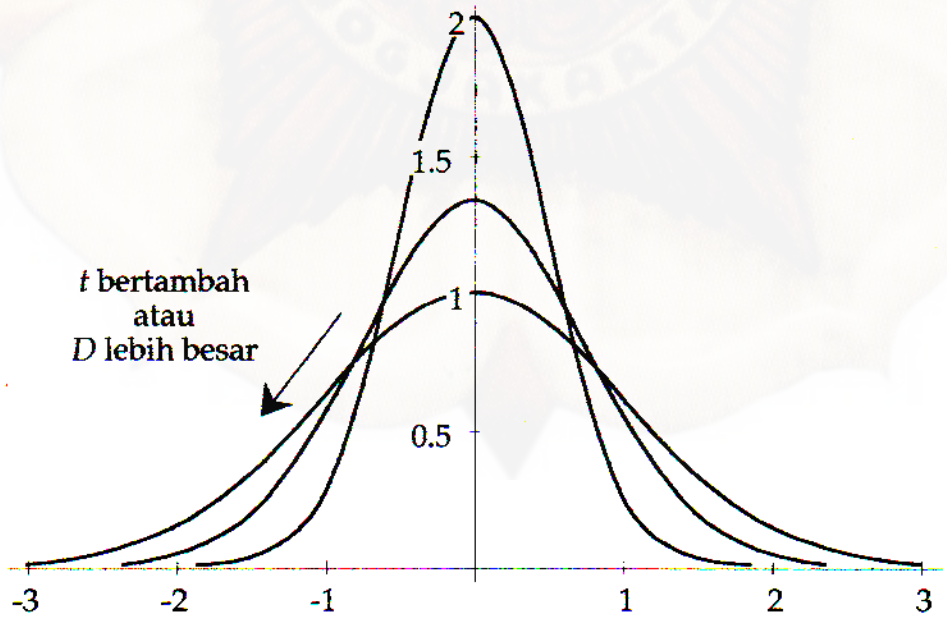
Tujuan:
membangun model numeris angkutan limbah

Persamaan dasar:

$$\frac{\partial C}{\partial t} + \frac{\partial (UC)}{\partial x} = D \frac{\partial^2 C}{\partial x^2}$$

Penyelesaian analitis:

$$C(x,t) = \frac{M}{\sqrt{4\pi Dt}} \exp\left\{ - \frac{(x - Ut)^2}{4Dt} \right\}$$



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Persamaan dasar:

$$\frac{\partial(AC)}{\partial t} + \frac{\partial(AUC)}{\partial x} = \frac{\partial}{\partial x}\left(AD\,\frac{\partial C}{\partial x}\right)$$

Adveksi

$$A\,\frac{\partial C}{\partial t} + AU\,\frac{\partial C}{\partial x} = 0$$

$$\frac{\partial C}{\partial t} + U\,\frac{\partial C}{\partial x} = 0$$

$$\frac{\partial C}{\partial t} + \frac{dx}{dt}\,\frac{\partial C}{\partial x} = 0$$

$$\frac{dC}{dt} = 0$$

→ Metode Karakteristik

Dispersi

$$A\,\frac{\partial C}{\partial t} = \frac{\partial}{\partial x}\left(AD\,\frac{\partial C}{\partial x}\right)$$

$$A\,\frac{\partial C}{\partial t} = \frac{\partial}{\partial x}\left(AD\,CX\right)$$

