.14	1.19▲	1.25	1.34	1.36	1.36
Aug. 20	"	0.21	0.30	0.35▲	0.38	0.42▲	0.45	0.47▲	0.50▲	0.51	0.51▲	0.51	0.51	0.52	0.52
Oct. 25	"	0.25	0.39	0.47▲	0.51	0.53▲	0.54	0.55▲	0.56▲	0.56	0.58▲	0.62	0.64	0.68	0.68
Apr. 2	1940	0.17	0.30	0.36▲	0.41	0.48▲	0.55	0.57▲	0.59▲	0.60	0.61▲	0.64	0.68	0.73	0.76
Aug. 10	"	0.31	0.34	0.37▲	0.39	0.47▲	0.56	0.58▲	0.59▲	0.60	0.63▲	0.68	0.75	0.90	1.21
Aug. 12	"	0.24	0.32	0.42▲	0.50	0.54▲	0.56	0.56▲	0.56▲	0.56	0.56▲	0.56	0.56	0.56	0.56
May 15	1941	0.16	0.25	0.35▲	0.44	0.46▲	0.47	0.48▲	0.49▲	0.50	0.50▲	0.51	0.52	0.53	0.53
Aug. 14-15	"	0.25	0.49	0.55▲	0.64	0.64▲	0.65	0.70▲	0.78▲	0.85	0.85▲	0.85	0.85	0.86	0.86
Aug. 30	"	0.27	0.30	0.33▲	0.35	0.37▲	0.39	0.39▲	0.40▲	0.40	0.40▲	0.40	0.41	0.41	0.41
Sept. 3	"	0.50	0.70	0.78▲	0.83	0.83▲	0.84	0.87▲	0.92▲	0.97	0.97▲	0.98	0.98	0.98	0.98
Sept. 9	"	0.35	0.39	0.44▲	0.48	0.53▲	0.57	0.61▲	0.65▲	0.70	0.70▲	0.71	0.71	0.82	0.88
Oct. 22	"	0.17	0.32	0.39▲	0.43	0.49▲	0.56	0.67▲	0.77▲	0.85	0.88▲	0.95	1.00	1.00	1.01
May 11	1942	0.31	0.39	0.41▲	0.42	0.43▲	0.45	0.45▲	0.45▲	0.45	0.46▲	0.46	0.46	0.46	0.46
May 31	"	0.25	0.41	0.62▲	0.78	0.85▲	0.91	1.04▲	1.16▲	1.26	1.30▲	1.35	1.35	1.35	1.35
July 3	"	0.34	0.57	0.71▲	0.78	0.80▲	0.81	0.81▲	0.82▲	0.82	0.82▲	0.82	0.82	0.82	0.82
Aug. 22	"	0.33	0.49	0.49▲	0.49	0.49▲	0.49	0.49▲	0.49▲	0.49	0.49▲	0.49	0.49	0.49	0.49
Sept. 5	"	0.28	0.35	0.37▲	0.40	0.44▲	0.49	0.51▲	0.53▲	0.54	0.57▲	0.66	0.68	0.68	0.68
Apr. 29	1943	0.26	0.29	0.33▲	0.38	0.40▲	0.41	0.41▲	0.41▲	0.41	0.41▲	0.41	0.41	0.41	0.41
June 3	"	0.26	0.32	0.33▲	0.33	0.33▲	0.33	0.33▲	0.34▲	0.34	0.35▲	0.36	0.47	0.48	0.51
June 13	"	0.25	0.28	0.29▲	0.29	0.29▲	0.29	0.29▲	0.29▲	0.29	0.29▲	0.29	0.29	0.29	0.29
July 6	"	0.58	0.92	1.15▲	1.32	1.45▲	1.57	1.76▲	1.97▲	2.17	2.38▲	2.81	2.96	3.36	3.67
Aug. 3	"	0.16	0.28	0.38▲	0.45	0.49▲	0.52	0.54▲	0.57▲	0.60	0.61▲	0.65	0.66	0.68	0.69
May 31	1944	0.50	0.66	0.67▲	0.67	0.70▲	0.72	0.72▲	0.72	0.72▲	0.72▲	0.72	0.74▲	0.83▲	1.02
June 12	"	0.32	0.54	0.72▲	0.82	0.87▲	0.91	0.92▲	0.92▲	0.93▲	0.93▲	0.94	0.94▲	0.94▲	0.95
July 8	"	0.16	0.24	0.36▲	0.45	0.46▲	0.47	0.47▲	0.48▲	0.48▲	0.48▲	0.48	0.48▲	0.48▲	0.48
Aug. 23	"	0.26	0.35	0.38▲	0.39	0.41▲	0.42	0.43▲	0.44▲	0.44▲	0.44▲	0.45			
Aug. 30	"	0.19▲	0.29▲	0.38▲	0.45▲	0.49▲	0.52▲	0.55▲	0.57▲	0.58▲	0.60▲	0.61▲	0.54▲	0.65▲	0.65
June 10	1945	0.42	0.65	0.76▲	0.84▲	0.90▲	0.96	1.00▲	1.03▲	1.08▲	1.08▲	1.12	1.25▲	1.40▲	1.57
Aug. 11	"	0.22	0.40	0.57▲	0.70	0.88▲	1.05	1.11▲	1.14▲	1.17▲	1.19▲	1.20	1.20▲	1.20▲	1.20
Sept. 22	"	0.20	0.29	0.34▲	0.39	0.40▲	0.42	0.43▲	0.44▲	0.44▲	0.45▲	0.46	0.53▲	0.57▲	0.61
