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TECHNICAL NOTE

An improvement on Hvorslev's shape factors

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INTRODUCTION

The shape factors listed by Hvorslev (1951) are widely used to measure in situ hydraulic conductivity (e.g. discussion in Ratnam *et al.*, 2001). The shape factor F [L] is defined by the general equation

$$F = \frac{Q}{KH} \tag{1}$$

where Q [$\mathrm{L^3T^{-1}}$] is the flow rate of water from a piezometer, K [$\mathrm{LT^{-1}}$] is the hydraulic conductivity of the aquifer, and H [L] is the constant applied head-difference between the piezometer and the unperturbed aquifer.

Lowther (1978) commented that the table of shape factors provided by Hvorslev (1951, fig. 12) is inconsistent. Specifically, consider the shape factors for Hvorslev's Cases 7 (well point or hole extended at impervious boundary), 2 (hemispherical soil bottom at impervious boundary) and 3 (soil flush with bottom at impervious boundary), F_7, F_2 and F_3 respectively:

$$F_7 = \frac{2\pi L}{\ln\left[(L/R) + \sqrt{1 + (L/R)^2}\right]} \tag{2}$$

$$F_2 = 2\pi R \tag{3}$$

$$F_3 = 4R \tag{4}$$

where L [L] and R [L] are the length and radius of the well-screen respectively.

When $L = R$, F_7 should equal F_2, and when $L = 0$, F_7 should equal F_3, but instead

$$\lim_{L \to R} F_7 = \frac{2R}{\ln\left(1 + \sqrt{2}\right)} \quad \text{and} \quad \lim_{L \to 0} F_7 = 2R \tag{5}$$

THE IMPROVEMENT

Youngs (1980) states that this discrepancy is due to the approximate nature of the Case 7 shape factor, which was derived on the basis of flow from a line source for which the equipotentials are hemispheroids (Dachler, 1936) (see Fig. 1) (note that a spheroid is a special case of an ellipsoid). However, the constant spheroidal equipotential problem was solved by Moon & Spencer (1961, p. 242) from which the exact solution for Case 7 can be obtained:

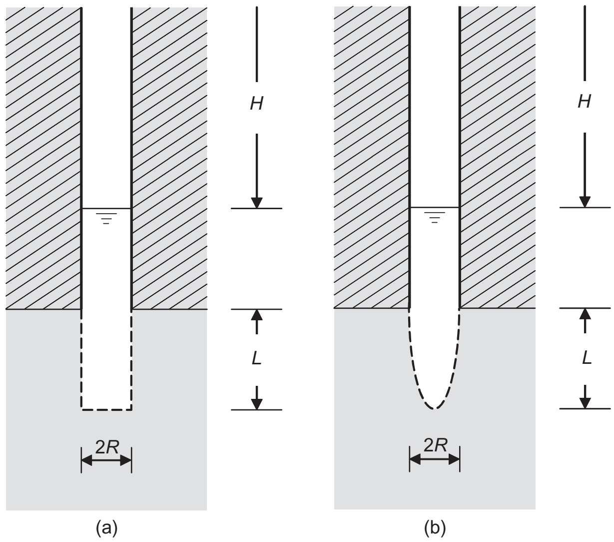

Fig. 1. Comparison between the actual and simulated geometries for Hvorslev's (1951) Case 7: (a) well point or hole extended at impervious boundary; (b) well-screen replaced with an equivalent hemispheroid

$$F_7 = \frac{2\pi R}{\sinh[\operatorname{arctanh}(R/L)]\ln\left\{\coth\left[\frac{1}{2}\operatorname{arctanh}(R/L)\right]\right\}} \tag{6}$$

In contrast to equation (2), equation (6) has the correct limits:

$$\lim_{L \to R} F_7 = 2\pi R, \quad \text{and} \quad \lim_{L \to 0} F_7 = 4R \tag{7}$$

Furthermore, for large L/R, equation (6) reduces to

$$F_7 = \frac{2\pi L}{\ln(2L/R)}, \ \frac{L}{R} \gg 1 \tag{9}$$

which also describes the asymptotic behaviour of equation (2).