$$\frac{\partial C}{\partial x} = CX$$

→ Skema Preissmann

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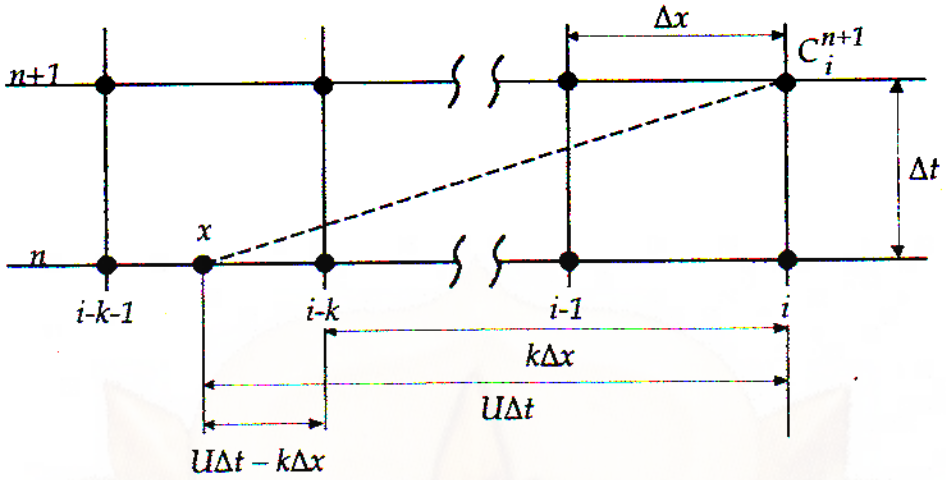
Metode Karakteristik



$$C_i^{n+1} = C_\xi^n$$

$$C_\xi^n \approx y(\alpha) = A\alpha^3 + B\alpha^2 + D\alpha + E$$

$$\alpha = \frac{x_{i-k} - x}{\Delta x} = \frac{U\Delta t - k\Delta x}{\Delta x} = Cr - k$$

$$Cr = \frac{U\Delta t}{\Delta x}$$

Interpolasi Hermite:

$$y(0) = C_{i-k}^n \qquad y(1) = C_{i-k-1}^n$$

$$\left.\frac{dy}{dx}\right]_{a=0} = CX_{i-k}^n \qquad \left.\frac{dy}{dx}\right]_{a=1} = CX_{i-k-1}^n$$

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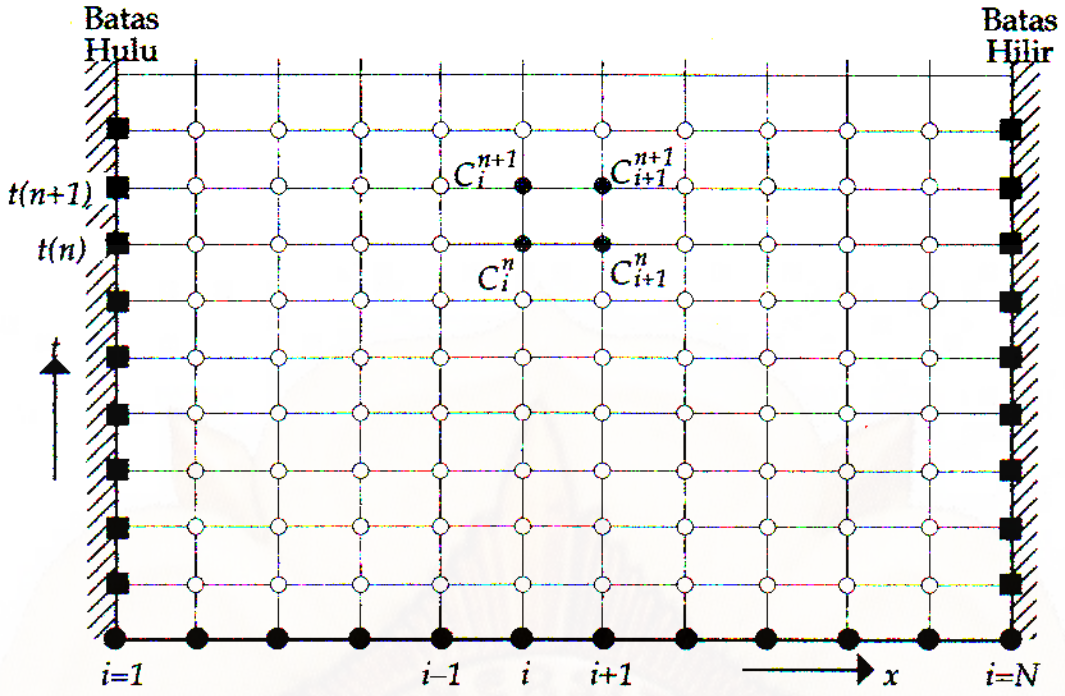
Skema Preissmann