June 12	1946	0.31	0.48	0.59▲	0.70▲	0.77▲	0.83	0.86▲	0.88▲	0.90▲	0.92▲	0.95	1.03▲	1.11▲	1.18
June 27	"	0.19	0.32	0.35▲	0.38▲	0.40▲	0.42	0.42▲	0.43▲	0.43▲	0.43▲	0.43	0.43▲	0.43▲	0.43
June 30	"	0.40	0.40	0.40▲	0.40▲	0.40▲	0.40	0.40▲	0.40▲	0.41▲	0.41▲	0.43	0.50▲	0.57▲	0.73
June 9	"	0.37	0.63	0.80▲	0.97▲	1.12▲	1.26	1.43▲	1.60▲	1.76▲	1.95▲	2.15	2.35▲	2.40▲	2.43
Aug. 9	"	0.25	0.50	0.63▲	0.70▲	0.75▲	0.78	0.79▲	0.80▲	0.80▲	0.80▲	0.80	0.85	0.95▲	1.13
Apr. 4-5	1947	0.33	0.58	0.73▲	0.86	0.98▲	1.06	1.10▲	1.15▲	1.19	1.26▲	1.48	1.53	1.75	1.91
July 6	"	0.38	0.60	0.76▲	0.86	1.10▲	1.29	1.43▲	1.55▲	1.66	1.74▲	1.92	2.28	2.32	2.37
July 13	"	0.31	0.44	0.57▲	0.62	0.64▲	0.66	0.68▲	0.70▲	0.72	0.73▲	0.75	0.81	0.84	0.84
Aug. 29	"	0.36	0.60	0.72▲	0.77	0.81▲	0.83	0.85▲	0.87▲	0.89	0.90▲	0.91	0.91	0.91	0.91
Sept. 11	"	0.25	0.50	0.72▲	0.77	0.77▲	0.78	0.78▲	0.78▲	0.78	0.78▲	0.78	0.78	0.78	0.78
Sept. 21	"	0.13	0.23	0.33▲	0.42	0.50▲	0.57	0.62▲	0.67▲	0.72	0.77▲	0.85	0.98	1.13	1.24
Oct. 26	"	0.19	0.29	0.38▲	0.43	0.45▲	0.46	0.48▲	0.49▲	0.49	0.50▲	0.53	0.55	0.56	0.56

APPENDIX IV

MAXIMUM EXCESSIVE PRECIPITATION DATA AT CHICAGO, ILLINOIS, 1913-1947, EXTENDED BY THE EXTENDED DURATION PRINCIPLE

Date		Duration (min)													
		5	10	15	20	25	30	35	40	45	50	60	80	100	120
July 14	1913	0.16	0.29	0.40	0.50	0.59	0.67	0.74	0.79	0.79	0.79	0.79	0.79	0.79	0.79
Aug. 7*	"	0.31	0.37	0.37	0.37	0.37	0.37	0.37	0.37	0.37	0.37	0.37	0.37	0.37	0.37
Aug. 7-8*	"	0.30	0.44	0.56	0.56	0.56	0.56	0.56	0.56	0.56	0.56	0.56	0.56	0.56	0.56
Aug. 18	"	0.28▲	0.49	0.63	0.67	0.67	0.67	0.67	0.67	0.67	0.67	0.67	0.67	0.67	0.67
Apr. 27	1914	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27
May 27	"	0.18	0.33	0.41	0.49	0.49	0.49	0.49	0.49	0.49	0.49	0.49	0.49	0.49	0.49
June 4	"	0.21	0.35	0.40	0.40	0.40	0.40	0.40	0.40	0.40	0.40	0.40	0.40	0.40	0.40
July 16	"	0.33	0.66	0.79	0.97	1.21	1.48	1.61	1.61	1.61	1.61	1.61	1.61	1.61	1.61
Aug. 9	"	0.35	0.62	0.83	0.91	0.91	0.91	0.91	0.91	0.91	0.91	0.91	0.91	0.91	0.91
Aug. 13	"	0.19	0.36	0.50	0.60	0.68	0.68	0.68	0.68	0.68	0.68	0.68	0.68	0.68	0.68
Sept. 1	"	0.14	0.27	0.38	0.40	0.40	0.40	0.40	0.40	0.40	0.40	0.40	0.40	0.40	0.40
May 15,	1915	0.17	0.25	0.32	0.40	0.48	0.48	0.48	0.48	0.48	0.48	0.48	0.48	0.48	0.48
June 12	"	0.18	0.31	0.46	0.56	0.72	0.82	0.89	0.92	0.98	0.98	0.98	0.98	0.98	0.98
July 7	"	0.20	0.33	0.45	0.45	0.45	0.45	0.45	0.45	0.45	0.45	0.45	0.45	0.45	0.45
July 18	"	0.21	0.29	0.37	0.37	0.37	0.37	0.37	0.37	0.37	0.37	0.37	0.37	0.37	0.37
Aug. 3	"	0.16	0.29	0.44	0.52	0.58	0.58	0.58	0.58	0.58	0.58	0.58	0.58	0.58	0.58
Aug. 23	"	0.19	0.36	0.45	0.45	0.45	0.45	0.45	0.45	0.45	0.45	0.45	0.45	0.45	0.45
