HISTORY OF WATER-HAMMER

by

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SYNOPSIS

This report traces the theoretical development of the phenomenon of water-hammer from the days of the natural scientists of the seventeenth century to the middle of the twentieth century.

INTRODUCTION

Water-hammer is the phenomenon of the transmission of pressure and velocity surges along a pipe or conduit when a change in either the velocity or the pressure is imposed at some section in the pipe or at its ends. This change may be imposed by some mechanical device such as a valve or by a variation in the performance of some hydraulic machinery attached to the pipe, such as a pump or hydraulic turbine.

The inception of study, and its pursuit, in any field of scientific thought is dependent on several factors. An essential one is the recognition of the physical facts which are involved, i.e. of their existence. Another factor is the need for a solution of the problems which arise from these physical facts. A third factor is curiosity aroused regarding their cause, their significance, and their impact on other fields. A fourth factor, which is necessary in the development of any study, is the ability to cope with the problem and its solution. This last involves ability to recognize the significance and inter-dependence of the physical phenomena and on the mental tools available for their study, (such as mathematical, physical, and logical).

The study of water-hammer, due to its relatively late arrival on the scene, gives us a good opportunity to note how these factors have played their part in the recognition and development of the subject.

In the larger field of hydraulics, early development was based on problems of transportation of water, and on the measurement of flow for irrigation and for domestic use in the centres of population.

This involved devices for pumping, for delivery along canals and conduits, and for the measurement of quantities used by individuals. The theory and designs had to do with hydrostatic pressures, friction losses, and discharge coefficients, most of which could be handled by experimental studies. There was no need to consider cases of unsteady flow, either in open channels or in closed pipes and conduits. The action of waves on shore lines, in bays and ports, and along canals, was the only problem similar to water-hammer known in early times, and the mathematical and physical knowledge in those days was unable to cope with this problem except by construction of breakwaters for protection of harbours and shipping. This branch of hydraulics, now designated as coastal engineering, which deals with tidal and surface waves, has become a very important field, particularly in this 20th century.

CHAPTER I   EARLY STUDIES UP TO 1700 A.D.

The earliest study of the subject of water-hammer was made by Euler [5] in 1775 when he attempted a solution of the phenomenon of flow of blood through the arteries. The solution eluded him, although it was almost within his grasp, as the mathematical and physical tools were at his command. It is instructive and appropriate that we show how these mathematical and physical tools were developed.

Analagous fields which occupied the attention of mathematicians and physicists (natural scientists) in early times, and whose study became the foundation on which the theory of water-hammer is based, were -

(1)   the propagation of waves on shallow water;

(2)   the propagation of sound waves in air;

(3)   the flow of blood in the arteries.

None of these problems held much hope of detailed solution
until the development of the calculus and the solution of partial
differential equations. This is because they involve rates of change
of pressure and velocity in elastic media. It was necessary for the
mathematicians to develop a tool for studying rates of change and
for the physicists to develop a theory of elasticity which would
co-ordinate these rates of change.

Almost all of the earlier students were equally facile in
mathematics and science, and it is difficult to separate the work
done into these two distinct disciplines. Developments will be shown
in chronological order and this chapter will cover the work done up
to the time of Newton and Leibniz.

Descartes, Réné (1596-1650).

He opened the new field of Analytic Geometry by inventing
cartesian co-ordinates. This made it possible to put Geometry on an
analytical (algebraic) basis, and led to the development of
Trigonometry and Analytical Geometry. All of our later development
is indebted to Descartes for this powerful tool which is the
foundation of mathematical analysis.

Cavalieri, B. (1598-1647)

His idea of "indivisibles" (developed about 1635) which was
probably suggested to him by much earlier work by the Athenian
Xenocrates, was that there are indivisible (Geometric) quantities
called points and the sum of an infinite number of points constitute

a line. Then the sum of an infinite number of lines gives a surface and the sum of an infinite number of surfaces gives a volume. His studies lacked clarity and were not satisfactory to a scientific mind. However, he undoubtedly stimulated Leibniz and other contemporaries in the development of the integral calculus.

Barrow, Isaac (1630-1677).

In 1663 he used what is now called the differential triangle (Fig. 1) and thus introduced the idea of limits in a geometric sense. As Δx and with it Δy are continuously reduced, the chord AB approaches the tangent AT. The ratio $\Delta y/\Delta x$ approaches as a limit the slope of the tangent AT, and this is the rate of change of y with respect to x at the point A on the curve. He communicated this study to Newton. It is also known that Leibniz corresponded with him and knew of this work.
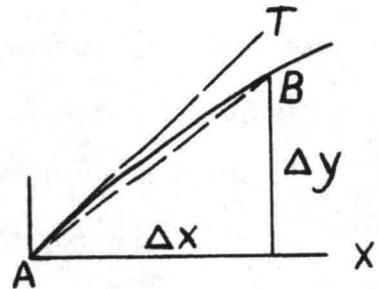
FIG. 1.

Newton, Isaac (1642-1727).

There are two of Newton's studies which are particularly significant for us. One of these is his theory of fluxions, the basis of the calculus.

In his earlier work he considered infinitely small quantities, similar to Cavalieri, but he discarded this as he recognized it was not mathematically sound. In 1664-1666 he developed his theory of "fluxions". This was based at first on the time rate of movement of a point (i.e. the velocity), "flowing" along a curve. Thus his study had a geometric and physical basis. He studied the fluxions

of various functions, and the ratio of these fluxions. Thus the ratio of the fluxion of y with respect to the fluxion of x was the fluxion of y with respect to x. In our notation this is written

$$\frac{dy}{dt} \div \frac{dx}{dt} = \frac{dy}{dx} .$$

In Newton's "dot" notation, where $\dot{y}$ signifies differentiation with respect to time, the fluxion of y (with respect to time), this ratio is written $\dot{y} \div \dot{x}$

Newton later studied the inverse operation on functions, their integration or quadrature. Reference 1 dealing with these studies is in Vol. I, pp. 145-244, 400-446.

The second of Newton's studies significant for us is his study of the propagation of sound in air. This is given in his Principia [2] published in 1687. All of Newton's work in this monumental effort is based on synthetic geometry. In Book I he deals with the general motion of bodies subjected to various types of forces and in particular to the motion of the planets. In Book II he considers the motion of bodies in resisting media (i.e. the effect of friction). Here are also the studies of the oscillating pendulum, the propagation of waves in canals and the propagation of sound waves in the air. In Book III he gives his "System of the World" which is based on Book I and includes a more systematic and complete statement of the motion of heavenly bodies.

It is instructive to study Newton's development of the theory of propagation of water waves in canals and of sound waves in air. First (Book I Prop 52) he studied the oscillation of a pendulum, (Fig. 2). The pendulum cord OP is restricted by the two half-

cyclords OP and OR, each of length
L, and the pendulum bob oscillates
along the cyloid PQSR. The force
of attraction is toward the centre
C of the circle PBR. He found that
the force causing acceleration along
the path PQSR is proportional to the
distance along the path from S.
This gives us simple harmonic motion
along the path and the pendulum
motion can be related to the motion
of a particle in a circular path
with constant angular velocity
(i.e. the motion of the projection
of this particle on a diameter of
the circle).
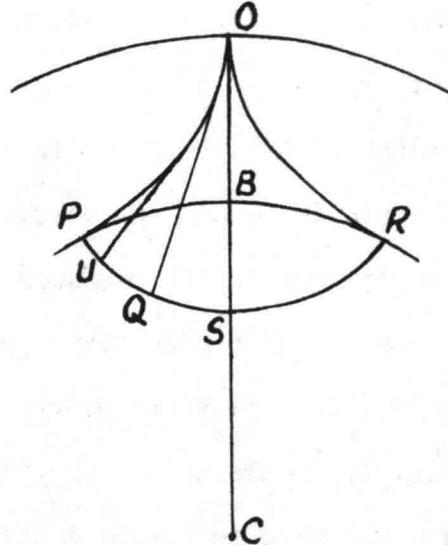
FIG. 2.

He extended his study to the case where the circle PBR has an
infinite radius, so that the centripetal forces become parallel, and
deduced the formula for the period of oscillation of a simple
pendulum at the earth's surface

$$\tau = 2\pi\sqrt{L/g} \ .$$

To find the period of waves in a canal (Book II Prop. 44,45,46)
he first showed that the period of oscillation of a liquid in a U-
tube of total length L was equal to that of a pendulum of length
L/2. Then he used the analogy (erroneously, as Lagrange later pointed
out) that the distance from crest to trough on a water wave

corresponded to the length of liquid in a U-tube, the accelerating force in each case acting on these two corresponding lengths of liquid. Hence the period of a wave in a canal is $\pi\sqrt{L/g}$, where L is the wave length. This gives, erroneously, a wave velocity $\frac{1}{\pi}\sqrt{L/g}$. For the case of sound waves in air, Newton (Book II Prop. 47 to 50) compared them to the oscillation of a simple pendulum. He assumed the density to be inversely proportional to the pressure. If h is the height of an air column of uniform density equal to that at the earth's surface which would give a pressure at its base equal to the barometric pressure (e.g. 30 inches of mercury), he deduced that the time taken for a pressure wave to travel a distance equal to the circumference of a circle of radius h (i.e. to travel a distance $2\pi h$) was equal to the period of a pendulum of length h. Hence the wave velocity is $\sqrt{gh}$.

Using a barometric pressure of 30'' Hg. he finds h = 29725 feet and the velocity of sound in air is 979 ft/sec. This is the same result obtained later by Lagrange. It is interesting to note how Newton and Lagrange tried to reconcile this theoretical value with the experimental values of approximately 1142 ft/sec. Lagrange said the difference was due to experimental error. Newton, however, had confidence in the experimental value. He gave two reasons for the error in the theoretical value.

(1) The "crassitude" of the solid particles of air.

These, being approximately of the same density as water or salt must be distant from each other approximately 10 diameters, since water is 870 times heavier than

air. The sound (pressure surge) travels instantaneously through the solid particles and all of the time is consumed in the movement from particle to particle. This will give a corrected velocity of

979 x 10/9 = 1088 ft/sec.

(2) The presence of "vapours". These latter are of another "spring" or type than the air particles and have a different tone and will hardly partake of the motion of the true air in which the sound is propagated. Thus if the vapours remain unmoved, the propagation will be faster through the true air. Taking the ratio of air to vapours as 10 to 1, the velocity, which depends on the square root of the density, will be increased to $1088\sqrt{11/10}$
= 1142 ft/sec.

Leibniz, Wilhelm (1646-1716)

He established his version of the calculus independently of Newton, although we know that he had access to Newton's method in 1673 when he visited London. By 1675 he had established his notation d for the differential and $\int$ for the summation or integral. This latter symbol is related to the "summa" of the "indivisibles" of Cavalieri. In 1684 and 1686 he published his method. His symbols were later adopted by everyone, including the English mathematicians. Thus he wrote

$$\int y\,dy = \tfrac{1}{2}y^2$$

Leibniz appears to have developed the integral calculus first, and thought of the differential as the inverse of the integral.

This is the reverse of Newton's thinking, who concerned himself first with differentials and later with the inverse operation of integration or quadrature.

CHAPTER II   DEVELOPMENT IN THE 18th CENTURY

Bernoulli, Jean I (1667-1748)

He was one of the greatest mathematicians of his time and a pioneer in putting the integral calculus on a sound foundation.

Taylor, Brooks (1685-1731)

Worked on infinite series and derived Taylor's Series in 1712, one of our fundamental theorems.

Maclaurin, Colin (1698-1746)

Published his series in 1742. This is a particular case of Taylor's Series. It was proved much earlier in 1730 by James Stirling.

Euler, Leonhard (1707-1783)

He was the pioneer in developing a detailed theory of the propagation of sound waves in air and of elastic waves travelling along a plucked cord.

In his article on the propagation of sound [3], he states first that he would not have been led to his solution if he had not read Lagrange's analysis of the problem [4].

Euler found Lagrange's treatment prodigiously difficult to follow. However, Euler's analysis is quite different from Lagrange's. He uses discontinuous functions for the first time on record, as he had already done in his study of the vibrating cord, maintaining

that the very nature of the sudden disturbances which caused the
waves required the use of discontinuous functions. Lagrange , on the
other hand, used only continuous functions, assuming a system of
finite particles of air rather than a continuous medium. He solved
the case of the vibrating cord similarly by assuming a set of finite
masses. Both Lagrange and D'Alembert rejected the validity of using
discontinuous functions. The controversy persisted even though
Euler demonstrated that his solutions agreed with the others, where
continuous functions could be used. In these introductory remarks
Euler expresses the hope that "in time" these conservative Geometers
will accept these new functions.