CONSIDERATION OF HVORSLEV'S CASE 8

Equation (6) can be modified to deal with Hvorslev's Case 8 (well point extended in uniform soil) (see Fig. 2) by replacing the radius R for the well-screen diameter, $D = 2R$, such that

$$F_8 = \frac{2\pi D}{\sinh[\operatorname{arctanh}(D/L)]\ln\left(\coth\left[\frac{1}{2}\operatorname{arctanh}(D/L)\right]\right)} \tag{10}$$

which has the limits

$$\lim_{L \to D} F_8 = 2\pi D \quad \text{and} \quad \lim_{L \to 0} F_8 = 4D \tag{11}$$

The limit when $L \to D$ corresponds exactly with Hvorslev's Case 1 (spherical intake or well point in uniform soil). The limit when $L \to 0$ should correspond with Hvorslev's Case 4 (soil flush with bottom in uniform soil) (i.e. $F_4 = 2{\cdot}75\ D$, which was obtained empirically: Harza (1935) and Taylor

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Discussion on this paper closes on 1 June 2007, for further details see p. ii.
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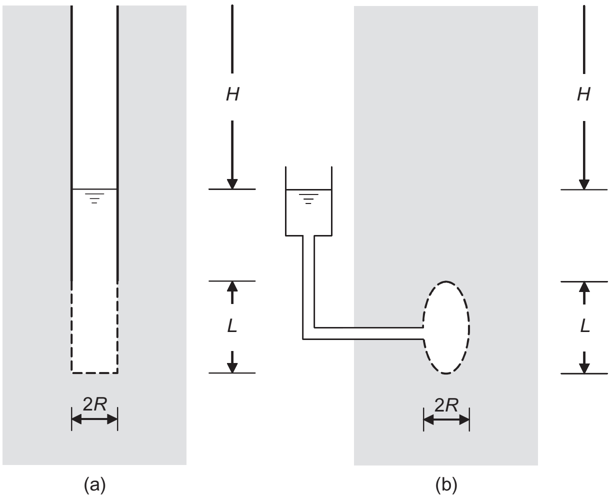

Fig. 2. Comparison between the actual and simulated geometries for Hvorslev's (1951) Case 8: (a) well point or hole extended in uniform soil; (b) well-screen replaced with an equivalent spheroid

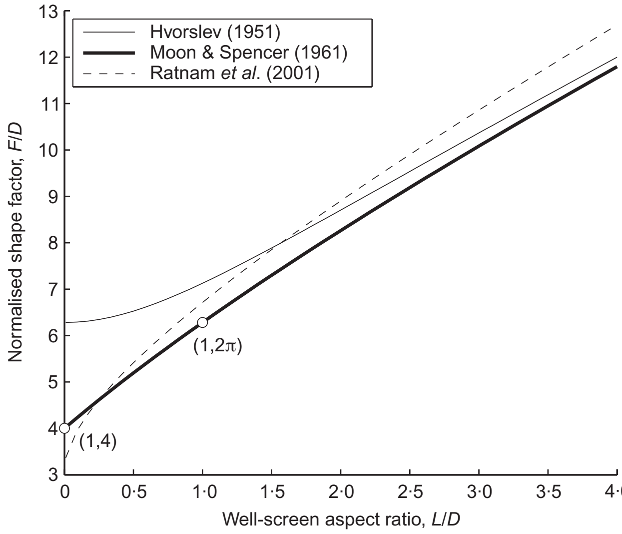

Fig. 3: Comparison of Hvorslev (1951, equation (12)), Moon & Spencer (1961, equation (10)) and Ratnam *et al.* (2001, equation (13))

(1948)). This is not the case, because the spheroidal approximation fails to incorporate the presence of the impermeable casing (see Fig. 2).

Note that Hvorslev's formula for Case 8 is

$$F_8 = \frac{2\pi L}{\ln\left[(L/D) + \sqrt{1 + (L/D)^2}\right]} \tag{12}$$

Ratnam *et al.* (2001) obtained a shape-factor formula through regression analysis of results from a finite element model for the geometry depicted in Fig. 2(a) as

$$F_8 = \left[1{\cdot}1872(L/D) + 2{\cdot}4135(L/D)^{1/2} + 3{\cdot}1146\right]D \tag{13}$$

Figure 3 compares equations (10), (12) and (13). It can be seen that equation (10) passes through the points defined by equation (11) and approaches equation (12) for large *L/D*. The exact solution (equation (10)) corresponds much more closely with equation (13) (derived from finite element analysis), although there are still discrepancies owing to Hvorslev's assumption that the well-screen geometry can be approximated as a spheroid.

SUMMARY

Hvorslev's (1951, fig. 12) shape factors have been widely used for deriving in situ measurements of hydraulic conductivity. However, the shape factors for Hvorslev's Cases 7 and 8 are incorrect as they are incompatible with the special Cases 1, 2 and 3. In this paper, correct solutions for Hvorslev's 'approximated' (spheroidal) geometries associated with these two cases are presented (equations (6) and (10)). Furthermore, these provide much better correspondence with the finite element analysis of Ratnam *et al.* (2001).

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