Sept. 10	"	0.26	0.32	0.37	0.37	0.37	0.37	0.37	0.37	0.37	0.37	0.37	0.37	0.37	0.37
May 14	1916	0.30	0.32	0.32	0.32	0.32	0.32	0.32	0.32	0.32	0.32	0.32	0.32	0.32	0.32
July 19	"	0.35	0.58	0.69	0.84	0.89	0.93	1.05	1.08	1.08	1.08	1.08	1.08	1.08	1.08
Aug. 20	"	0.30	0.36	0.36	0.36	0.36	0.36	0.36	0.36	0.36	0.36	0.36	0.36	0.36	0.36
Sept. 5	"	0.17	0.31	0.37	0.43	0.43	0.43	0.43	0.43	0.43	0.43	0.43	0.43	0.43	0.43
May 19	1917	0.12	0.22	0.26	0.37	0.47	0.47	0.47	0.47	0.47	0.47	0.47	0.47	0.47	0.47
June 28	"	0.24	0.32	0.32	0.32	0.32	0.32	0.32	0.32	0.32	0.32	0.32	0.32	0.32	0.32
July 26	"	0.23	0.41	0 57	0.57	0.57	0.57	0.57	0.57	0.57	0.57	0.57	0.57	0.57	0.57
July 24	1918	0.24	0.38	0 41	0.43	0.43	0 43	0.43	0.43	0.43	0.43	0.43	0.43	0.43	0.43
July 28	"	0.23	0.47	0.53	0.53	0.53	0.53	0.53	0.53	0.53	0.53	0.53	0.53	0.53	0.53
June 11	1919	0.29	0.42	0.42	0.42	0.42	0.42	0.42	0.42	0.42	0.42	0.42	0.42	0.42	0.42
June 14	"	0.19	0.35	0 39	0.39	0.39	0.39	0.39	0.39	0.39	0.39	0.39	0.39	0.39	0.39
June 19	"	0.21	0.33	0 40	0.40	0.40	0.40	0.40	0.40	0.40	0.40	0.40	0.40	0.40	0.40
Sept. 10	"	0.24	0 38	0 47	0.47	0.47	0.47	0.47	0.47	0.47	0.47	0.47	0.47	0.47	0.47
Oct. 5	"	0.41	0.74	0 93	1.10	1 29	1.44	1.53	1.53	1 53	1.53	1.53	1.53	1.53	1.53
Oct. 9-10	"	0.11	0.19	0.27	0.34	0.43	0.50	0.56	0.61	0.61	0.61	0.61	0.61	0.61	0.61
June 14	1920	0.22	0.41	0.41	0.41	0.41	0.41	0.41	0.41	0.41	0.41	0.41	0.41	0.41	0.41
June 14	"	0.21	0.39	0.43	0.43	0.43	0.43	0.43	0.43	0.43	0.43	0.43	0.43	0.43	0.43
June 29	"	0.53	0.53	0.53	0.53	0.53	0.53	0.53	0.53	0.53	0.53	0.53	0.53	0.53	0.53
July 13	"	0.16	0.25	0.32	0.37	0.50	0.54	0.54	0.54	0.54	0.54	0.54	0.54	0.54	0.54
Sept. 5	"	0.39	0.52	0 69	0.88	1.05	1.12	1.16	1.22	1.22	1.22	1.22	1.22	1.22	1.22
July 7	1921	0.39	0.76	1 08	1.35	1.45	1.49	1.49	1.49	1.49	1.49	1.49	1.49	1.49	1.49
Aug. 19	"	0.22	0.38	0.58	0.58	0.58	0.58	0.58	0.58	0.65	0.74	1.12	1.45	1.45	1.45
Sept. 2	"	0.23	0.42	0.48	0.48	0 48	0.48	0.48	0.48	0.48	0.48	0.48	0.48	0.48	0.48
Sept. 29	"	0.28	0.32	0.32	0.32	0.32	0.32	0.32	0.32	0.32	0.32	0.32	0.32	0.32	0.32
Sept. 29	"	0.25	0.34	0.34	0.34	0.34	0.34	0.34	0.34	0.34	0.34	0.34	0 34	0.34	0.34
Apr. 10	1922	0.15	0.26	0.34	0.40	0.40	0.40	0.40	0.40	0.40	0.40	0.40	0.40	0.40	0.40
Apr. 10	"	0.15	0.30	0.39	0.50	0.55	0.60	0.65	0.69	0.73	0.78	0.78	0.78	0.78	0.78
July 11	"	0.25	0.42	0.61	0.72	0.82	0.88	0.92	0.98	1.06	1.12	1.21	1.21	1.21	1.21
July 22	"	0.19	0.36	0.41	0.41	0.41	0.41	0.41	0.41	0.41	0.41	0.41	0.41	0.41	0.41
Aug. 24	"	0.29	0.57	0.69	0 77	0.79	0.79	0.79	0.79	0.79	0.79	0.79	0.79	0.79	0.79