Euler assumed a disturbance within a limited length of a
horizontal air column in a cylindrical tube and the transmission of
this disturbance in both directions. This disturbance is a
discontinuous function, being zero on each side and of any desired
form initially. Assuming that the density varies inversely as the
pressure , he develops the partial differential equation for
the wave propagation

$$\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2} , \quad \text{-----------} \quad \text{I}$$

where a^2 = gh .

Here   x   is the equilibrium position of the particle,
y   its displacement at time   t, and   h   is the height of a column
of air of constant density which exerts atmospheric pressure at its
base. Euler gave the functional solution of this equation

$$y = F(x+at) + f(x-at), \quad \text{-----} \quad \text{II}$$

the velocity of propagation being $a = \sqrt{gh}$, the same as obtained by Newton and Lagrange.

It is not difficult to show that this solution agrees with present one $a = \sqrt{Kg/\gamma}$. We note that the atmospheric pressure P is equal to γ h. Then, using the assumption PV = constant, where V is the volume of a given mass, we obtain

$$P\,dV + V\,dP = 0, \quad \text{or} \quad dP/P = -dV/V.$$

But we define the modulus K by $dP/K = -dV/V$. Hence we may substitute P = K giving

$$gh = gP/\gamma = Kg/\gamma.$$

Euler proceeded to discuss the significance of these functions F and f, which represent waves travelling up and down the pipe with velocity a; and he solves for cases where -

    (1)   the initial displacement,   and

    (2)   the initial velocity,

are given. He noted the difference between this theoretical velocity of sound and the experimental value and surmised that this was due to the extra factor, $1 + \left(\frac{dy}{dx}\right)^2$, which was in his general equation, but which he had considered negligible. As he said, this would not be negligible for strong disturbances.

Euler, in conclusion, gives priority credit to Lagrange for the development of the partial differential equation I and its functional solution II, as applied to continuous functions. Later, the use of these functional expressions was studied extensively by Gaspard Monge (1746-1818) [8].

For our purpose this is a sufficient summary of Euler's
text, but the reader will be rewarded by a more detailed study.

In 1775, Euler wrote a second article [5] on the flow of
blood through the arteries but did not succeed in obtaining any
satisfactory solution. (Note that Harvey, 1578-1657, discovered
the circulation of the blood.)

> Euler's concluding remarks, made in a religious vein, were:
> "In explaining, then, the motion of the blood, we run
> into the same unsuperable difficulties which prevent
> us from examining more accurately all the things
> that are clearly works of the Creator ...... since
> not even the greatest human genius is able to
> understand and explain the true structure of the
> most insignificant worm".

Euler published his works on analysis 1748-1770 and Hankel pays
him the tribute that "he freed the analytic calculus from all
geometric bonds, thus establishing analysis as an independent science".

To avoid confusion, the research results of all who followed
Euler will be expressed in a common nomenclature as given in the
table of notation wherever this is possible.

Lagrange, Joseph, L.   (1736-1813)

He succeeded Euler in 1766 as mathematical director in the
Berlin Academy, which was a flourishing institution under the
patronage of Frederick the Great. He obtained solutions for the
movement of incompressible fluids and compressible fluids in his
text Mecanique Analytique [6].

(1) For the former he used the velocity potential $\emptyset$ and his

method is rather cumbersome, being based on approximations of Taylor's expansion $\phi = \phi' + z\phi'' + z^2\phi'''$ etc.

In the case of waves in a canal, assumed shallow enough so that the vertical dimension Z may be neglected and vertical planes in the medium remain vertical, he develops the relation

$$\frac{\partial^2 \phi'}{\partial t^2} = gh\left[\frac{\partial^2 \phi'}{\partial x^2} + \frac{\partial^2 \phi'}{\partial y^2}\right], \quad \text{--------- I(a)}$$

where h is the canal depth (p. 334). Reducing this to a single space dimension, he finds the velocity of the wave $a = \sqrt{gh}$.

(2) For compressible fluids, such as air, (p. 339), he adds a term in his fundamental relations for incompressible fluids, to allow for the compressibility, and deduces the same solution as I(a) above. Here h is the pressure head on the air mass, as defined earlier by Newton and Euler. To solve this, he reduces again to one space dimension and obtains the functional solution II.

Lagrange in a similar manner studied vibrating cords (Ref. 6, Vol. I pp. 382-422).

Note on the solution of waves in open channels.

Let us now summarize the solutions for waves in open channels, using for reference Coulson's condensed text on waves [7].

(1) Surface waves, or waves in relatively deep water. Here a general treatment is necessary, since vertical components of the variables cannot be neglected, and the disturbance does not extend far below the surface. To this group belong -

(a) waves promoted by wind,

(b) waves of short length for which surface tension becomes a significant factor.

(a) neglecting surface tension, the velocity of propagation a
is given by

$$a^2 = \frac{g\lambda}{2\pi} \tanh\frac{2\pi h}{\lambda} \; ,$$

where λ is the wave length and h the depth of water.

At the two limits of the quantity $2\pi h/\lambda$ we find

(i) For very deep water, as $h \longrightarrow \infty$, $a^2 \longrightarrow \frac{gh}{2\pi}$;

(ii) For very shallow water, as $h \longrightarrow 0$, $a^2 \longrightarrow gh$.

This last is the case considered by Newton and Lagrange.

(b) Allowing for surface tension, we have

$$a^2 = \left[\frac{g\lambda}{2\pi} + \frac{2\pi T}{\lambda\rho}\right] \tanh\frac{2\pi h}{\lambda} \; ,$$

where T is the surface tension and ρ the mass density.

As the wave length λ decreases, the surface tension becomes
more important.

Monge, Gaspard.

In 1789 outlined his graphical integration of partial
differential equations [8]. Later he published in the same Journal
his "Memoire sur l'integration graphique des équations aux derivées
partielles". This appeared in Vol. 23 new series, and he used the
term "method of characteristics". Monge was the founder of the
École Polytechnique.

Laplace, Pierre, S. (1749-1827)

He was another great mathematician and scientist, a contemporary
of Lagrange. He developed the Laplace equation, the criterion of
equilibrium of homogeneous fluids (Ref. 9, Vol. I, p. 206)

$$\nabla^2 \phi = \left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right] \phi = 0 \; .$$

About 1808 he explained the difference between the experimental and the theoretical values of the velocity of sound in air. The theoretical value, based on Boyle's law (see Newton, Euler, Lagrange) was $a = \sqrt{P/\rho}$. He stated that this law did not apply since the temperature was not constant under varying pressure. For adiabatic conditions $a^2 = \dfrac{C_p}{C_v} \cdot \dfrac{P}{\rho}$, where C_p and C_v are the specific heats. Using $C_p/C_v = 1.41$, this increases the theoretical velocity by about 20%.

Laplace's work included celestial mechanics [9] and he used analytical methods throughout in his physical studies, as compared with Newton, who used Synthetic Geometry in his "Principia".

Cauchy, A.L. (1789-1857)

He is of interest because he put the differential on a sound analytical basis [10], not dependent on a geometric relationship between chords and tangents. He expressed the infinitesimal i as a product α h, where h is an arbitrarily chosen finite quantity and α another infinitesimal. Then

$$\lim_{\alpha \to 0} \frac{f(x+\alpha h) - f(x)}{\alpha h} = f'(x),$$

by definition of the derivative.

Hence

$$\lim_{\alpha \to 0} \frac{f(x+\alpha h) - f(x)}{\alpha} = h.f'(x).$$

If we denote the arbitrary quantity h by dx, and call it the differential of x, then the left hand side above may be denoted by dy the differential of y, so that

$$dy = \lim_{\alpha \to 0} \frac{f(x+\alpha h) - f(x)}{\alpha} = d\big[f(x)\big] = f'(x)dx = y'dx.$$

By the turn of the century, the mathematical tools were well developed, but the physical tools, dealing with the elastic properties of materials, were still lacking somewhat for the requirement of rigorous and detailed analysis.

In review we note that the work of Newton, Leibniz and their contemporaries gave rise to a great surge in mathematical and scientific development. In the latter half of the 18th century Western Europe was a closely knit community of savants. National Institutes, Academies and Schools flourished in Italy, France, Austria and Germany, as well as in England and Russia. The teaching and writing of such great men as the Bernoullis, D'Alembert, Lagrange, Laplace, Legendre and Euler became well known everywhere. Physical and mathematical concepts could now be expressed in the form of integral equations and ordinary and partial differential equations. Later came the complex variable and the calculus of variations.

It is difficult for us, at the present time, to be certain as to who initiated some of the new ideas in the 19th century. We can only record the results of our research, indicating the general development.

CHAPTER III   DEVELOPMENT IN THE 19th CENTURY

Young, Dr. Thomas

In 1808 he made some hydraulic studies, in preparation for the Croonian Lecture, on the flow of the blood stream [11]. He studied friction losses, including losses in bends, as well as the propagation of the pressure wave. He discussed DuBuat's text, Principles of Hydraulics, Vol. II, 1786. His physical arguments appear unsound

and unfortunately he did not discuss the subject further in his lecture or subsequently. Much experimental work on air and water surges in pipes was carried on in the first half of the century, notably Wertheim [12] in 1848. However, no satisfactory theory was developed for the propagation of the pressure wave in water pipes.

Helmholtz, Hermann von (1821-1894)

He was editor for Pogg (Annalen der Physik und Chimie) and had wide interests. He appears to have been the first to explain (in 1848) the fact that the velocity of wave propagation in a pipe containing water was less than the velocity in water not confined. He explained, correctly, that this was due to the elasticity of the pipe walls. Wertheim, in the same year, offered the same explanation but made no theoretical study.

Riemann, B.

In 1860 he published a paper [13] at Gottingen on sound waves and later in 1869 he published his text on Partial Differential Equations [14]. He refers to the earlier work by Euler in 1759.

Riemann developed and applied the three dimensional equation of motion and its simplified one dimensional form, in several fields, notably for vibrating strings and sound waves, as do others before him. However, he used neater definitions and expressions for the elastic properties of the media. Thus for air he expressed the relations between pressure and density by the equations

$$\rho / \Delta = 1 + s \;\; ; \quad P = \phi(\rho).$$

Here Δ is the density in the equilibrium state, ρ is the variable density, P the pressure and s the compression ratio.

His general equation was

$$\frac{\partial^2 s}{\partial t^2} = a^2 \left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right] s .$$

His particular studies are based on the functional solution for a single space dimension, II.

Weber, Wilhelm studied the theory of flow of an incompressible fluid in an elastic pipe [15]. He had already made some experiments in 1850 [16] to find the velocity of propagation. Using the two linear relations for elasticity and acceleration which he developed,

$$\frac{\partial V}{\partial x} = - \frac{2}{r} \cdot \frac{\partial r}{\partial t} \quad ; \qquad \frac{\partial V}{\partial t} = - \frac{1}{P} \cdot \frac{\partial P}{\partial x} ,$$

he obtained the second order equation

$$\frac{\partial^2 r}{\partial t^2} = \frac{r}{2 k \rho} \cdot \frac{\partial^2 r}{\partial x^2} .$$

Here $k = \frac{dr}{dP}$, his elastic modulus; V is the velocity, r the radius and P the pressure. He deduced the velocity of propagation $a = \sqrt{\frac{r}{2 k \rho}}$. Thus he related the elasticity of the pipe walls directly to the variations of pressure and radius. This has been found necessary in the recent study of surges in rubber and plastic pipes.

Using our notation

$$K_P = \frac{dP}{dA} \cdot A = \frac{1}{2} \frac{dP}{dr} \cdot r .$$

his modulus $k = \frac{r}{2 K_P}$, so that his velocity of propagation becomes

$$a = \sqrt{K_P / \rho} = \sqrt{\frac{K_P g}{\gamma}} ,$$

which agrees with present-day theory.

His tests on a rubber hose 16.5 mm radius, which expanded 2.75 mm in radius under a pressure of 3500 mm head, gave an experimental value of the velocity of propagation $a = 11255$ mm/sec, whereas the theoretical value of wave propagation velocity is

10033 mm/sec. He charges the discrepancy to inaccuracy of measurement of the strain and of the period. It is certainly not due to neglecting the elasticity of the water.

Mention should be made of other experimenters, such as Wertheim [12], Kundt [17] and Dvorak [18].

