The Stability of Throttled Surge Chambers

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Note 1: Oscillations in a throttled surge chamber not conforming to the conditions of stability enunciated by Thoma. Theory established during a study of the surge chamber of the Yablanica station, Yugoslavia.

When a power station is feeding an electrical network under constant load, the action of the governors tends to create an oscillation of the water level in the surge chamber. On the other hand, the dissipation of energy due to the loss of head in the supply tunnel is a factor tending to damp out these oscillations. Thoma has shown that these oscillations are damped out, maintained, or amplified indefinitely according as the product $2p_o h_o$ is greater than, equal to, or less than unity.

We are about to show that in the case of throttled surge chambers this conclusion is not entirely correct; for $2p_o h_o < 1$ the oscillations are not amplified indefinitely but are merely maintained at a stable value.

First, let us consider a shaft in which, instead of an actual constriction giving rise to a head loss according to the square law $r = r_o v^2$, there is a hypothetical constriction in which the head loss is in the form $r = mv$ where m is a constant. We will examine the case of operation at full load, the turbine absorbing the maximum stable discharge Q_o. Let us set out the following fundamental equations, using standard notation:—

$$v\frac{dv}{dz} + \frac{1}{2\pi}\frac{du}{dt'} + z \pm p \pm r = 0$$

$$z = x - p_o,\ p = p_o w^2,\ r = r_o v^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$u = 1 - \frac{p_o}{h_o} - \frac{z}{h_o} \mp \frac{r}{h_o}$$

and let us substitute $r = mv$ for $r = r_o v^2$.

From this we deduce, using the same approximations as in Thoma's theory, the following linear differential equation of the second order, with constant coefficients:—

$$(h_o - m)\frac{d^2x}{dt'^2} + 2\pi[2p_o h_o - 1 + m(h_o - 2p_o)]\frac{dx}{dt'} + 4\pi^2 (h_o - 2p_o)x = 0$$

Let us confine ourselves to the case $2p_o h_o < 1$ for surge chambers not satisfying Thoma's conditions, and let us take account of the conditions

$$h_o > 1, \quad h_o - m > 0, \quad h_o - 2p_o > 0$$

which are always found in practice. The solution of the differential equation will then give the following results:—

for $m < r_1$: oscillations indefinitely amplified

for $m = r_1$: oscillations maintained at a cycle period of $\sqrt{\dfrac{h_o^2 - 1}{h_o - 2p_o}}$

for $r_1 < m < r_2 = r_1 + \dfrac{2(h_o - 1)}{h_o - 2p_o}$: damped oscillations

for $m > r_2$: dead-beat condition in which

$$r_1 = \frac{1 - 2p_o h_o}{h_o - 2p_o} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

From this general discussion we would merely make use of the following relationships: that the amplitude of the oscillations

is indefinitely amplified when $r < r_1$
is damped and dies away when $r > r_1$
remains constant when $r = r_1$

Now, the hypothetical constriction imposes a limit corresponding to a loss of head $r = r_1 v$, and gives rise to a damping effect which, associated with that causing the loss of head in the supply tunnel, exactly balances the amplifying action of the governor; this results in a pure and simple maintenance of the oscillations without amplification or damping.

Let us return to the actual constriction $(r - r_o v^2)$ giving rise to a loss of pressure $rv = r_o v^3$ and compare its action with that produced by the hypothetical constriction limit $(rv = r_1 v^2)$ corresponding to the maintenance of the oscillations. If a small oscillation occurs, curve C $(v, r_o v^3)$ in Fig. 1, being below curve