Sept. 10	"	0.23	0.44	0.65	0.66	0.67	0.79	0.94	1.10	1.14	1.23	1.53	1.55	1.55	1.55
June 25	1923	0.17	0 30	0.35	0.35	0.35	0.35	0.35	0.35	0 35	0.35	0.35	0.35	0.35	0.35
July 6	"	0.14	0.27	0.34	0.42	0.45	0 45	0.45	0.45	0.45	0.45	0.45	0.45	0.45	0.45
Aug. 7	"	0.20	0.30	0.30	0.30	0 30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30
Aug. 11	"	0.45	0.80	1.12	1 39	1.49	1.61	1.71	1 77	1.80	1.85	2.02	2.30	2.30	2.30
Aug. 11	"	0.27	0.50	0.66	0.82	0.82	0.82	0.82	0.82	0.82	0.82	0.82	0.82	0.82	0.82

Date		Duration (min)													
		5	10	15	20	25	30	35	40	45	50	60	80	100	120
Aug. 27	1923	0.21	0.31	0.40	0.40	0.40	0.40	0.40	0.40	0.40	0.40	0.40	0.40	0.40	0.40
Aug. 27	"	0.29	0.45	0.56	0.56	0.56	0.56	0.56	0.56	0.56	0.56	0.56	0.56	0.56	0.56
July 20	1924	0.39	0.66	0.86	1.05	1.25	1.43	1.59	1.75	1.75	1.75	1.75	1.75	1.75	1.75
Aug. 4	"	0.36	0.59	0.69	0.87	0.96	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02
Aug. 5	"	0.23	0.35	0.47	0.57	0.69	0.80	0.88	1.08	1.12	1.15	1.22	1.54	1.54	1.54
Aug. 5	"	0.16	0.27	0.36	0.45	0.45	0.45	0.45	0.45	0.45	0.45	0.45	0.45	0.45	0.45
Aug. 6	"	0.19	0.34	0.41	0.41	0.41	0.41	0.41	0.41	0.41	0.41	0.41	0.41	0.41	0.41
Aug. 8	"	0.34	0.34	0.34	0.34	0.34	0.34	0.34	0.34	0.34	0.34	0.34	0.34	0.34	0.34
Apr. 13	1925	0.26	0.38	0.38	0.38	0.38	0.38	0.38	0.38	0.38	0.38	0.38	0.38	0.38	0.38
May 16	"	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
June 15	"	0.21	0.40	0.52	0.57	0.57	0.65	0.69	0.70	0.82	0.87	0.87	0.87	0.87	0.87
June 17	"	0.23	0.40	0.53	0.53	0.53	0.53	0.53	0.53	0.53	0.53	0.53	0.53	0.53	0.53
June 24	"	0.19	0.36	0.51	0.58	0.69	0.78	0.96	1.13	1.25	1.25	1.25	1.25	1.25	1.25
Aug. 13	"	0.41	0.68	0.84	1.05	1.21	1.31	1.38	1.38	1.38	1.38	1.38	1.38	1.38	1.38
Sept. 8	"	0.20	0.32	0.41	0.47	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52
May 18	1926	0.38	0.68	0.82	0.95	1.04	1.04	1.04	1.04	1.04	1.04	1.04	1.04	1.04	1.04
June 11	"	0.22	0.32	0.43	0.52	0.62	0.67	0.67	0.67	0.67	0.67	0.67	0.67	0.67	0.67
June 11	"	0.30	0.47	0.54	0.54	0.54	0.54	0.54	0.54	0.54	0.54	0.54	0.54	0.54	0.54
June 11	"	0.17	0.34	0.34	0.34	0.34	0.34	0.34	0.34	0.34	0.34	0.34	0.34	0.34	0.34
June 13	"	0.33	0.59	0.87	1.13	1.34	1.55	1.77	1.85	1.85	1.85	1.85	1.85	1.85	1.85
June 25	"	0.35	0.53	0.69	0.77	0.93	1.03	1.09	1.09	1.09	1.09	1.09	1.09	1.09	1.09
Sept. 23	"	0.13	0.26	0.33	0.37	0.45	0.50	0.55	0.59	0.63	0.68	0.81	0.81	0.81	0.81
May 9	1927	0.23	0.38	0.44	0.44	0.44	0.44	0.44	0.44	0.44	0.44	0.44	0.44	0.44	0.44
May 24	"	0.11	0.21	0.30	0.39	0.46	0.46	0.46	0.46	0.46	0.46	0.46	0.46	0.46	0.46
July 1	"	0.30	0.55	0.77	0.94	1.05	1.11	1.11	1.11	1.11	1.11	1.11	1.11	1.11	1.11