Weber appears to have been the first to develop the two first order equations for the elasticity of the pipe walls and the acceleration of the water column. We may designate these the continuity and dynamic equations respectively. These are the basis of all our theoretical studies involving the magnitudes of the pressure and velocity in water-hammer.

In the past, investigators were concerned only with the velocity of propagation (as was Weber himself) and so the mathematics developed was based on the second order equation in one independent variable, which led directly to the velocity of propagation. The two first order equations were lost in obtaining this second order equation. Weber happened to discover these two first order equations because of his detailed study of the elasticity of the pipe walls. The two main fields of study of wave propagation in pipes were still quite distinct, one with an elastic medium (air) in an in-elastic pipe, and the other with an in-elastic medium (water) in an elastic pipe (rubber).

Marey, Dr. published in 1875 an account of his experimental work on the propagation of water (and mercury) waves [19]. He conducted medical research on the flow of blood in small animals (turtles, frogs) and to attempt to develop a theory for the propagation of pulse beats he set up experiments in his hydraulic laboratory and

published accounts of his research over a period of several years.

He had a pipe line of rubber with six short branches spaced evenly along its length and leading to his six chronographs. These recorded continuously the pressure oscillations caused by the operation of a piston at one end. This piston could apply an impulsive injection of fluid or, alternatively, it could work in reverse. Both the amount of injection and the rate could be varied, with either water or mercury. His laboratory technique was excellent. In fact he criticised the work of earlier experimenters (Newton, Weber, Poisson, Biot) saying that their methods and instruments were imperfect and their results sometimes contradictory.

He was able to formulate several general conclusions, e.g.

(1) the wave velocity is independent of the impulsive blows ;

(2) the wave velocity for mercury is approximately three times that for water ;

(3) the wave velocity is proportional to the elastic force of the tube ;

(4) the reflected wave from the closed end has the same velocity.

He did not have the mathematical knowledge to develop any analytical results, but his contemporary Resal (editor of the Journal de Math. pures et appliquées) verified his experimental results by developing his theory of propagation.

Resal, H. in his article in the Journal [20] develops the continuity and acceleration equations and from these the second order wave equations, assuming an incompressible fluid and an elastic pipe. Hence he finds the wave velocity identical with the work of Weber [15],

but expressed in terms of Young's modulus, i.e. $a = \sqrt{\dfrac{Ebg}{D\gamma}}$.

Rayleigh, J.W.S., Lord (1842-1919)

published his text in 1877 on the Theory of Sound [21], which brought up to date the result of all the studies of the past, and of his own research.

Korteweg, D.J. was the first to solve for the wave velocity

considering the elasticity of both the pipe wall and the fluid [22]. Those before him considered either one or the other of the two inelastic or incompressible. He also took into account the radial acceleration of the pipe walls in his general equation. His development is very close to present day analysis, both in thought and procedure. However, he was concerned only with the velocity of propagation and not with the transient pressure velocity relationship.

He neglected -

(1) the effect of friction, since, like all other writers, he dealt with pipes in which only the vibrations due to sound and wave propagation existed, and no study made of the effect of change in flow;

(2) longitudinal stresses due to bending of the pipe walls, as he considered the wave length considerably large with respect to the pipe diameter;

(3) variation in the modulus of elasticity with pressure.

His basic equations were -

$$\frac{dP}{K} + \frac{\partial u}{\partial x} + \frac{2 \cdot dr}{r} = 0, \text{ for continuity;}$$

$$\frac{\partial^2 u}{\partial t^2} = - \frac{g}{\gamma} \cdot \frac{\partial P}{\partial x} , \text{ for acceleration of the fluid;}$$

$$\frac{\partial^2 r}{\partial t^2} = \frac{g}{b\gamma}\left[dP - \frac{Eb}{r^2}\, dr\right]$$, for radial acceleration;

Korteweg stated the radial acceleration could be neglected due to the low inertia of the pipe walls. (This has been verified later [58].) Hence he found from the last equation

$$dP = \frac{Eb}{r^2}\cdot dr \quad \text{for the elastic equation of the pipe walls.}$$

From these equations he obtained

$$\frac{\partial^2 u}{\partial x^2} = \frac{\gamma}{g}\left[\frac{1}{K} + \frac{2r}{bE}\right]\frac{\partial^2 u}{\partial t^2} ,$$

whence the velocity of propagation

$$a = \sqrt{\frac{Kg}{\gamma}} \div \sqrt{1 + \frac{2Kr}{Eb}} .$$

This is identical with present day theory. Korteweg stated that, for Wertheim, Kundt and Dvorak, the correction factor for the effect of the elastic walls was $\sqrt{1 + \dfrac{2Kr}{bE}}$.

He also verified that Resal's and Weber's solution agreed with his if the fluid elasticity factor $\dfrac{1}{K}$ was neglected.

We note that Korteweg's first equation is our acceleration equation, but he did not develop its interlocking equation by differentiating his second equation with respect to time.

Lamb, Horace in 1878 published his text book "Motion of Fluids" [23], which is similar to that of Lord Rayleigh. Later in 1910 he published his "Theory of Sound" which is a revision of the earlier work.

Michaud, Jules in 1878 published an article [24] which to the author's knowledge is the first to deal with the problem of water-hammer.

He does not develop or use any of the theoretical solutions for the wave propagation (at that time no relationship of the pressure and velocity surges had been discovered) but he studied extensively the design and use of air-chambers and safety valves in the pipe lines to ameliorate the effects of sudden and gradual closures of gates and valves.

Gromeka, V.I.

In May 1883 he presented a paper [25] on water-hammer surges and seems to be the first to consider the effect of friction, apart from the experiments of Marey and the latter's general conclusions.

Gromeka criticised much of the earlier work, mainly because it neglected friction. However, there are several assumptions of his which are open to criticism also. He based his study first on the assumption that the pipe walls are thin and act as a cylindrical membrane. Then he applied membrane theory to determine its oscillations. He assumed the liquid is incompressible and also that the friction force is proportional to the velocity (i.e. the velocities are low and flow is laminar). In conclusion he stated that the terms dependent on friction made the equations too difficult to solve and he limited himself to the effect of the inertia of the pipe walls. He is throughout thinking of two waves being propagated, one in the pipe walls and the other, dependent on the former, in the fluid, and that there must be equilibrium between these "inner and outer pulses". When he reduced his formula to the case of negligible pipe inertia he obtained Resal's solution.

During the years 1885 to 1899 several engineers in the United States undertook experiments in water-hammer, with and without

air chambers, and some attempted to develop a theoretical relation-
ship between the velocity reduction and the corresponding pressure
rise. There was little success. In many cases this was due to the
shortness of the pipe lines and the slow closure of the valves.

Three of the most notable of these engineers were -

(1) Weston, E.B. [26] at Providence, R.I.

(2) Church, I.P. [27,28] at Cornell, and

(3) Carpenter, R.C. [29] at Cornell, who worked with

   two graduate students.

In his discussion, Carpenter attempted a theory based on the
elasticity of both the water and the pipe walls. He also used
relatively short pipes 375 feet long, both with and without air
chambers.

Frizell, J.P.

He presented a paper [30] in October 1897 which gave the first
known analytical treatment of the pressure and velocity surges due to
water-hammer. This paper was the result of his studies as consulting
engineer for the Hydro-Electric development at Ogden, Utah, where
they were concerned with the effect of a 20% reduction of the power
demand on the pressure surges in the penstock 31,000 feet in length.
These surges were found to interfere with the speed regulation of
the turbines (see also Church, Ref. 27).

Frizell developed the fundamental formulae for the velocity
of the shock wave and for the intensity of pressure due to an
instantaneous reduction of the flow. It is remarkable that he was
able to do this, apparently without knowledge of the studies which
had been made in Europe, resulting in the equation for the wave

propagation I and its functional solution II.

His treatment is worth our study. He considered a piston being pushed along a pipe. As it moved it compressed the fluid and also increased the cross-sectional area of the pipe in its neighbourhood. The piston must continue to move in order to maintain the pressure, and more and more of the fluid and of the pipe length will become strained.
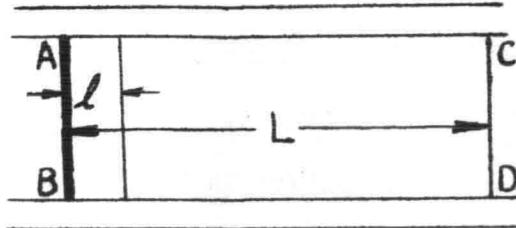
FIG.3.

Using our notation, and referring to Fig. 3, let the piston move a distance ℓ in a time t, and let the fluid between sections AB and CD, over a length L, be compressed during this time. Then due to the elastic properties of the water and pipe walls, he found that

$$\ell = L P \left[\frac{2r}{Eb} + \frac{1}{K} \right] .$$

The velocity of the piston is therefore

$$V = \frac{\ell}{t} = \frac{LP}{t} \left[\frac{2r}{Eb} + \frac{1}{K} \right] .$$

Frizell now considers the ratio of this velocity, (which is the velocity of the whole compressed column ABCD and is the result of the force applied to the piston), to the velocity gt which the column would have if acted on by gravity, stating that the ratio of these velocities is equal to the ratio of the forces applied.

We interject here the remark that, alternatively, he could have used the equation for impulse equal to the change in momentum.

The resulting wave velocity is

$$a = \frac{L}{t} = \sqrt{\frac{g}{\gamma} \div \left[\frac{2r}{Eb} + \frac{1}{K} \right]} \quad ,$$

which is identical with that obtained by Korteweg and others.

To find the pressure rise due to a sudden gate closure, with a corresponding velocity rejection of V in the column, he considered the volumn of Fig. 3 to be moving to the left and stopped by the piston AB. This led to the solution, obtained by considering the volumetric compression,

$$P = \frac{V}{a} \div \left[\frac{2r}{Eb} + \frac{1}{K} \right] .$$

If we express the bracketed expression in terms of the wave velocity a, this becomes $P = \left[\frac{a\gamma}{g} \right] V$, which is identical with the solutions of later investigators.

This result could have been obtained directly from his original study relating V to L, P and t, or, more elegantly perhaps, he could have equated the impulse of the force on the gate to the change in momentum of the mass, obtaining

$$PAt = \frac{AL\gamma}{g} \cdot V, \quad \text{whence} \quad P = \left[\frac{a\gamma}{g} \right] V \quad .$$

Without doubt Frizell understood the action of water-hammer thoroughly. His analysis is fundamentally the same as that of Korteweg and others, including his contemporaries Joukowski and Allievi. Frizell did not use the relationship between water-hammer and sound waves, as did Joukowski. He did, however, state that the wave velocity, as the pipe modulus $E \longrightarrow \infty$, was that of sound in unrestricted water.

His work was criticised and not accepted by his American contemporaries. One of them "saw no reason why this coincidence (with the velocity of sound)" should reassure Frizell regarding the validity of his results.

Frizell also considered the effects of branch lines and wave reflection and discussed cases of slow closure, explaining why earlier studies and experiments had failed to develop theories because the gate closure times were greater than the period. He discussed the effect of successive waves on speed regulation and suggested tests to check his theory.

It is difficult to realize at this present time why Frizell's work was not appreciated and why he has not been given a place of distinction in the development of the subject. One reason is perhaps the lack of communication across the Atlantic.

Joukowski, N.

During the summer of 1897 and the following winter, at almost identically the same time as Frizell's studies, Joukowski made extensive experiments in Moscow in his capacity as consulting engineer for the municipal water-works. As a result of these tests and of his theoretical studies he published a report [31] in the spring of 1898 on the basic theory of water-hammer.

This is a classic piece of work as it verifies the theoretical relations between the pressure and velocity changes during the surges with experimental tests. He tested three loops of cast-iron pipe, the flow being stopped almost instantaneously. These were -

    (1)   25,000 feet of 2" diameter;

    (2)   1,000 feet of 4" diameter;

(3) 1,000 feet of 6" diameter.

Unfortunately, the author has at hand only the translation of this paper, so that there may be important and illuminating details in the original text of which he is unaware.

Joukowski was very well acquainted with the work that had already been done, as he mentions the earlier work of Marey, Gromeka and Korteweg, as well as the experiments of the engineers in the U.S., including the most recent paper by Frizell (i.e. according to the translation of 1904).

He developed independently the formula for the wave velocity, taking into consideration the elasticity of both the water and the pipe walls. His formula agrees identically with those of Korteweg and Frizell. He also developed the relation between the velocity and pressure surges, using two methods, one the conservation of energy and the other the continuity condition. The latter is given in the translation and agrees identically with Frizell's formula.