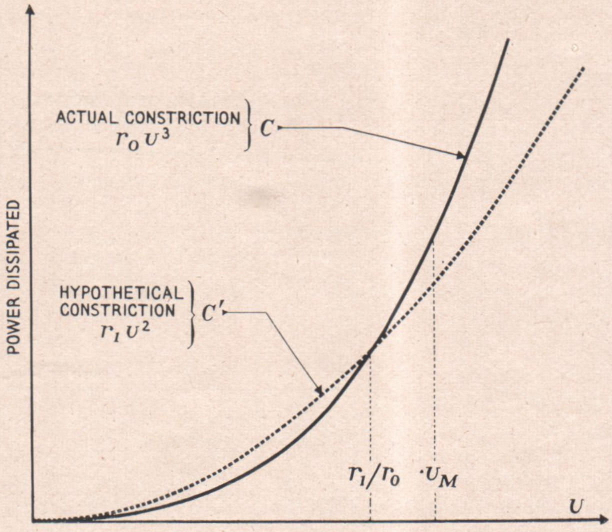


Fig. 1

C' (v, r_1v^2) beneath the point of intersection $(r_1/r_o, r_1^3/r_o^2)$, the loss of energy in the actual constriction is insufficient to neutralise the action of the governor. The amplitude of v progressively increases, passes r_1/r_o the abscissa of the point of intersection of the two curves C and C', and stabilises at a value v_m such that during each cycle the loss of energy in the actual constriction is the same as that in the hypothetical constriction already considered:—

$$\int_O^T r_o v^3 dt = \int_O^T r_1 v^2 dt \quad \text{..........................(3)}$$

On the other hand, commencing with an initial oscillation of high amplitude and considering the energy balance in the same manner, it can be established that damping occurs until the condition of stable oscillation just defined is reached.

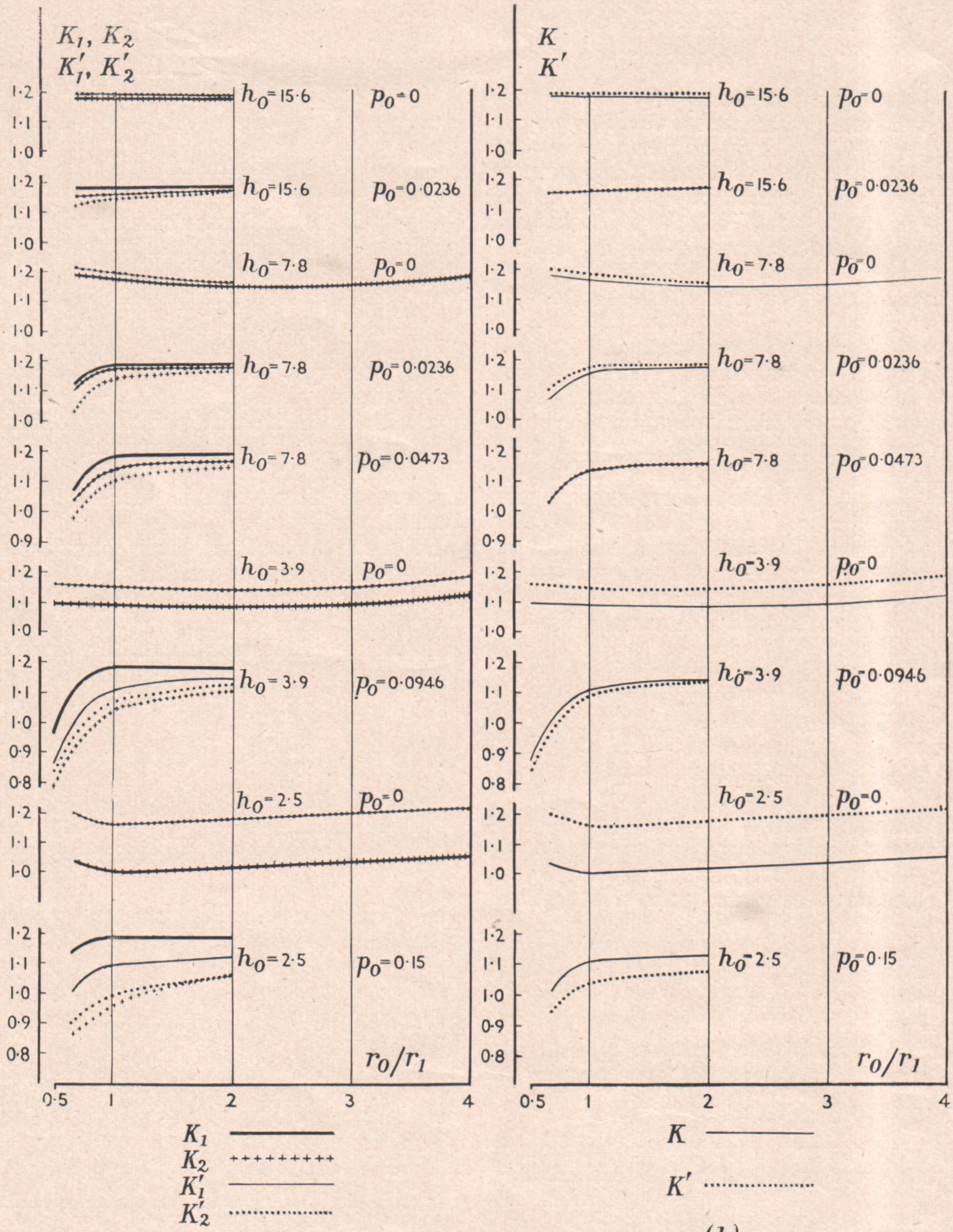
If, on a first approximation, the oscillations are assumed to be sinusoidal ($v = v_m \sin \omega t$), the energy balance expressed in equation (3) gives:—