July 6	"	0.24	0.32	0.32	0.32	0.32	0.32	0.32	0.32	0.32	0.32	0.32	0.32	0.32	0.32
Aug. 8	"	0.48	0.61	0.73	0.79	0.87	0.87	0.87	0.87	0.87	0.87	0.87	0.87	0.87	0.87
Sept. 17	"	0.32	0.51	0.63	0.69	0.75	0.81	0.81	0.81	0.81	0.81	0.81	0.81	0.81	0.81
Sept. 17	"	0.17	0.26	0.40	0.49	0.58	0.64	0.69	0.70	0.74	0.80	0.97	0.97	0.97	0.97
June 20	1928	0.50	0.88	1.16	1.38	1.58	1.70	1.86	1.92	2.04	2.06	2.30	2.30	2.30	2.30
June 24	"	0.16	0.31	0.37	0.41	0.47	0.57	0.64	0.67	0.71	0.78	0.99	0.99	0.99	0.99
July 2	"	0.24	0.41	0.41	0.41	0.41	0.41	0.41	0.41	0.41	0.41	0.41	0.41	0.41	0.41
July 3	"	0.22	0.43	0.54	0.64	0.70	0.78	0.78	0.78	0.78	0.78	0.78	0.78	0.78	0.78
Aug. 3	"	0.27	0.47	0.55	0.61	0.61	0.61	0.61	0.61	0.61	0.61	0.61	0.61	0.61	0.61
Aug. 29	"	0.28	0.48	0.59	0.83	1.02	1.20	1.30	1.30	1.30	1.30	1.30	1.30	1.30	1.30
Sept. 14	"	0.15	0.29	0.40	0.50	0.54	0.61	0.61	0.61	0.61	0.61	0.61	0.61	0.61	0.61
Mar. 31	1929	0.12	0.21	0.28	0.34	0.42	0.50	0.56	0.64	0.72	0.79	0.94	1.23	1.23	1.23
June 11	"	0.24	0.40	0.52	0.63	0.72	0.77	0.77	0.77	0.77	0.77	0.77	0.77	0.77	0.77
June 11	"	0.28	0.39	0.46	0.68	0.78	0.84	0.84	0.84	0.84	0.84	0.84	0.84	0.84	0.84
June 27	"	0.30	0.49	0.54	0.54	0.54	0.54	0.54	0.54	0.54	0.54	0.54	0.54	0.54	0.54
Aug. 10	"	0.21	0.39	0.46	0.46	0.46	0.46	0.46	0.46	0.46	0.46	0.46	0.46	0.46	0.46
Aug. 26	"	0.26	0.46	0.63	0.74	0.74	0.74	0.74	0.88	0.88	0.88	0.88	0.88	0.88	0.88
Sept. 28	"	0.25	0.39	0.48	0.51	0.58	0.58	0.58	0.58	0.58	0.58	0.58	0.58	0.58	0.58
June 5	1930	0.20	0.32	0.32	0.32	0.32	0.32	0.32	0.32	0.32	0.32	0.32	0.32	0.32	0.32
July 5	"	0.19	0.33	0.40	0.41	0.43	0.55	0.58	0.63	0.67	0.72	0.82	0.96	0.96	0.96
June 23	1931	0.26	0.49	0.67	0.76	0.88	0.96	0.99	1.00	1.00	1.02	1.13	1.22	1.36	1.53
July 19	"	0.32	0.55	0.76	0.91	0.98	1.04	1.17	1.24	1.31	1.40	1.40	1.40	1.40	1.40
July 21	"	0.28	0.40	0.54	0.66	0.66	0.66	0.66	0.66	0.66	0.66	0.66	0.66	0.66	0.66
Aug. 10-11	"	0.37	0.43	0.55	0.63	0.73	0.78	0.83	0.92	1.05	1.17	1.27	1.27	1.27	1.27
Aug. 11	"	0.51	0.96	1.22	1.39	1.39	1.39	1.39	1.39	1.39	1.39	1.39	1.39	1.39	1.39
Aug. 27	"	0.42	0.74	0.96	1.21	1.27	1.27	1.27	1.27	1.27	1.27	1.27	1.27	1.27	1.27
Sept. 2	"	0.22	0.43	0.55	0.66	0.69	0.78	0.85	0.90	0.90	0.90	0.90	0.90	0.90	0.90
Sept. 25	"	0.24	0.47	0.62	0.70	0.74	0.81	0.81	0.81	0.81	0.81	0.81	0.81	0.81	0.81
May 7	1932	0.22	0.37	0.48	0.54	0.59	0.59	0.59	0.59	0.59	0.59	0.59	0.59	0.59	0.59
June 5	"	0.23	0.46	0.59	0.69	0.78	0.78	0.78	0.78	0.78	0.78	0.78	0.78	0.78	0.78
June 26	"	0.55	0.94	1.16	1.16	1.16	1.16	1.16	1.16	1.16	1.16	1.16	1.16	1.16	1.16
June 30	"	0.24	0.43	0.43	0.43	0.43	0.43	0.43	0.43	0.43	0.43	0.43	0.43	0.43	0.43