In his experimental work he used Marey's chronograph to measure the time to .01 second and Crosby Indicators to record the pressures. These two sets of instruments were correlated by means of a pendulum which made an electric contact every half second.

Joukowski does not use the functional solution I I of the wave equation. However, he shows very clearly how the pressure wave travels along the pipe and the nature of the reflections at the ends. Thus for a sudden gate closure, he shows that the wave of positive pressure and zero velocity reaches the upper (reservoir) end at time $\frac{L}{a}$, and for the return wave, the pressure has returned to normal (that obtaining at the reservoir) and the velocity becomes

full negative, i.e. flow is back into the reservoir. This and following surges are shown in Fig. 4.

$\frac{3}{4}\frac{L}{a}$ $V=0; P=+; \quad \rightarrow \quad \begin{matrix} V=+ \\ P=0 \end{matrix}$

$\frac{11}{4}\frac{L}{a}$ $V=0; P=-; \quad \rightarrow \quad \begin{matrix} V=- \\ P=0 \end{matrix}$

$\frac{5}{4}\frac{L}{a}$ $V=0; P=+; \quad \leftarrow \begin{matrix} V=- \\ P=0 \end{matrix}$

$\frac{13}{4}\frac{L}{a}$ $V=0; P=-; \quad \leftarrow \begin{matrix} V=+ \\ P=0 \end{matrix}$

$\frac{7}{4}\frac{L}{a}$ $\begin{matrix} V=0 \\ P=+ \end{matrix} \leftarrow \quad V=-; P=0;$

$\frac{15}{4}\frac{L}{a}$ $\begin{matrix} V=0 \\ P=- \end{matrix} \leftarrow \quad V=+; P=0;$

$\frac{9}{4}\frac{L}{a}$ $\begin{matrix} V=0 \\ P=- \end{matrix} \rightarrow \quad V=-; P=0;$

$\frac{17}{4}\frac{L}{a}$ $\begin{matrix} V=0 \\ P=+ \end{matrix} \rightarrow \quad V=+; P=0;$

Time   GATE   FIG. 4.   RES'R   Time   GATE   RES'R
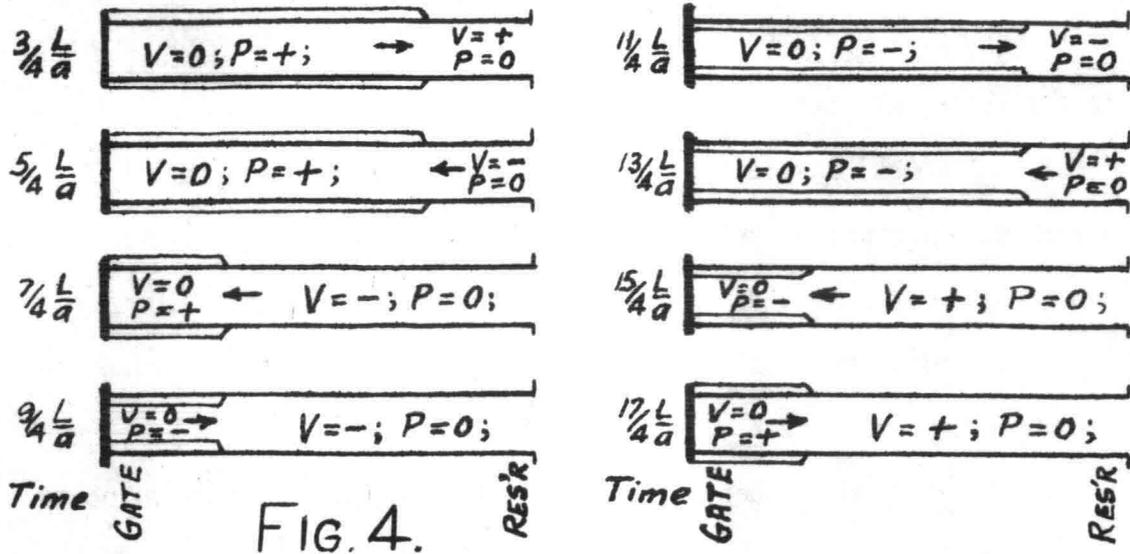
On the reservoir side of the surge front the pressures are normal, as imposed by the reservoir, and the velocities alternate between positive and negative as the surge is reflected. On the gate side of the surge front the velocities are zero as imposed by the gate, and the pressures alternate between positive and negative as the surge is reflected. A complete cycle of surges occurs in two periods, i.e. in a time interval of $\frac{4L}{a}$.

Amongst the phenomena Joukowski studied and his conclusions were:

(a) Passage of the pressure wave into a smaller pipe with a dead end.

(b) Reflection of the pressure wave from an open end of a branch pipe.

(c) Variation of the time of closure of the gate or valve, verifying that pressures are maximum for closing times $t \lessgtr \frac{2L}{a}$.

(d) Effect of air chambers and of large water chambers (e.g. a 12" diameter pipe).

(e) Spring safety valves were found very effective and used later at all important junctions in the water-works.

(f) Detection of leaks in the system could be detected by depressions in the pressure diagram and located by the position of these drops in the chart.

These two consulting engineers, Frizell and Joukowski, developed the practical theory of pressure surges for two different reasons. Frizell was concerned with the safety and speed regulation of hydro-electric plants. Joukowski was concerned with the safety and operation of municipal water works.

Until their time, investigations of these waves were for scientific reasons and particularly concerned with the wave velocity. From this time on, engineers would be assuming the important role in developing the science and in its practical application.

It would have been a simple step for Weber and his contemporaries to find the relation between the pressure and velocity surges but they did not have the practical urge. Thus, using Weber's continuity equation and his elastic modulus and considering an element of pipe dx which is traversed by the wave in a time dt we obtain

$$dV = -\frac{2}{r} \cdot \frac{dr}{dt} \cdot dx = -\frac{2}{r} a.dr = -\frac{2ka}{r} \cdot dP = -\frac{a}{K_P} \, dP,$$

which can be reduced to the form $dP = -\frac{a\gamma}{g} \cdot dV.$

Allievi, L. in 1902 and 1913 published his texts on the general (32) theory. These were both translated into French and English. In his

1902 paper he developed the theory from first principles, similar to Korteweg. However, he obtained a more accurate fundamental first order equation for the acceleration

$$-\frac{\partial P}{\partial x} = \frac{\gamma}{g}\left[\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x}\right] = \frac{\gamma}{g}\frac{dV}{dt} \; ,$$

where he used the total derivative of the velocity with respect to time.

The continuity equation was unchanged, and it was not until 1937 [51] that this latter equation was expressed with the total derivative $\frac{dP}{dt} = \frac{\partial P}{\partial t} + V\frac{\partial P}{\partial x}$ replacing the partial $\frac{\partial P}{\partial t}$.

Allievi showed that the term $V\frac{\partial V}{\partial x}$ could be dropped because of its relative unimportance and so obtained the second order equation and its solution in functional form I and II. He introduced two dimensionless quantities called the characteristics, defined as

$$\rho = \frac{a V_0}{2 g y_0} \; ; \quad \theta = \frac{a \tau}{2 L} \; ; \quad Note \quad \frac{\rho}{\theta} = \frac{L V_0}{g y_0 \tau} \; .$$

Here ρ is half the ratio of the kinetic energy of the water and the potential energy stored in the water and pipe walls due to the hydraulic head y_0 , and varies from .1 for high heads and low velocities to 10 for low heads and high velocities. It is a pipe flow characteristic.

θ is a gate operation characteristic, τ signifying the equivalent time of gate closure. Thus if τ is five times the period $\frac{2L}{a}$, $\theta = 5$. Using uniform gate closure in time T, he obtained the general solution for maximum pressure rise (Halmos p. 30).

$$\frac{h}{H_0} = \frac{1}{2}\left[\frac{\rho}{\theta}\right]^2 \cdot \left[1 + \sqrt{4\left(\frac{\theta}{\rho}\right)^2 + 1}\right] \; .$$

Allievi's work is too monumental to be studied in any detail
here. It covers the whole field of operation, including gradual
gate closures, and his charts and tables are all-embracing. This
fact is perhaps a drawback in the sense that one must become
immersed in the work in order to derive full benefit.

CHAPTER IV   DEVELOPMENT IN THE 20th CENTURY UP TO 1925

The study of water-hammer has now assumed a broader world-
wide aspect, carried on mostly by engineers and concerned mainly
with its practical importance. Fields of study have become more
specialized and the methods are more accurate and in greater detail,
with experimental tests and the determination of the hydraulic
characteristics of the various components playing an important role.

It becomes increasingly difficult to consider all the
specialized studies being carried on throughout the world. In this
history we will now limit ourselves to the continued development of
the fundamental ideas and of their practical use in the various fields,
rather than attempting a history of the work done by all workers in
all specialized fields. Originality of development will be the key
to recognition, although an immense amount of work of extremely high
practical value has been done by outstanding engineers in the various
specialized fields, much of this work being of a consulting nature
unavailable for publication.

During the first twenty years of the century there was a
great amount of work in applying the theory developed and published
by Joukowski and Allievi to the practical design of water-works and
hydro-electric plants. In Europe the authority was Allievi, due

mainly to the French translations. In North America Joukowski's text was the key, partly because Miss Simin's translation was available in 1904, whereas Halmos' translation of Allievi did not appear until 1925.

These early designs were in the main concerning penstocks and surge tanks. For example, Warren, M.M. presented an important paper on Penstock and Surge Tank Problems [33], which reviewed the fundamental theory, stating the wave equation and its functional solution, and giving the formulae for the wave velocity and pressure rise due to a sudden velocity rejection.

The difficulty encountered at this time was in finding the pressure rise for slow gate closure (the term used here was "ordinary water-hammer"). Warren assumed that for slow closure times T, greater than the period $\frac{2L}{a}$ the maximum pressure surge occurred just when the first wave returned to the gate and that thereafter the negative (reflected) surge would cancel any increase in pressure due to continued gate closure, friction finally causing the pressure to return to normal. On this assumption he found the maximum pressure rise for slow closure was

$$H_R = \frac{LV}{g(T - L/a)}$$

The reader should consult Allievi on this point.

Constantinescu, G., in 1920 described his invention of a mechanism to transmit mechanical energy by use of the water-hammer wave [34]. The energy involved could be large and the transmission distance long, for very small displacements of the liquid, (usually an oil). He gave many examples of the application of this method in various fields.

The author recalls that during the First World War, British fighter planes were equipped with the Constantinescu gear, a device for remote control of the firing of the machine guns. These were timed to shoot between the propeller blades. This gear was later superseded by a purely mechanical gear, partly because any malfunction of the mechanism could (and often did) result in the propeller blades being shot off near the hub.

Gibson, N.R. presented a paper [35] in 1920 on pressure surges due to gradual gate closures, basing his study on Joukowski. In all these and subsequent studies of surges in simple pipe lines due to closure of gates at the lower end, the relation between the discharge and the pressure loss through the partially closed gate has been based on some assumed law which seems reasonably applicable, e.g. that at any particular gate position the head loss through the gate varies as the square of the discharge rate (i.e. the velocity). In the case of a hydraulic turbine, the whole unit below the control mechanism is usually assumed to obey this law, the head loss being the drop in pressure head between the control and the tailrace. Gibson's treatment is now given in some detail as it will be of use in later comparisons.

It is assumed that the gate movement is uniform, i.e. that the (equivalent or hypothetical) area of the gate opening B is reduced uniformly with time during the time of gate closure T. It is also assumed that the velocity in the penstock just above the gate is proportional to the gate area and to the square root of the head loss through the gate. Hence the head loss through the gate and the penstock velocity at any intermediate time t during the first period are related by the conditions -

$$V_t = \left[1 - \frac{t}{T} \right] B_o \sqrt{H_o + h_t} \ ; \quad h_t = \frac{a}{g} \left(V_o - V_t \right) ;$$

where B_o is defined by the conditions at time $t = 0$:-

$$V_o = B_o \sqrt{H_o} \ ; \quad h_o = 0 , \quad \text{and} \quad V_t , H_t$$

are velocity and head loss through the gate at time t .

In the example of the article, successive sudden partial closures of the gate occurred at times $t = (1,2,3,4)$ units of $\mathcal{T}/4$; i.e. there were four partial closures of the gate during the first period, and during each succeeding period until closure.

During succeeding periods until complete closure, the first relation for V_t above is unchanged, and at complete closure, when $t = T$, $V_t = 0$, remaining zero thereafter.