$$v_m = 1{\cdot}18 \frac{r_1}{r_o} = 1{\cdot}18 \frac{1 - 2p_o h_o}{h_o - 2p_o} \quad \text{..............(4)}$$

Thus, in a throttled surge chamber having a section less than the limiting value given by Thoma, the foregoing theory points to the existence of stable oscillations having an amplitude v, given, on a first approximation, by equation (4).

Note 2: Graphical determination of the amplitude of stable oscillations of the water level in a throttled surge chamber not complying with Thoma's conditions; approximate expression for this amplitude.

In the foregoing note we have demonstrated anal-



(a) Fig. 2 (b)

tically the existence of stable oscillations of the water level in throttled surge chambers having a section below the limiting value laid down by Thoma.

Seeing that certain assumptions were made in the calculations—in particular that the loss of head p in the supply tunnel was linear—it is important to prove that the results obtained remain valid not only for small oscillations but equally for large ones.

We have been able to obtain this result by means of a graphical method based on equations (1) in our previous note, and the systematic study of 44 examples has enabled us in every case to verify the existence of a stable condition of maintained oscillations.

These oscillations are not exactly sinusoidal, and this leads us to lay down the following equations:—

$$x_{1m} = -K_1\frac{r_1}{r_o},\quad x_{2m} = K_2\frac{r_1}{r_o},\quad K = \frac{K_1+K_2}{2}$$

$$v_{1m} = -K_1'\frac{r_1}{r_o},\quad v_{2m} = K_2'\frac{r_1}{r_o},\quad K = \frac{K_1'+K_2'}{2}$$

where v_{1m}, v_{2m}, x_{1m}, x_{2m} are the maximum amplitudes of the negative or positive alternations of v and x, the values being related to the size of the water surface above the stable dynamic level corresponding to the maximum discharge Q_o.

Fig. 2a gives the values of K_1, K_2, K_1', K_2' and Fig. 2b those of K and K'.

Within the limits obtaining in industrial applications, the values obtained are all between 1 and 1·8, and generally are much nearer to the second figure than to the first. We can thus retain the approximate expression:—

$$x_m = x_m \,.\, Z_X = 1{\cdot}18\frac{r_1}{r_o}Z_X = \frac{1{\cdot}18}{r_o}\,.\,\frac{1-2p_oh_o}{h_o-2p_o}\,.\,Z_X$$

for the amplitude of the oscillations, as the error, which is always in excess of the true value, is invariably below 15 per cent. and is always on the side of safety.