July 7	"	0.29	0.56	0.76	0.76	0.76	0.76	0.76	0.76	0.76	0.76	0.76	0.76	0.76	0.76
July 7	"	0.20	0.31	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35
July 26	"	0.15	0.23	0.33	0.39	0.49	0.57	0.72	0.78	0.78	0.78	0.78	0.78	0.78	0.78
June 29	1933	0.37	0.68	0.80	0.96	1.18	1.38	1.53	1.81	2.03	2.03	2.03	2.03	2.03	2.03
July 2	"	0.41	0.80	1.00	1.16	1.16	1.16	1.16	1.16	1.16	1.16	1.16	1.16	1.16	1.16
June 22	1934	0.26	0.44	0.63	0.72	0.72	0.72	0.72	0.72	0.72	0.72	0.72	0.72	0.72	0.72
Aug. 15	"	0.36	0.62	0.82	0.95	1.07	1.19	1.21	1.23	1.23	1.23	1.23	1.23	1.23	1.23
Aug. 23	"	0.22	0.35	0.40	0.45	0.48	0.50	0.50	0.51	0.51	0.51	0.51	0.51	0.51	0.51
Sept. 29	"	0.21	0.35	0.53	0.64	0.72	0.78	0.85	0.88	0.88	0.88	0.88	0.88	0.88	0.88
May 12	1935	0.21	0.42	0.47	0.60	0.67	0.77	0.89	0.95	0.95	0.95	0.95	0.95	0.95	0.95
June 16*	"	0.15	0.28	0.39	0.45	0.45	0.46	0.46	0.47	0.47	0.47	0.47	0.47	0.47	0.47
June 16*	"	0.11	0.22	0.31	0.38	0.47	0.47	0.47	0.47	0.47	0.47	0.47	0.47	0.47	0.47
June 18	"	0.38	0.67	0.86	0.94	0.95	0.96	1.02	1.03	1.03	1.03	1.03	1.03	1.03	1.03
June 26	"	0.27	0.28	0.28	0.28	0.28	0.28	0.28	0.28	0.28	0.28	0.28	0.28	0.28	0.28
July 5	"	0.26	0.41	0.46	0.48	0.49	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
July 23	"	0.21	0.30	0.36	0.37	0.38	0.39	0.40	0.40	0.40	0.40	0.40	0.40	0.40	0.40

Date		Duration (min)													
		5	10	15	20	25	30	35	40	45	50	60	80	100	120
July 25	1935	0.19	0.31	0.32	0.32	0.32	0.32	0.32	0.32	0.32	0.32	0.32	0.32	0.32	0.32
July 28	"	0.20	0.36	0.48	0.58	0.59	0.59	0.60	0.60	0.60	0.60	0.60	0.60	0.60	0.60
Aug. 2	"	0.37	0.71	0.87	0.96	1.03	1.05	1.05	1.06	1.06	1.06	1.06	1.06	1.06	1.06
Aug. 17	"	0.22	0.30	0.37	0.38	0.38	0.38	0.39	0.39	0.39	0.39	0.39	0.39	0.39	0.39
May 1	1936	0.21	0.38	0.45▲	0.50	0.57▲	0.63	0.64▲	0.65▲	0.65	0.66▲	0.67	0.69	0.72	0.72
June 29	"	0.18	0.32	0.36▲	0.38	0.38▲	0.39	0.39▲	0.39▲	0.39	0.40▲	0.40	0.40	0.41	0.41
Aug. 16	"	0.33	0.46	0.46▲	0.46	0.46▲	0.46	0.46▲	0.46▲	0.46	0.46▲	0.47	0.48	0.49	0.49
Sept. 11	"	0.24	0.40	0.54▲	0.66	0.76▲	0.86	0.90▲	0.93▲	0.94	0.95▲	0.95	0.98	1.04	1.09
Sept. 13	"	0.61	1.11	1.29▲	1.45	1.64▲	1.81	1.90▲	1.98▲	2.04	2.08▲	2.15	2.20	2.26	2.30
Sept. 15	"	0.29	0.38	0.41▲	0.43	0.44▲	0.44	0.44▲	0.45▲	0.48	0.50▲	0.57	0.80	0.84	1.02
Oct. 6	"	0.35	0.54	0.63▲	0.68	0.71▲	0.73	0.76▲	0.79▲	0.82	0.83▲	0.84	1.00	1.08	1.21
June 4	1937	0.21	0.33	0.35▲	0.37	0.40▲	0.41	0.41▲	0.42▲	0.42	0.42	0.42	0.42	0.42	0.42
June 13	"	0.28	0.43	0.46▲	0.47	0.48▲	0.49	0.50▲	0.51▲	0.51	0.52▲	0.53	0.54	0.55	0.55
June 13	"	0.41	0.57	0.60▲	0.62	0.62▲	0.62	0.62▲	0.63▲	0.63	0.63	0.63	0.63	0.63	0.63