During the second period, where $t = (5,6,7,8)$ $\mathcal{T}/4$, the reflection of the first waves must be allowed for, and the condition for h_t is

$$h_t = h_{t-1} + \frac{a}{g} \left(V_{t-1} - V_t \right) - \frac{2a}{g} \left(V_{t-5} - V_{t-4} \right) ; \quad t = 5,6,7,8 \ units.$$

During the third period there are two return waves imposing their reflections, and hence

$$h_t = h_{t-1} + \frac{a}{g} \left(V_{t-1} - V_t \right) - \frac{2a}{g} \left(V_{t-5} - V_{t-4} \right) + \frac{2a}{g} \left(V_{t-9} - V_{t-8} \right) ;$$

$$t = 9,10,11,12 \ units.$$

This procedure continues until closure, after which time

$$V_t = 0 ; \quad h_t = h_{t-1} + \frac{a}{g} \left(V_{t-1} - V_t \right) - \frac{2a}{g} \left(V_{t-5} - V_{t-4} \right) + \cdots - \cdots$$
$$etc.$$

It is seen that solutions involve quadratic equations in h_t.

Allowing for friction, the quantity $F\left(V_o^2 - V_t^2\right)$

is added to the expression under the square root sign in the V_t

expressions, as this is the value of the regained friction head.

This is the first instance of solutions including non-linear friction.

It was possible at that time only by using arithmetic integration, the

fore-runner of modern computer methods.

For non-uniform gate motion a coefficient C_g may be used,

and this coefficient included in the gate opening parameter B_o. It can be

made to fit any non-uniform closure, if necessary being expressed in

tabulated form.

This paper of Gibson's is particularly valuable because of the

discussions by eminent engineers, e.g. Eugene Halmos, who gave an

excellent synopsis of Allievi's theory. He pointed out common errors

in using Allievi's formulae, due to carelessness in following the

instructions regarding their conditions of operation.

In 1923 Gibson invented an apparatus for measuring the

discharge of hydraulic turbines, using the pressure-time surges

caused by the closure of the gates. He used it for running efficiency

tests on new power plants. Another method invented by Prof. Allen

and known as the "salt velocity" method, was equally accurate and

these two methods were in general use in North America during the

1920's and later.

CHAPTER V DEVELOPMENT - 1925 to 1955

Strowger and Kerr in 1926 presented a paper [36] on the speed

regulation of hydraulic turbines, using the method of computing the

pressure rise, and corresponding velocities due to the gate movement,

given by Gibson [35]. They took into account the efficiency of the turbine at the different gate positions and considered both uniform and non-uniform gate movement. From the values of head, velocity and efficiency, they obtained the excess or deficiency in H.P. input and calculated the speed variation.

The discussion of this paper, as in the case of Gibson's, was of great value. P.F. Kruse discussed the effect in low-head plants due to the relatively large proportion of the total ΣLV which was in the draft tube passages. He used as a basis for his calculations of the total pressure rise the total ΣLV. Then he determined the pressure surges above and below the gates by using the ΣLV above and below as a criterion. He noted also that for such installations the draft tube design involved not only the regain of energy but the effect of the inertia of the water column on the speed regulation.

T.H. Hogg and J.J. Traill gave valuable data obtained from regulation tests on the 55000 H.P. units at Queenston and the 2200 H.P. South Falls units, both operated by the H.E.P.C. of Ontario. In addition to the comparison of test and calculated regulation, they pointed out that in some of the Queenston tests there was a separation of the water column, causing violent shocks felt and heard near the unit. With slower gate closure the shocks did not occur. It was because of this separation rather than excessive pressure rise that the governor traversing time was increased.

The author of this history also presented a discussion in which he developed the graphical method. This will now be illustrated by using the example already discussed in Gibson's paper [35].

It is not difficult to show that the linear relation between head and velocity during the second and succeeding periods may be simplified to the form

$$h_t = -h_{t-4} + \frac{a}{g}(V_{t-4} - V_t).$$

This eliminates the inclusion of all the waves and their reflections since the start of the gate closure and shows that all of these waves and reflections are automatically incorporated in the single wave which left the gate one period earlier.

If we use Joukowski's chart (Fig. 4) as a guide, the wave which left the gate one period earlier is reflected from the reservoir one half period earlier. There it has had its pressure changed by an amount $-h_{t-4}$ and the velocity has been changed by a corresponding amount $-\frac{g}{a}h_{t-4}$. On its return to the gate it is now a wave having

$$h = 0; \quad V = V_{t-4} - \frac{g}{a}h_{t-4},$$

which is superposed on the new wave at time t, to give us the equations

$$V_t = (1 - \frac{t}{T})B_o\sqrt{H_o + h_t}; \quad h_t = \frac{a}{g}\left[(V_{t-4} - \frac{g}{a}h_{t-4}) - V_t\right].$$

In Fig. 5, Gibson's example is illustrated by the graphical representation of the above relations. The velocity expressions are represented by parabolas $V = G_t\sqrt{H}$. In order to standardize, the head and velocity are made dimensionless by putting

$$h = \frac{H}{H_o} = 1 + \frac{h_t}{H_o}; \quad v = \frac{V}{V_o}.$$

The horizontal line OA represents constant head $h = 1$, $H = H_o$;
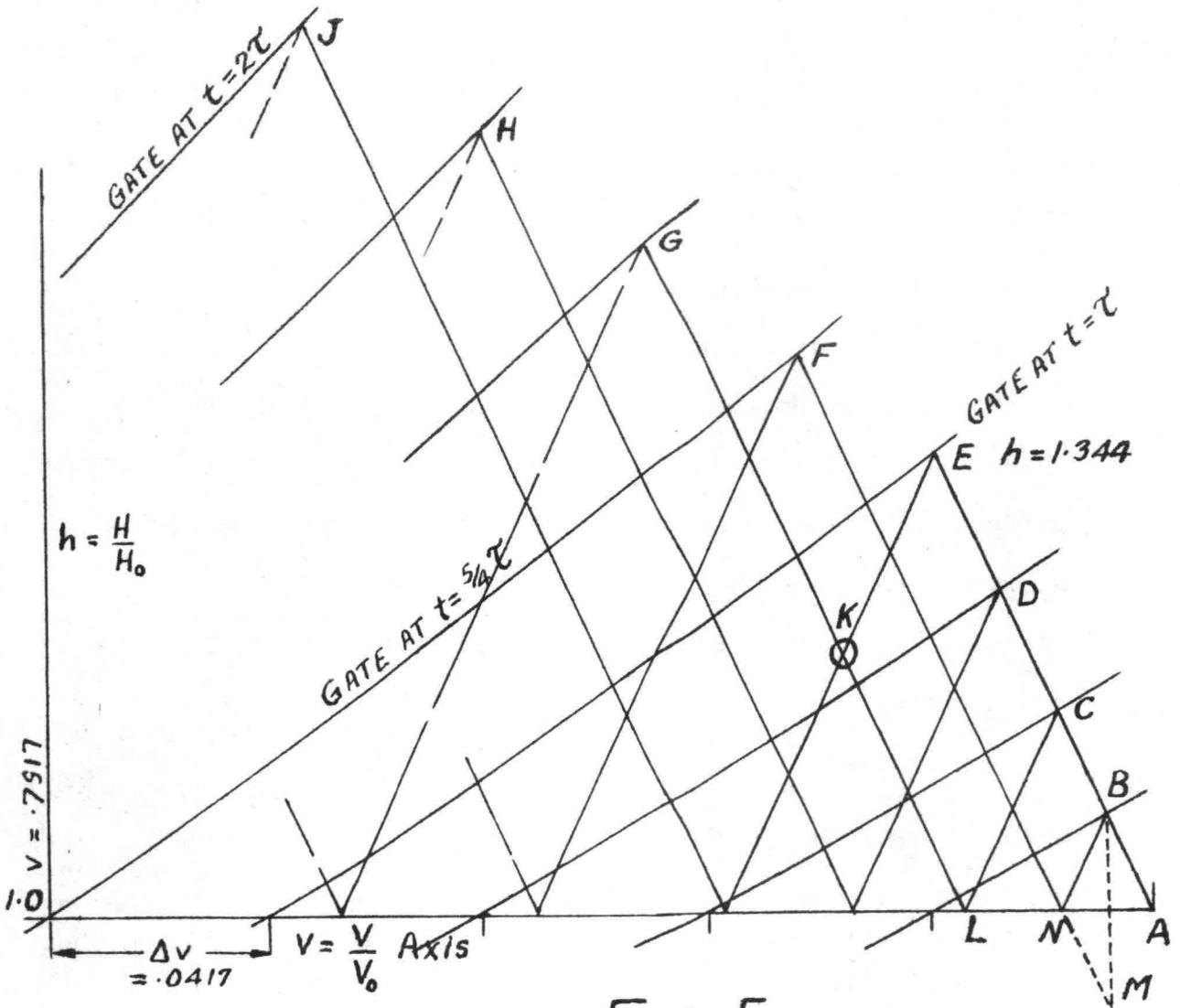
FIG. 5.

the vertical line through 0 represents zero velocity $v = 0$, $V = 0$. The vertex of the parabolas is at $v = 0$, $h = 0$, i.e. a unit distance below 0. For uniform gate closure the parabolas are evenly spaced along OA for uniformly spaced instants of time.

The linear expressions for h_t (note that this is Gibson's h_t and is not dimensionless), are now, in dimensionless form,

$$\frac{h_t}{H_o} = \frac{a\,V_o}{g\,H_o}\left[\frac{V_{t-4}}{V_o} - \frac{V_t}{V_o}\right] - \frac{h_{t-4}}{H_o}.$$

They are represented by straight lines having a slope of $-\dfrac{a\,V_o}{g\,H_o}$. The gate parabolas in dimensionless form are

$$v = \beta\sqrt{h}$$

where β is a new dimensionless partial gate characteristic.

In the diagram the parabolas are drawn for time intervals of quarter periods. The condition at the gate at times $t = (1,2,3,4)\dfrac{\tau}{4}$, i.e. up to the instant τ, are given by the points B,C,D,E. The condition at time $1.25\,\tau$ must satisfy the straight line MNF and also the parabola for $t = 1.25\,\tau$. These constructions meet at F.

In our dimensionless $\dfrac{h_t}{H_o}$ equation above, the h_{t-4} term is represented by the point M, and the $(V_{t-4} - V_t)$ term by the horizontal distance between M and F. The point M, a reflection of point E across the OA axis, is used if we employ Gibson's modified expressions. However, if we follow the preceding (t - 4) wave up the pipe to the reservoir, the returning reflected wave is represented by the point N, drawn from B with slope $+\dfrac{a\,V_o}{g\,H_o}$. Then the reflection of this wave at the gate will give us the line NF to be satisfied by the gate condition, the parabola through F.

In order to use a single chart applicable to each particular runner model, regardless of head and discharge and horse power for any particular plant, this dimensionless h and v chart was used as a base for each runner, and curves of equal H.P. excess, (or deficiency) were added. These curves are based on the model runner performance (efficiency) at the part-gates. Thus, by reading off the excess (deficiency) of H.P. at successive instants of time, the

average H.P. may be computed and the speed change found.

Löwy, R. in 1928 published his text on water-hammer [37] which covered the analytical step-by-step solution of the surges in the pipe line, and also (pp. 84 and following) his graphical method, which is identical in development and operation with that of the writer. There is no doubt that the two studies were developed independently of each other.

Löwy also studied resonant surges due to periodic oscillations of the gate, (this period being $\frac{2L}{a}$, that of the pipe line), and surges due to gradual opening of the gate. He also considered the effect of friction, as did Gibson [35], but his attack is more analytical, using the partial differential equations with friction terms added.

Löwy's work became the basis of the extension of the graphical method by European engineers during the following years. It was not until 1935 when Prof. Angus published a paper [42] at the E.I.C. meeting in Toronto, which was based on the work of Schnyder, that the earlier graphical study by the writer, which was a discussion of the paper by Strowger and Kerr [36], was recognized.

From this time on, engineers became more and more concerned with the solution of the transient performance of systems in which pipe lines were an integral part. The law governing the pressure surges in the pipe lines had been solved, at least to a good approximation, although it was not for some years that a more rigorous solution of the effect of friction and of low moduli of elasticity in the pipe walls was obtained. The main goal was to tie in the characteristics of the various hydraulic elements connected

with the pipe lines, such as pumps, surge tanks, relief valves, air chambers and turbines, as well as to develop solutions for compound pipes and pipe networks.

Since the various contributors who worked and published during this period had ready access to the literature, and since the time lag between research and publication is continually increasing, it is difficult to determine priority for any new development.