$$\Delta x = x_i - x_{i-1}$$

$$\Delta C_i = C_i^{n+1} - C_i^n \qquad \Delta CX_i = CX_i^{n+1} - CX_i^n$$

Persamaan dispersi

$$A\frac{\partial C}{\partial t} = \frac{\partial}{\partial x}(AD\,CX)$$

$$\frac{\partial C}{\partial x} = CX$$

Diubah menjadi sistem persamaan linier

$$\left.\begin{aligned} a_1\Delta C_i + a_2\Delta CX_i + a_3\Delta C_{i-1} + a_4\Delta CX_{i-1} + a_5 = 0 \\ b_1\Delta C_i + b_2\Delta CX_i + b_3\Delta C_{i-1} + b_4\Delta CX_{i-1} + b_5 = 0 \end{aligned}\right\}$$

untuk $i = 2, 3, \ldots, N$

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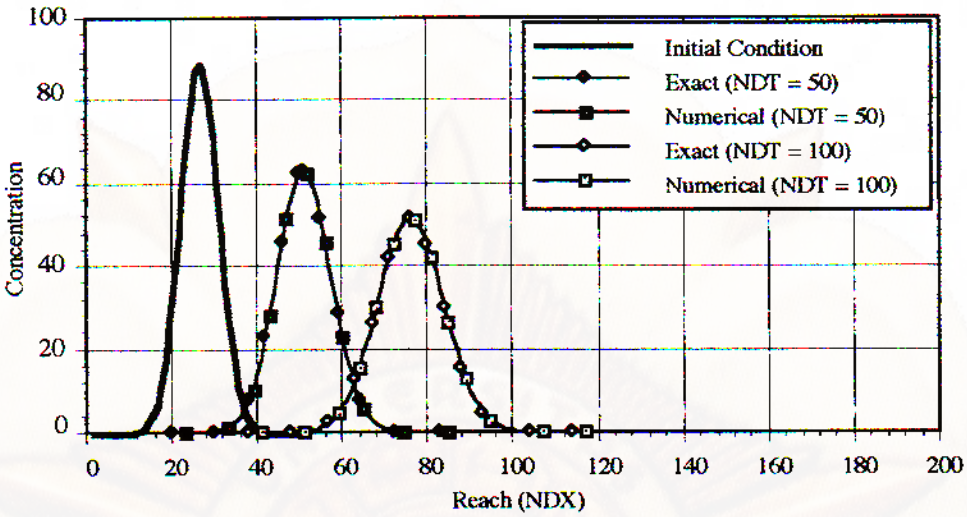
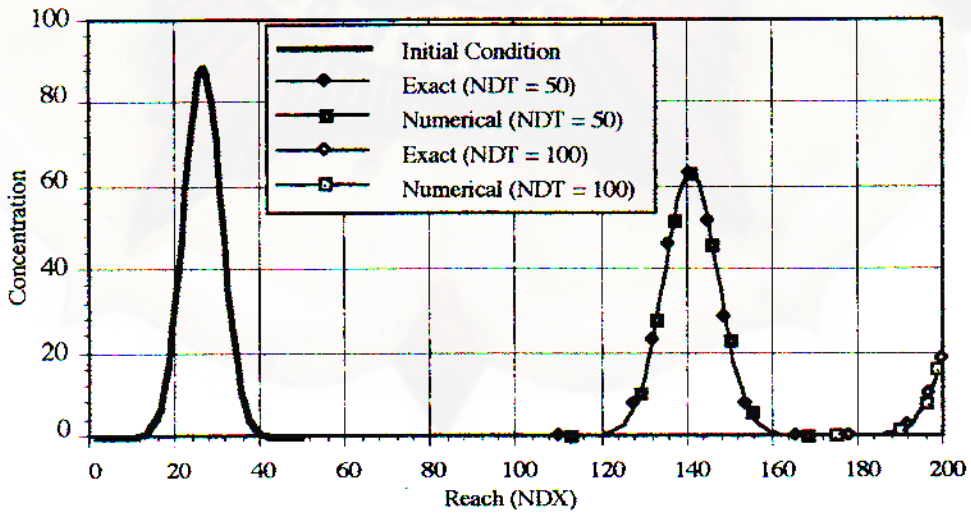
Verifikasi Model