June 21	"	0.34	0.64	0.85▲	1.00	1.10▲	1.20	1.26▲	1.31▲	1.34	1.35▲	1.36	1.37	1.38	1.38
Aug. 19	"	0.21	0.28	0.38▲	0.48	0.51▲	0.53	0.55▲	0.56▲	0.57	0.58▲	0.60	0.61	0.61	0.62
Oct. 4	"	0.22	0.37	0.48▲	0.59	0.66▲	0.72	0.80▲	0.85▲	0.91	0.93▲	0.97	0.98	1.00	1.02
June 1	1938	0.30	0.48	0.54▲	0.58	0.60▲	0.61	0.61▲	0.62▲	0.62	0.62	0.62	0.62	0.62	0.62
June 6	"	0.27	0.39	0.50▲	0.57	0.64▲	0.68	0.71▲	0.72▲	0.73	0.74▲	0.76	0.82	0.85	0.87
July 22	"	0.37	0.52	0.57▲	0.61	0.64▲	0.67	0.71▲	0.74▲	0.76	0.76	0.76	0.76	0.76	0.76
July 25	"	0.35	0.48	0.56▲	0.62	0.64▲	0.66	0.70▲	0.72▲	0.73	0.78▲	0.89	0.91	0.91	0.93
Aug. 5	"	0.27	0.42	0.47▲	0.50	0.53▲	0.54	0.54▲	0.55▲	0.55	0.58▲	0.64	0.67	0.68	0.69
Sept. 9	"	0.20	0.33	0.45▲	0.56	0.61▲	0.66	0.75▲	0.81▲	0.86	0.88▲	0.91	1.05	1.36	1.46
Sept. 11	"	0.23	0.41	0.56▲	0.65	0.68▲	0.69	0.69▲	0.70▲	0.70	0.71▲	0.73	0.77	0.78	0.79
May 27	1939	0.37	0.64	0.72▲	0.78	0.81▲	0.85	0.93▲	1.05▲	1.15	1.20▲	1.28	1.34	1.34	1.74
June 10	"	0.35	0.49	0.65▲	0.77	0.83▲	0.88	0.91▲	0.94▲	0.96	0.98▲	0.99	1.00	1.01	1.01
July 6	"	0.25	0.39	0.48▲	0.55	0.64▲	0.78	0.91▲	1.03▲	1.14	1.19▲	1.25	1.34	1.36	1.36
Aug. 20	"	0.21	0.30	0.35▲	0.38	0.42▲	0.45	0.47▲	0.50▲	0.51	0.51▲	0.51	0.51	0.52	0.52
Oct. 25	"	0.25	0.39	0.47▲	0.51	0.53▲	0.54	0.55▲	0.56▲	0.56	0.58▲	0.62	0.64	0.68	0.68
Apr. 2	1940	0.17	0.30	0.36▲	0.41	0.48▲	0.55	0.57▲	0.59▲	0.60	0.61▲	0.64	0.68	0.73	0.76
Aug. 10	"	0.31	0.34	0.37▲	0.39	0.47▲	0.56	0.58▲	0.59▲	0.60	0.63▲	0.68	0.75	0.90	1.21
Aug. 12	"	0.24	0.32	0.42▲	0.50	0.54▲	0.56	0.56	0.56	0.56	0.56	0.56	0.56	0.56	0.56
May 15	1941	0.16	0.25	0.35▲	0.44	0.46▲	0.47	0.48▲	0.49▲	0.50	0.50▲	0.51	0.52	0.53	0.53
Aug. 14-15	"	0.25	0.49	0.55▲	0.64	0.64▲	0.65	0.70▲	0.78▲	0.85	0.85▲	0.85	0.85	0.86	0.86
Aug. 30	"	0.27	0.30	0.33▲	0.35	0.37▲	0.39	0.39▲	0.40▲	0.40	0.40▲	0.41	0.41	0.41	0.41
Sept. 3	"	0.50	0.70	0.78▲	0.83	0.83▲	0.84	0.87▲	0.92▲	0.97	0.97▲	0.98	0.98	0.98	0.98
Sept. 9	"	0.35	0.39	0.44▲	0.48	0.53▲	0.67	0.61▲	0.65▲	0.70	0.70▲	0.71	0.71	0.82	0.88
Oct. 22	"	0.17	0.32	0.39▲	0.43	0.49▲	0.56	0.67▲	0.77▲	0.85	0.88▲	0.95	1.00	1.00	1.01
May 11	1942	0.31	0.39	0.41▲	0.42	0.43▲	0.45	0.45▲	0.45▲	0.45	0.46▲	0.46	0.46	0.46	0.46
May 31	"	0.25	0.41	0.62▲	0.78	0.85▲	0.91	1.04▲	1.16▲	1.26	1.30▲	1.35	1.35	1.35	1.35
July 3	"	0.34	0.57	0.71▲	0.78	0.80▲	0.81	0.81▲	0.82▲	0.82	0.82	0.82	0.82	0.82	0.82
Aug. 22	"	0.33	0.49	0.49	0.49	0.49	0.49	0.49	0.49	0.49	0.49	0.49	0.49	0.49	0.49
Sept. 5	"	0.28	0.35	0.37▲	0.40	0.44▲	0.49	0.51▲	0.53▲	0.54	0.57▲	0.66	0.68	0.68	0.68
Apr. 29	1943	0.26	0.29	0.33▲	0.38	0.40▲	0.41	0.41	0.41	0.41	0.41	0.41	0.41	0.41	0.41