For this reason a synopsis of the important publications is given in chronological order. Any omissions therefore which are apparent to the reader should be blamed on the writer's ignorance.

<u>1929 Schnyder, O.</u> applied the graphical method to the study of pipe lines connected to centrifugal pumps, using complete pump characteristics as determined by tests [38].

<u>1931 Bergeron, Louis</u> in an article [39] extended the graphical method to express an analytical relationship between the pressures and velocities at any two points on the pipe line. His treatment is a model of clarity. From the basic functional solution II he showed that for two positions (sections) in the pipe P and Q which are related in position and time by the condition

$$(x - at)_P = (x - at)_Q \, , \quad or \quad x_P - x_Q = a\left(t_P - t_Q\right),$$

their pressures and velocities are related by the equation

$$\left(H_P - H_Q\right) = - \frac{a}{g}\left(V_P - V_Q\right).$$

Similarly for two positions R,S, which are related by the condition

$$(x - at)_R = -(x - at)_S \, , \quad or \quad x_R - x_S = -a\left(t_R - t_S\right),$$

the equation relating their pressures and velocities is

$$\left(H_R - H_S\right) = + \frac{a}{g}\left(V_R - V_S\right).$$

Thus, if the values of H,V are known at two positions at a given time, these variables can be determined at the section midway between them at a later time, later by the time required for waves to reach it simultaneously from the two given positions. As an example, referring to Fig. 5, if we are given the condition at the reservoir and gate at the time $t = \tau$, (represented by points L and E respectively), the line LK represents the first equation above, the point L corresponding to P and any other point on this line corresponding to Q at a suitable later time. Similarly, the line EK represents the second equation, the point E corresponding to R and any other point on this line corresponding to S at a suitable later time. Since these two lines intersect at K, and since the two pseudo-waves from L and E will reach the centre section at the same time $t = \frac{5}{4}\tau$, the point K gives the conditions at the centre section at time $t = \frac{5}{4}\tau$, and it can be considered as both positions Q and S in the two conditions. In this way the conditions at all sections in the pipe can be found if we know the conditions at the two ends.

This treatment is of great value when studying pipe networks and compound pipes. For these studies the additional laws necessary for solution of the conditions at the junctions and change-in-section are :—

      (1)   Continuity of flow, i.e. sum of all volumes of flow

              into a junction is zero;   and

(2) pressures in all the branches at a junction
are equal.

<u>1932 Schnyder</u> in his article [40] was the first, (using the graphical method) to allow for friction along the pipe line. He drew a friction curve below the velocity axis (i.e. the h = 1 axis), with ordinate $-FV^2$, (minus the friction head for the full pipe length, based on D'Arcy's parabolic law). His constructions started from and were reflected from this line instead of from the horizontal line h = 1.

This is an approximate method of lumped friction, all of the friction head being "lumped" at the reservoir (upstream) end, and is the graphical equivalent of Gibson's solution [35]. Schnyder, in this article, studied also the surges in compound pipes.

<u>1933 Symposium</u>. At the meeting of the ASME and ASCE during "Engineering Week" at the Chicago Exposition several papers [41] were presented by engineers from North and South America, and discussions by Engineers in Europe also. A summary of the existing theory for simple pipe lines was given. The papers presented were on high-head penstocks, compound pipes, surge tanks and centrifugal pump installations, equipped with air chambers and relief valves.

<u>1935 Angus, R.W.</u> presented a paper [42] covering the basic theory and some applications of the graphical method, including the use of "lumped" friction and the study of pump installations.

<u>1937 July, August. Bergeron</u> wrote a general paper [43] covering all the theory of plane elastic waves in various media, longitudinal bars, vibrating strings, torsional oscillations with and without attendant masses and flywheels, electric transmission lines and water-hammer with "lumped" friction. This was followed in 1938 by a paper

of a similar nature at the 5th International Congress of Applied
Mechanics.

1937 December. Second Water-Hammer Symposium ASME, ASCE, and AWWA,
held at the annual meeting of the ASME in New York. Many papers were
presented by Engineers from both America and Europe.

Allievi presented a paper in absentia on the use of Air Chambers in
discharge lines [44]. This was his last paper (translated by Halmos).

Angus presented a paper on the effect of air chambers and valves in
pump discharge lines [45].

De Juhasz, K.J. presented a paper in Fuel Injection Systems [46].
This was very instructive, using the graphical method, with stereo-
grams to illustrate the operation of the waves.

Knapp, F. presented a paper on emergency shut-off valves [47].

Knapp, R.T. presented a paper [48] in which he discussed the complete
characteristics of Centrifugal Pumps. This paper is similar to, but
in much more detail, than the earlier paper by Schnyder [38] and
indicates how the pump characteristics can be tied in with the surges
in the pipe line.

Schnyder presented another paper comparing calculated and test results
on water-hammer surges in pump lines [49].

Strowger presented a paper [50] on water-hammer in a hydro-electric
plant with governor-controlled relief valve, both with and without
allowance for friction.

The writer presented a paper [51] using Heaviside's operational
calculus. Included was the analytical solution using a linearized

friction term. The fundamental continuity and dynamic equations were developed, the former including for the first time the total derivative $\frac{dP}{dt} = \frac{\partial P}{\partial t} + V \frac{\partial P}{\partial x}$.

The friction term was expressed as

$$\frac{2f\gamma}{gD} \cdot V^2 = \frac{2f\gamma}{gD} \left(k_f V_m \right) V,$$

where V_m is the maximum velocity and k_f some suitable constant. The solution showed the velocity and pressure surges gradually died down to the final steady state. It was found that a dimensionless quantity, called the friction modulus, was a criterion for similarity of solutions throughout the field (i.e. for the simplified case of linear friction and neglect of the $V \frac{\partial V}{\partial x}$ and $V \frac{\partial P}{\partial x}$ terms) and is the only parameter required. This modulus is

$$\phi = \frac{2fL}{D} \cdot \frac{k_f V_m}{a}.$$

This symposium was very interesting and instructive due to the presence of most of the leading workers in the field. Louis Bergeron was present and discussed almost all the papers as soon as they were presented. He sat beside Pierre Danel, who translated the papers for him, noted his comments and then delivered this discussion to the meeting. This included the result of rapid graphical analyses. Bergeron, in fact, stole the show.

1938 Angus presented a paper [52] considering compound and branched pipe lines. He also studied breaches of the water column in pump lines, and the effect of friction, suggesting, for greater accuracy, dividing the pipe length into several sections and using "lumped" friction in each section.

1940 Boulder Canyon Project. The report [53] compared theory with test results of water-hammer surges in the penstocks.

1944 Rich, G.R. presented a paper [54] in which he used the Laplace transform, solving in particular the case of friction using a linearized approximation, and compound and branched pipe lines. The solution for the case of friction was in Bessel functions and their integrals. In comparison, the use of the Heaviside Calculus [51] leads to cumbersome and sometimes very slowly convergent series.

The writer in 1951 wrote a paper [55] which was a general study of the graphical method applied to elastic surges in various media. This was similar to the earlier general paper by Bergeron [43].

1953 Lupton, H.R. presented a paper [62] in which he studied surges in pump discharge lines with reference to the separation of the water column.

November 1955. Several papers were presented at the Diamond Jubilee Annual meeting in Chicago, where the ASME, ASCE and AWWA all participated, as in the earlier symposiums in 1933 and 1937. Two of these papers dealt with pump discharge lines.

Richards, R.T. [63] compared theory with tests in several pump discharge lines, with particular regard to the water-separation. His simplest example, involving a practically horizontal pipe line, showed a theoretical surge of 4% in excess of the test surge, when the vacuous space collapsed after the first separation, and the test showed rapidly decreasing surges with time. The theory was based on Angus and Bergeron.

Kittredge, C.P. presented a more general paper [64] which covered theoretical solutions for pump discharge lines using the complete pump characteristics, and comparing the solution for rigid and elastic column theories. Reference is given here also to his earlier paper using rigid column theory in 1933. He stated that the elastic column theory gave higher peak surges but that in cases where the friction losses were considerable, the rigid column theory, neglecting friction, could be expected to give solutions within a few per cent of the test values.

While on this subject of water-column separation, the following references are appropriate:

Duc, F. published an article in 1959 [65] in connection with tests on a transparent pipe 3300 feet long. He found that where gravity could exert its influence in steep sections of the line, there was no visible cavity in the column even though there were present all the signs of vacuum formation.

Sharp, B.B. is conducting some recent work of which the writer has just become aware. He presented a paper in 1966 [66] in which he discussed the effects of the separation of the water column.

The abstract of these papers on column separation shows that more detailed study is required, considering several significant factors such as the profile and length of the line. Special laboratory tests are indicated and it appears that low pressure ranges should be used in order to make visual observations in conjunction with the regular surge instrumentation. Due to limited space it seems necessary to use pipes of low modulus of elasticity

and corresponding low wave velocity of the order of 100 to 500 ft/sec.

The problem of water-hammer in steam pipes is attracting more attention and it is evident that it offers particular difficulties for solution due to the mixture of gas and liquid, i.e. the interaction between the steam and condensate.

<u>Signor, C.W. and Ashrae, J.</u> have written a recent article [67] on this matter. Some of their conclusions are -

1) removal of condensate, as soon as it forms, reduces surges,

2) water-hammer occurs when the condensate is cooler than the steam, and

where the line is either horizontal or inclined upward in the direction of flow.

CHAPTER VI DEVELOPMENT SINCE 1955

There have not been many workers, in this period, on fundamental research of surges in pipe lines, due to the fact that the theory has been well developed and approximations for the effect of friction and other non-linear factors are reasonably accurate in the field of hydro-electric and pumping plants, where the friction losses are kept low and the pipes are metal. The need has been to improve our knowledge of the interaction between the various hydraulic devices (pumps, turbines, valves, etc.) and the pipe lines, i.e. the need to improve our knowledge of the boundary conditions of the pipe line surges.

Any refinement in the theory has been for cases of low moduli of elasticity in the pipe walls and high friction losses.

The addition of the non-linear terms $V \frac{\partial V}{\partial x}$ and $V \frac{\partial P}{\partial x}$ in the fundamental equations precludes solution in neat closed mathematical form, and the use of step-by-step arithmetic integration, with finite differences, is necessary, and this has proved to be arduous, time consuming and complicated in execution.

It was not until the late 1950's that the electronic computer became a possible tool for use in this field, and during the past ten years this has become of great value in the development.

In the following, which deals mainly with these non-linear studies, the writer wishes to apologise for the lack of depth in the record of work by other workers, particularly outside North America. The work of each researcher will be presented in turn and in sufficient detail for the reader to appreciate the work without studying the literature first-hand.

Research by the writer. A paper was printed [56] in January 1958 in which surges in pipe lines and surge tanks were studied by the graphical method, including non-linear friction. Using Bergeron's wave relations [39] for illustration, these were modified to the form

$$H_Q - H_P + \oint_P^Q FV\,dx = - \frac{a}{g}\left(V_Q - V_P\right) ;$$

$$H_S - H_R + \oint_R^S FV\,dx = + \frac{a}{g}\left(V_S - V_R\right) .$$

Here the factor $F = \frac{f}{2gD}|V|$, using D'Arcy's f, so that the friction term will reverse in sign for a reversal in the direction of flow. Also the "directed" integral $\oint$ signifies integration along the positively directed wave, travelling from P to Q. This takes the sign of V and can be approximated by using the average value

of FV along this wave, and multiplying by the distance dx = PQ.
In the calculation the value of V_Q is determined by trial and error.
For the negatively directed integral $\twoheadleftarrow$ from R to S, dx is
negative.

These interlocking equations were reduced to dimensionless
form, using $h = \dfrac{H}{H_o}$; $v = \dfrac{V}{V_o}$, and the construction lines having the
modified slopes $\pm \dfrac{a V_o}{g H_o}$.

For a completely graphical solution the friction can be
included by superposing curved lines above and below the inclined
construction lines having slopes of $\pm \dfrac{a V_o}{g H_o}$, the vertical distance
between the two sets being equal to $\dfrac{FV}{V_o^2}$. This makes it possible to
find the unknown condition S = Q without trial and error. However,
the constructions are time-consuming and it is fair to say that the
graphical method is most valuable for cases where friction is
neglected. In this study it was found desirable to use the graphical
solution as a basis for an analytical solution. This gave greater
accuracy. Also by constructing a table on a position-time grid it
was possible to pursue the solution up and down the pipe and on in
time by listing the appropriate formulae for each operation. In fact
windows were cut in a stencil, for each operation, with suitable
factors written beside each window according to the operation formula.
In this way errors in calculation were minimized and greater speed
attained. This use of a position-time grid was used in later work
until the electronic computer came into use.