(a) Courant Number = 0.5



(b) Courant Number = 1.4

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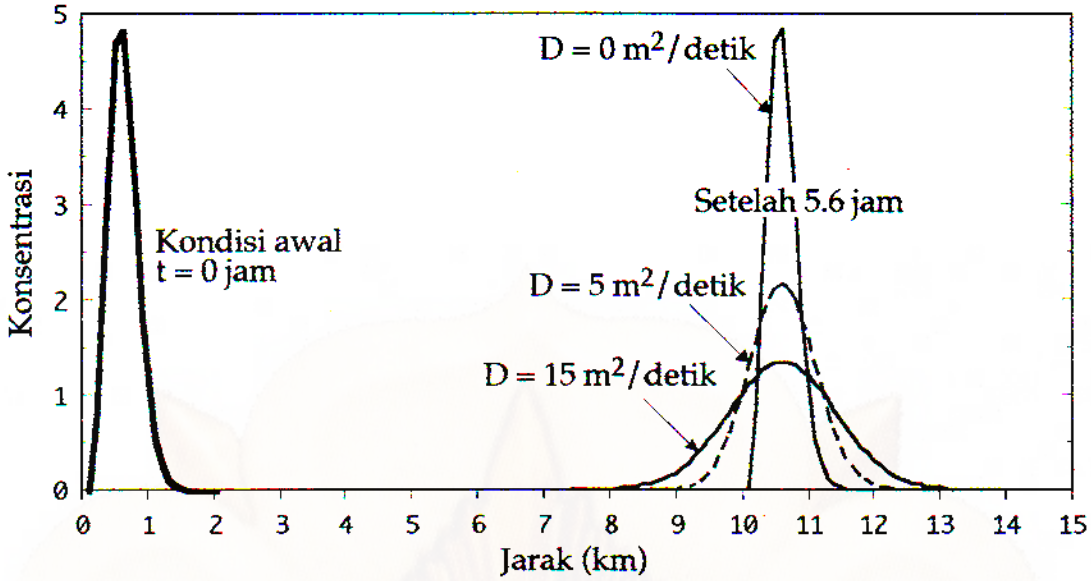
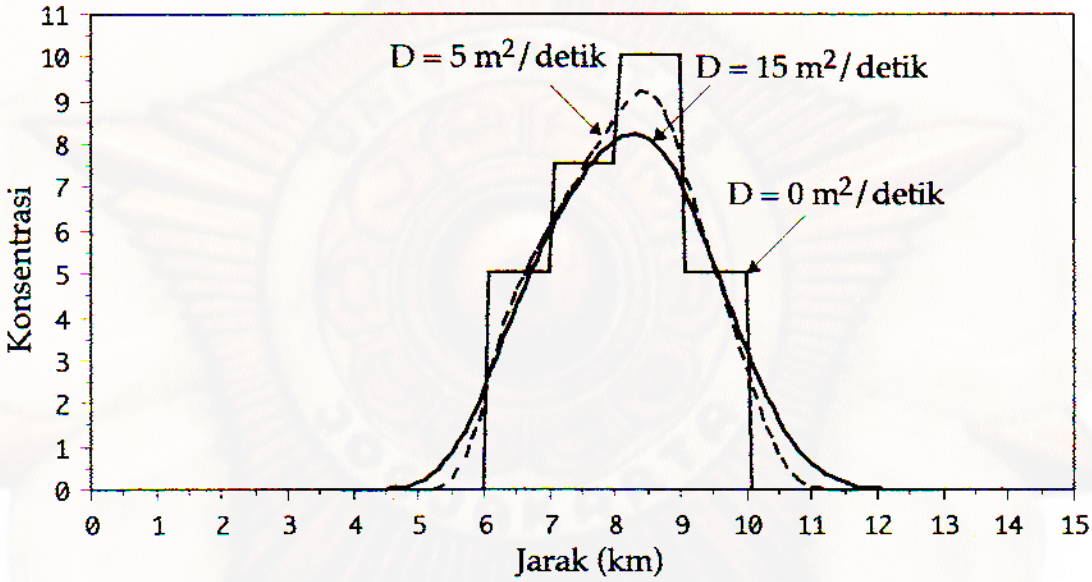
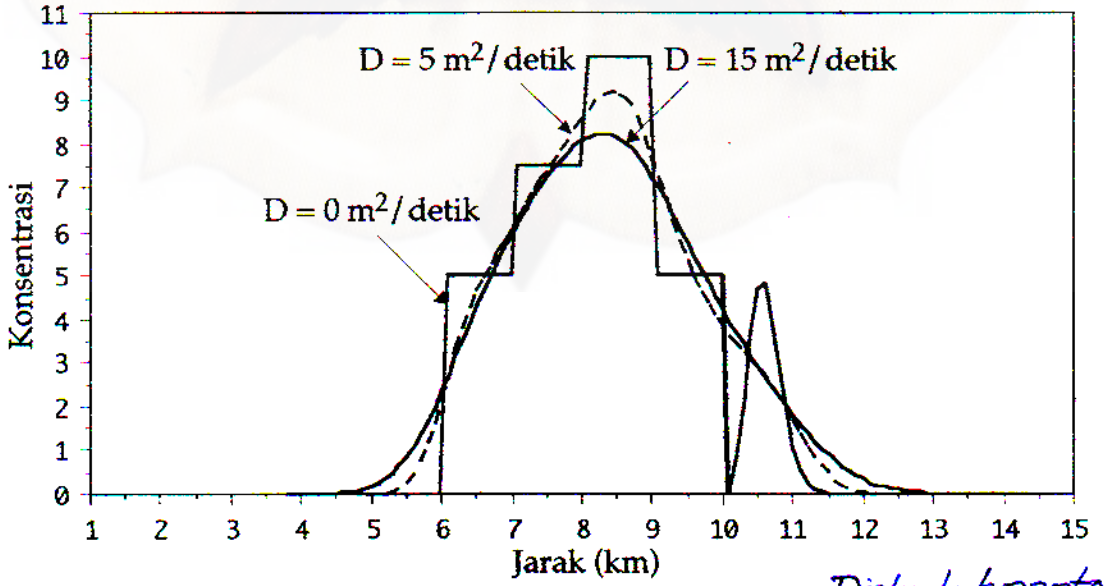
Hasil Model







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Kesimpulan dan Saran

- Walaupun model matematik yang digunakan masih relatif sederhana (hanya untuk kecepatan aliran seragam) tetapi model telah mampu menunjukkan unjuk kerja yang memuaskan.

- Model menunjukkan unjuk kerja yang memuaskan karena mampu mensimulasi mekanisme adveksi secara sempurna.

- Model ini membawa harapan untuk diterapkan pada model yang lebih komplek dimana hidraudinamika dari sungai dihitung terlebih dahulu dengan model numeris aliran tak tunak.

- Pengembangan model angkutan limbah ini, masih harus dilakukan seperti penambahan kemampuan dalam menangani limbah-limbah yang bereaksi kimia (seperti: COD dan BOD, dll)

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