June 3	"	0.26	0.32	0.33▲	0.33	0.33▲	0.33	0.33▲	0.34▲	0.34	0.35▲	0.36	0.47	0.48	0.51
June 13	"	0.25	0.28	0.29▲	0.29	0.29	0.29	0.29	0.29	0.29	0.29	0.29	0.29	0.29	0.29
July 6	"	0.58	0.92	1.15▲	1.32	1.45▲	1.57	1.76▲	1.97▲	2.17	2.38▲	2.81	2.96	3.36	3.36
Aug. 3	"	0.16	0.28	0.38▲	0.45	0.49▲	0.52	0.54▲	0.57▲	0.60	0.61▲	0.65	0.66	0.68	0.69
May 31	1944	0.50	0.66	0.67▲	0.67	0.70▲	0.72	0.72▲	0.72	0.72▲	0.72▲	0.72▲	0.74▲	0.83▲	1.02
June 12	"	0.32	0.54	0.72▲	0.82	0.87▲	0.91	0.92▲	0.92▲	0.93▲	0.93▲	0.94	0.94▲	0.94▲	0.95
July 8	"	0.16	0.24	0.36▲	0.45	0.46▲	0.47	0.47▲	0.48▲	0.48	0.48	0.48	0.48	0.48	0.48
Aug. 23	"	0.26	0.35	0.38▲	0.39	0.41▲	0.42	0.43▲	0.44▲	0.44▲	0.44▲	0.45	0.45	0.45	0.45
Aug. 30	"	0.19▲	0.29	0.38▲	0.45▲	0.49▲	0.52▲	0.55▲	0.57▲	0.58▲	0.60▲	0.61▲	0.64▲	0.65▲	0.65
June 10	1945	0.42	0.65	0.76▲	0.84▲	0.90▲	0.96	1.00▲	1.03▲	1.08▲	1.08▲	1.12	1.25▲	1.40▲	1.57
Aug. 11	"	0.22	0.40	0.57▲	0.70	0.88▲	1.05	1.11▲	1.14▲	1.17▲	1.19▲	1.20	1.20	1.20	1.20
Sept. 22	"	0.20	0.29	0.34▲	0.39	0.40▲	0.42	0.43▲	0.44▲	0.44▲	0.45▲	0.46	0.53▲	0.57▲	0.61
June 12	1946	0.31	0.48	0.59▲	0.70▲	0.77▲	0.83	0.86▲	0.88▲	0.90▲	0.92▲	0.95	1.03▲	1.11▲	1.18
June 27	"	0.19	0.32	0.35▲	0.38▲	0.40▲	0.42	0.42▲	0.43▲	0.43	0.43	0.43	0.43	0.43	0.43
June 30	"	0.40	0.40	0.40▲	0.40▲	0.40▲	0.40	0.40▲	0.40▲	0.41	0.41	0.43	0.50▲	0.57▲	0.73
June 9	"	0.37	0.63	0.80▲	0.97▲	1.12▲	1.26	1.43▲	1.60▲	1.76▲	1.95▲	2.15	2.35▲	2.40▲	2.43
Aug. 9	"	0.25	0.50	0.63▲	0.70▲	0.75▲	0.78	0.79▲	0.80▲	0.80▲	0.80▲	0.80	0.85	0.95▲	1.13
Apr. 4-5	1947	0.33	0.58	0.73▲	0.86	0.98▲	1.06	1.10▲	1.15▲	1.19	1.26▲	1.48	1.53	1.75	1.91
July 6	"	0.38	0.60	0.76▲	0.86	1.10▲	1.29	1.43▲	1.55▲	1.66	1.74▲	1.92	2.28	2.32	2.37
July 13	"	0.31	0.44	0.57▲	0.62	0.64▲	0.66	0.68▲	0.70▲	0.72	0.73▲	0.75	0.81	0.84	0.84
Aug. 29	"	0.36	0.60	0.72▲	0.77	0.81▲	0.83	0.85▲	0.87▲	0.89▲	0.90▲	0.91	0.91	0.91	0.91
Sept. 11	"	0.25	0.50	0.72▲	0.77	0.77▲	0.78	0.78	0.78	0.78	0.78	0.78	0.78	0.78	0.78
Sept. 21	"	0.13	0.23	0.33▲	0.42	0.50▲	0.57	0.62▲	0.67▲	0.72	0.77▲	0.85	0.98	1.13	1.24
Oct. 26	"	0.19	0.29	0.38▲	0.43	0.45▲	0.46	0.48▲	0.49▲	0.49	0.50▲	0.53	0.55	0.56	0.56

* Corrections for Dependency. (Owing to the dependent nature of the two storms occurring on the same day, the two storms are considered as one storm and the larger value of the two is used in the analyses.)

Aug. 7-8	1913	0.31	0.44	0.56	0.56	0.56	0.56	0.56	0.56	0.56	0.56	0.56	0.93	0.93	0.93
June 16	1935	0.15	0.28	0.39	0.45	0.47	0.47	0.47	0.47	0.47	0.47	0.47	0.57	0.94	0.94

APPENDIX V
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