By the year 1961 computer programmes had been run for the
dimensionless solutions including non-linear friction, using friction
modulus values ranging between ϕ = 0 to 1.0 (see Ref. 51), and

rejection of velocity at the gate (due to sudden gate closures)
of .25, .5, .75 and full. This work was not published as it showed
close agreement with the earlier work (Ref. 51 and 56).

In the study of surges in pipes having a low modulus of
elasticity, such as rubber and plastic, the non-linear terms $V \frac{\partial V}{\partial x}$
and $V \frac{\partial p}{\partial x}$ must be considered. For metal pipes (steel, copper,
etc.) these terms have a relative importance to the time-derivative
terms with which they are associated of $\frac{V}{a+V}$, and for wave
propagation velocities in metal pipes of the order of 3000 ft/sec.,
this relative importance is of the order of one tenth of a percent.
For non-metallic pipes, however, the wave velocity may be of the
order of 50 ft/sec, so that these non-linear terms have a relative
importance of the order of ten percent.

Several difficulties arose, during the progress of this
research, of a fundamental nature. To enumerate, these were:

(1) The cross-sectional area varied appreciably under
     varying pressure due to the low modulus of
     elasticity of the pipe walls. Even for steady flow
     in a horizontal pipe, the pressure drop due to
     friction losses affected the area and velocity.
     Similarly the profile had to be taken into account.

(2) Determination of the friction coefficient (e.g.
     D'Arcy's f) was difficult to obtain by test and the
     variation in area and velocity during steady flow
     runs required special treatment and programming.
     All equations and development had to be based on
     the conditions at the entrance under steady flow,

designated by the subscript zero.

(3) Determination of the modulus of elasticity was impossible by using the standard laboratory tests in tension or otherwise. For rubber hose this is partly due to the method of manufacture - by vulcanizing a strip of rubber in a spiral - so that the cross-section is not isotropic. For plastic hose, this is partly due to the effect of temperature and ageing on its properties. The solution of this difficulty was found by obtaining a volumetric strain in the hose under varying pressure, and when possible this was done with the hose in place ready for the surge tests. Thus the equivalent bulk modulus, water and pipe combined, was determined by the formula

$$K_e = Q \frac{\Delta P}{\Delta Q} .$$

(4) Entrance loss was found to be significant. Thus the pressure drop at entrance is due to the increase in kinetic energy and friction loss, and is of the order of $1.5 \frac{V_o^2}{2g}$ feet. In terms of the velocity modulus ς this is equal to $h_e = .75 \varsigma$.

(5) To consider the effect of the profile it was necessary to express the drop in elevation in terms of the friction modulus. Thus for a uniform slope giving a total drop of $.5 H_f$ this drop in elevation could be expressed by the dimensionless quantity $.5 \phi$.

(6) The slopes of the characteristics C_+ and C_- vary considerably for small a.

These studies were completed and an abridged edition of the results was published [57] in 1966. This article is unsatisfactory for a detailed study and a later, more complete, report was published [58] in January 1969. The details of this study will not be discussed further. To give the reader a general idea of the extent of the study, the fundamental equations are given in the List of Symbols and Figures 6 to 10 indicate the development and findings. It has been shown that the whole field of surges in pipe lines can be standardized to depend on two parameters only, viz. the friction modulus ϕ and the velocity or elastic modulus ζ.

Figure 6 - shows the pipe element in developing the theory.

Figure 7 - indicates the scheme of computer solution (the x-t grid).

Figure 8 - gives the pressures at gate, sudden closure, $\phi = \cdot2 \text{ to } \cdot8$; $\zeta = 0, \cdot15$.

Figure 9 - gives gate conditions under steady flow, $\zeta = \cdot05 \text{ to } \cdot25$

Figure 10 - gives theoretical and test results for surges in rubber hose.

FIG. 6.

FIG. 7.

Figure 8.

Pressure at Gate, Sudden Full Closure; ζ = 0 and .15
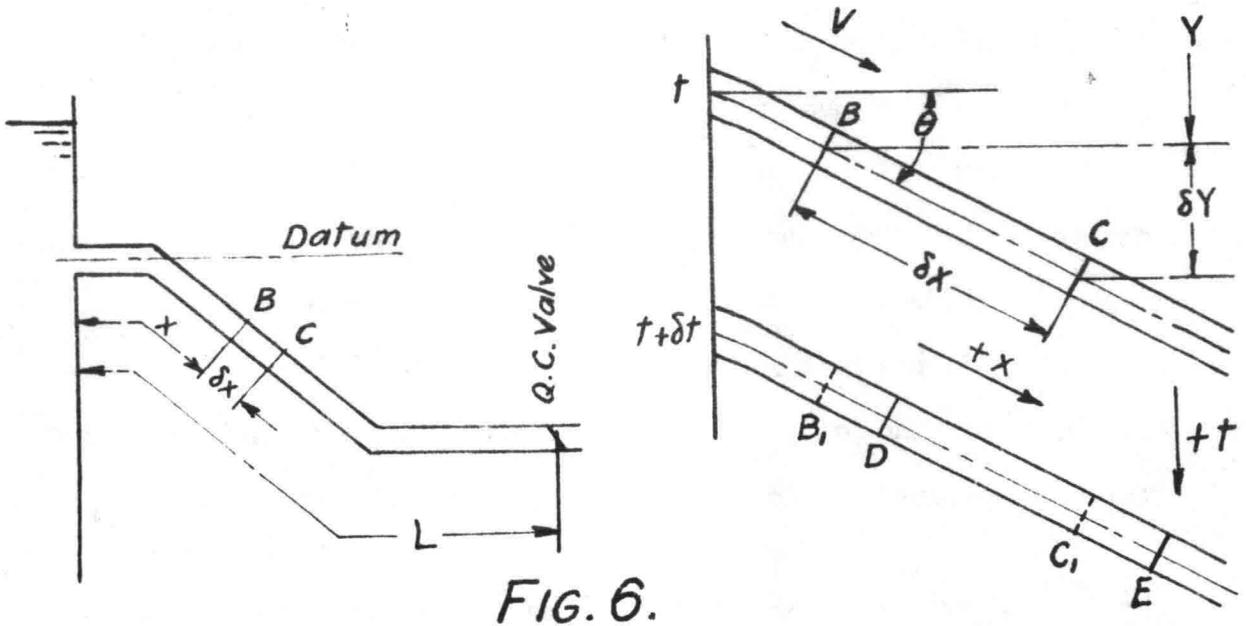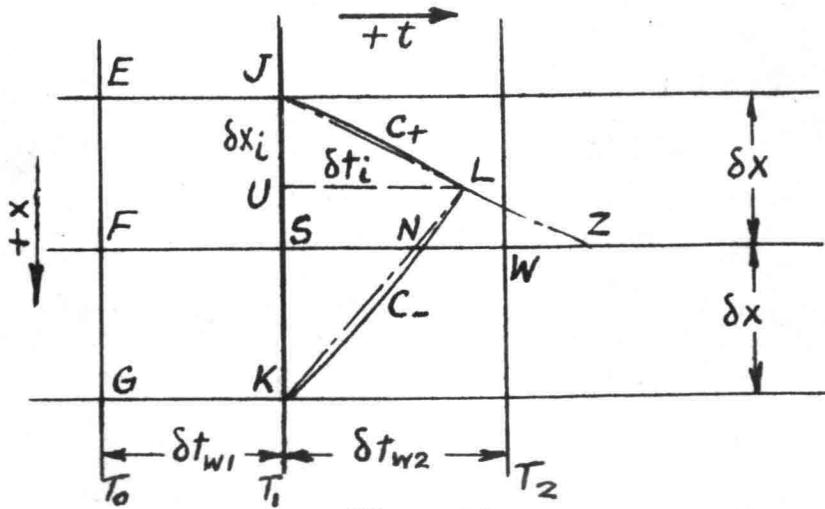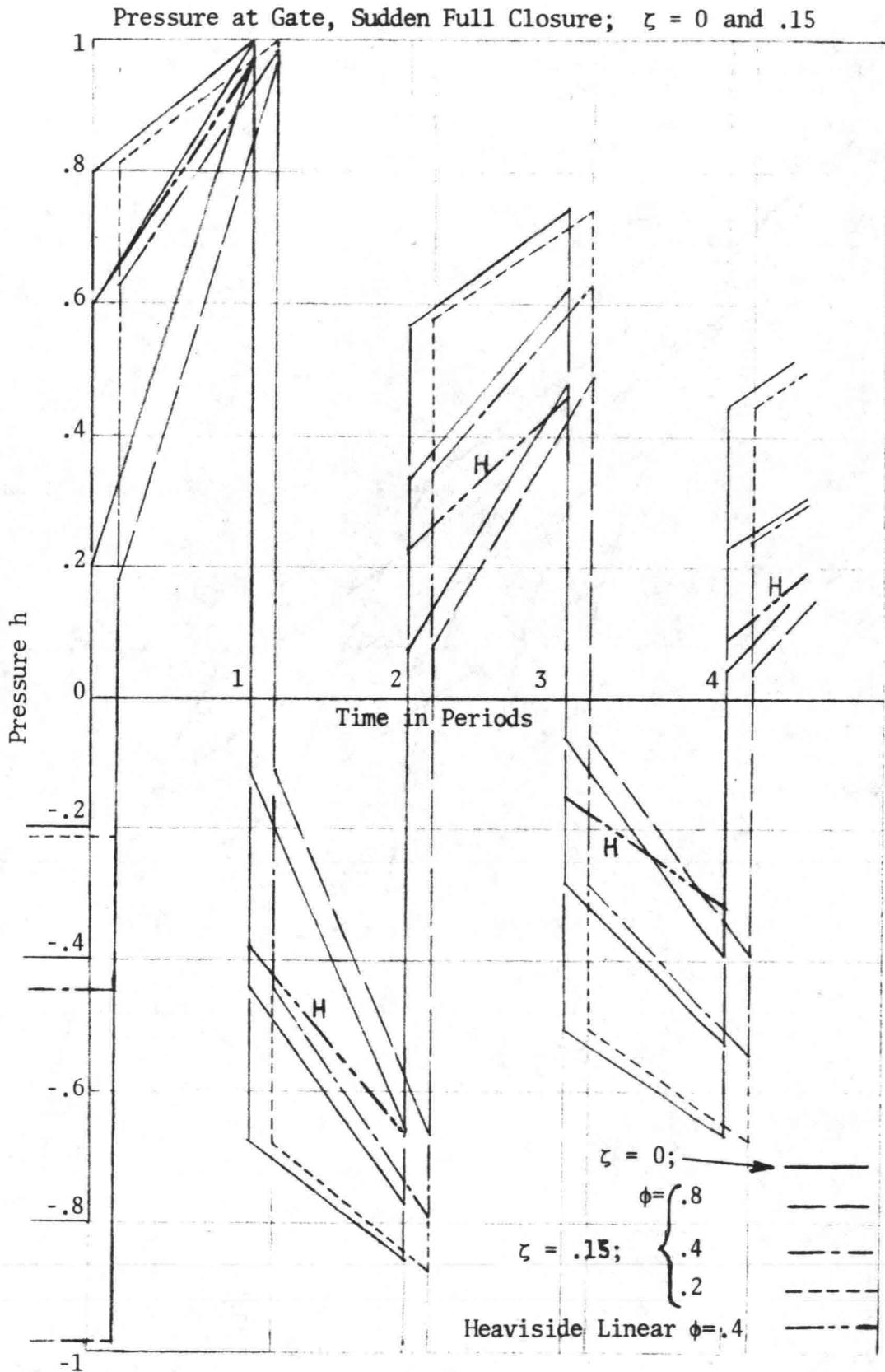
Figure 9.