We have also studied the case of a steady condition corresponding to a fraction $\frac{Q_o}{n}$ (n being greater than 1) of the maximum discharge Q_o. Stable oscillations are obtained the amplitude of which is given by the approximate expression:—

$$X_m'' = \frac{1{\cdot}18}{r_o}\,.\,\frac{Z_X}{n}\,.\,\frac{1-2p_oh_o\,[1+\frac{p_o}{h_o}(1-\frac{1}{n^2})]}{\begin{matrix}3\\ h_o+p_o(1-\frac{1}{n^2})\end{matrix}} < \frac{X_m}{n}$$

This amplitude is smaller, both absolutely and proportionately, than that apertaining to full flow.

Industrial Consequences of the Preceding Results

For throttled surge chambers it becomes possible to adopt, with complete safety, a section smaller than the limiting value laid down by Thoma on condition that a procedure is adopted of temporarily subordinating the electrical output to the hydraulic pressure eliminating the oscillations. A failure to carry out this procedure would not be accompanied by any major inconvenience, as the oscillations set up would have a limited amplitude instead of growing indefinitely as in an ordinary surge chamber.

Trief Slag-Cement Process for Glen Moriston

The North of Scotland Hydro-Electric Board, in an attempt to reduce constructional costs and save cement, have decided to use ground blast-furnace slag in the construction of the Loyne and Cluanie dams of the Glen Moriston hydro-electric scheme in Inverness-shire.

The process which the Board have decided to try out is called the Trief process and the contract for the construction of the dams has been placed with Mitchell Engineering Co. Ltd., of Peterborough, who have themselves also carried out investigations.

In 1948 the attention of the Hydro-Electric Board was drawn to the use being made of ground blast-furnace slag in the construction of a large dam by the French Electricity Board at Bort-les-Orgues in Central France. The blast-furnace slag was used in part replacement of Portland cement and the reports received were so encouraging that it was decided to send to France a deputation consisting of two engineers and a chemist to get fuller details. The process was attractive to the Board for two reasons: it held out the hope of economies in constructional costs and it might also save a considerable tonnage of cement which would become available for the export market.

The Trief process, which was patented by M. Victor Trief, a Belgian, consists of using blast-furnace slag (which is at present virtually a waste product) by grinding it wet at site in a rotary grinder. The slurry so produced is passed direct into the concrete mixers to be mixed with the appropriate quantities of Portland cement, aggregate and sand, to produce a concrete equal in strength to a similar mixture using wholly ordinary Portland cement. The French also claim that the slag cement concrete produces less heat when setting and also is more resistant to acid peaty waters than pure Portland cement concrete.

The blast-furnace slag available from Scottish steelworks contains not quite so much lime as the slag the French used, but the Highland experiment should nevertheless produce results which will be of value not only to the Board and to the country's export trade, but for other large dams at home.

The Cluanie dam is to be 112 ft. high and 2,165 ft. long and the Loyne dam 58 ft. high and 1,745 ft. long. The proportion of slag cement to Portland cement will probably be about one-third Portland and two-thirds blast-furnace slag, in which case the saving in Portland cement will amount to about 20,000 tons. In the construction of the dams, the Mitchell Engineering Company are also adopting another idea which was tried out at Bort, which is not to use shuttering but to use instead precast concrete slabs for the faces of the dams. By this means, neither steel nor timber shuttering will be needed and the demand therefore for these two scarce materials will be reduced.

Thew Lorain. A brochure in colour received from The Thew Shovel Company illustrates some striking examples of the firm's shovels and cranes in action in diverse conditions. The brochure also gives the firm's annual report and calls attention to the very satisfactory increase in the net annual sales during 1951.