Pressure and Velocities at Gate for Steady Flow

$\phi = .2$ to 1.0 and $\zeta = .05$ to $.25$
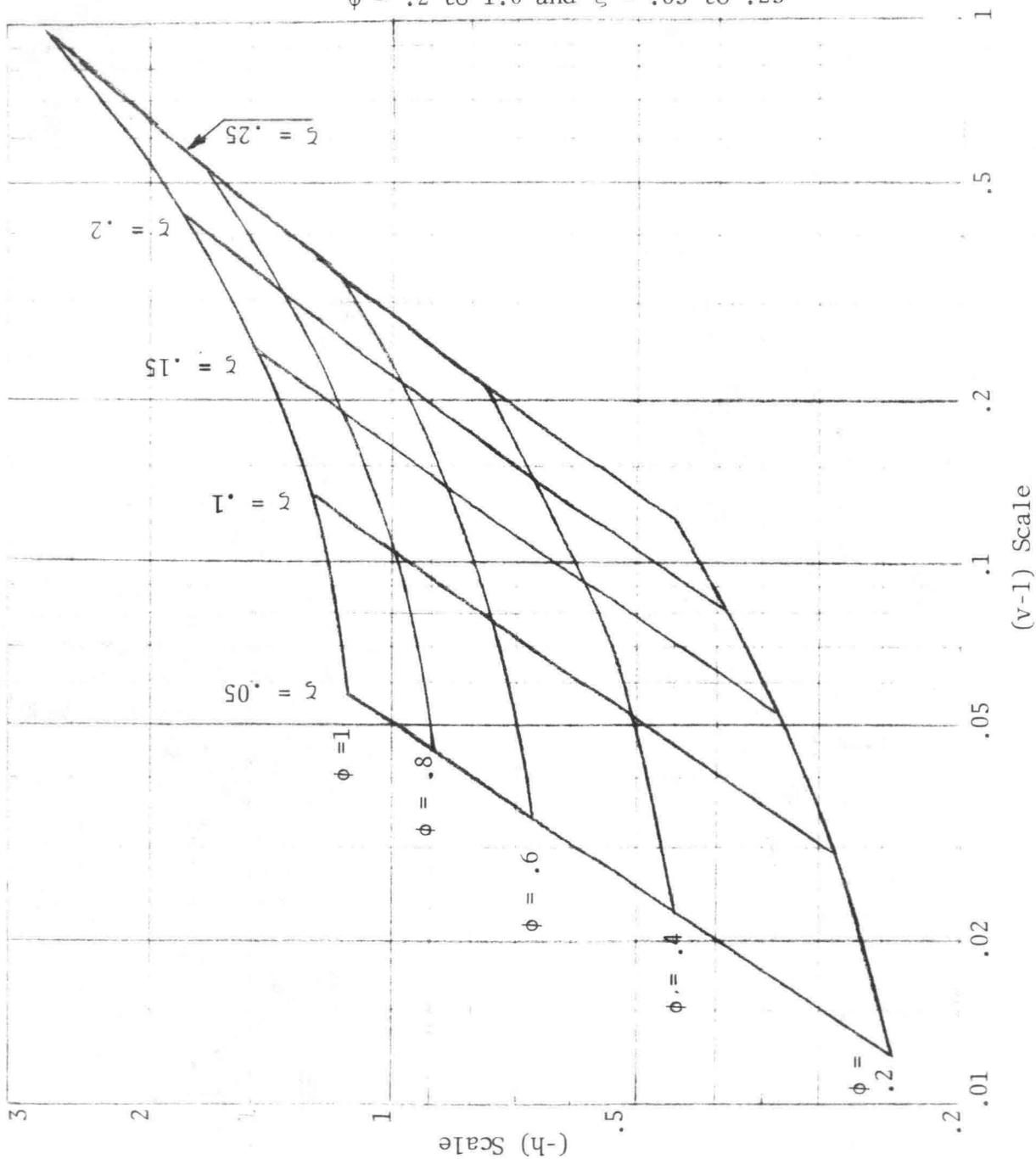
Figure 10.

Rubber Hose Solutions

	K_e	ϕ	ζ	Vo	H_r	a
	22.13	.810	.113	4.58	5.77	40.53
	22.13	.862	.120	4.87	6.14	40.53
	27.66	.715	.102	4.61	6.49	45.31
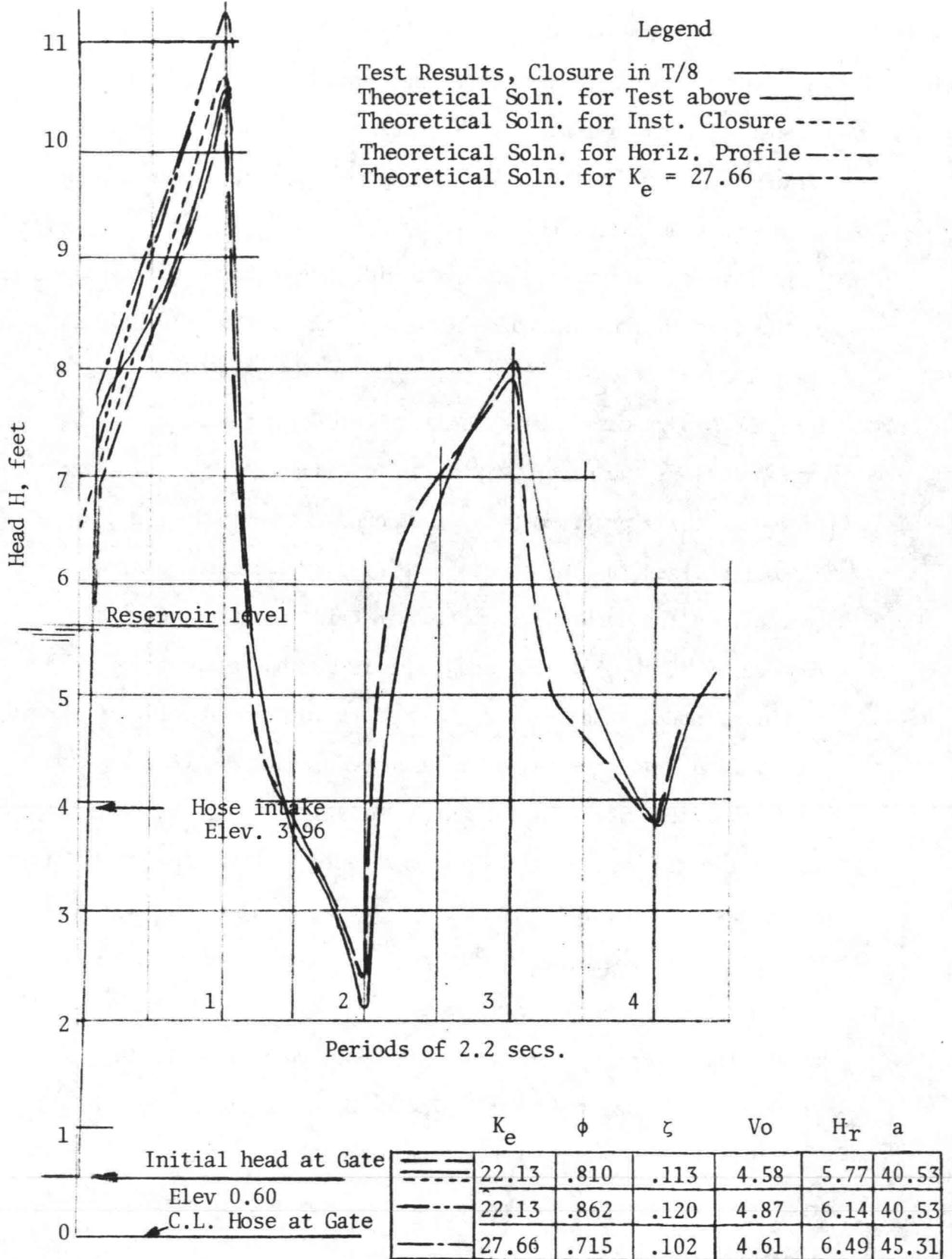
Research by V.L. Streeter

A paper was presented by Streeter and Lai [59] in 1962 which was the first published study using computer methods, necessary because of the non-linearity of several terms in the wave equations. Taking into account the $V\frac{\partial V}{\partial x}$ and $V\frac{\partial P}{\partial x}$ terms resulted in waves of propagation having velocities $V \pm a$, i.e. the velocities are $\pm a$ with respect to the moving fluid. They showed that the accuracy of the calculations increases with the reduction of the grid size, i.e. the reduction of Δx and Δt, and this was incorporated into the programmes to give the true values of the solutions by extrapolation, the error varying as the square of the grid size. (In the writer's programmes [58] he found that even in the case of discontinuities, as for sudden gate closures, the successive corrections for each 50% reduction in grid dimension were also reduced by 45 to 55%) The programme is set up to solve for both turbulent and laminar flow and solutions were found for both sudden and gradual gate closure. They assumed, regarding the elemental grid, that the slopes of the characteristics C_+ C_- were constant within the grid. The writer was unable to use this assumption for his cases of low elastic moduli and low wave velocity. A greater discrepancy between their tests and theory occurred in the case of laminar flow, which they considered as probably due to the assumption of constant velocity across the section. Unfortunately the characteristics of the copper pipe used in the tests made the $V\frac{\partial V}{\partial x}$ and $V\frac{\partial P}{\partial x}$ terms significant only to the order of 0.001, so that the discrepancy between theory and test was not affected by their inclusion.

In 1964 and 1966 Streeter presented papers [60,61] in which he treated a wide variety of applications, using computer programmes with non-linear friction. He neglected the non-linear velocity and pressure terms as his study was confined to metal pipes.

Amongst the problems considered in detail were -

(1) compound and parallel pipes and general networks,

(2) pump failures using dimensionless homologous complete pump data linked with the pipe characterics,

and

(3) resonance studies and valve operations to control the surges. This is a very complete treatment showing the applications of computer methods to pipe systems of all types.

Supplementary Remarks

With the use of electronic computers there is less or no need to use approximations for the operations of the various parts of hydraulic systems. In the past, such approximations were used to make available the mathematical tools for proceeding with the solution. In general our mathematical tools cannot solve non-linear problems in a closed form. Since practically all of our physical laws are non-linear, we are forced to use finite difference methods, with smaller differences for greater accuracy. The resulting increase in the calculations is now no deterrent to the use of smaller differences because of the speed of operation of the computer.

Thus in future we should search for accuracy in stating our physical laws of operation of each hydraulic element. If these laws do not fit into some mathematical pattern, or approximately so, they may be expressed in either graphical or tabulated form, which can be fed into the computer.

In the case of hydraulic turbine and pump characteristics, it is doubtful if individuals or independent research teams (such as university research programmes) can cope with this objective. It will most probably be undertaken by the manufacturers, who will determine with greater accuracy the characteristics of their products as the need arises for greater refinement in design.

Complete characteristics of centrifugal pumps have been used in relation with surges in pump pipe lines. This does not appear to have been done for turbines. For these latter, we need in our complete analysis the performance (i.e. discharge, efficiency and horsepower) for a range of head and R.P.M. at each of a series of gate positions.

There are still some features of surges in simple pipe lines which require more refinement. One is the effect of acceleration and deceleration on frictional resistance, for both turbulent and laminar flow. Some theoretical studies have been made on this topic, but the main difficulty is in developing accurate experimental programmes to vindicate the theory.

A second matter is that of the rupture of the water column when the pressure is reduced to the vapour pressure. Here again there is need for a broad experimental programme.

A third matter is the determination of the laws governing

the reflection of the surges at the pipe boundaries. Considerable study has been made on the operation of gates and valves, and this problem appears to be in good progress. In the case of the operation of hydraulic turbines and pumps, their complete characteristics should include the effect of any gates and valves present in the installation.

A fourth matter is the study of surges for laminar flow.

For those who wish to pursue the subject in more detail, particularly in any specialized branch, there are many bibliographies in the references. In addition there are several text books of recent date, such as those by Parmakian [68], Rich [69], and Streeter and Wylie [70].

ACKNOWLEDGMENT

NOTATION AND SYMBOLS

A   = Cross-sectional area

a   = Velocity of pressure wave propagation = $\sqrt{g K_e / \gamma}$

b   = Thickness of pipe walls

C_+, C_- are the characteristic curves at slopes $\dfrac{dx}{dt} = V \pm a$

D   = inner pipe diameter

E   = Young's Modulus of Elasticity for the pipe material

e   = strain in the pipe material

F   = $\dfrac{f}{2gD}|V|$, D'Arcy, or in general = $|\psi(V)|$

f   = friction coefficient, D'Arcy.

g   = acceleration due to gravity

H   = Pressure head = P/γ

H_f = friction head loss = $\dfrac{fL}{D} \cdot \dfrac{V^2}{2g}$. D'Arcy

H_{fo} = Total friction head loss for length L with constant $V_o D_o$

H_r = $\dfrac{a V_o}{g}$ = pressure head rise for velocity rejection V_o

K   = Volume modulus for the fluid

K_e = Volume modulus for pipe walls and fluid combined

K_p = Volume modulus for pipe walls alone

L   = Pipe length

n   = Number of equal pipe segments

P   = Pressure

Q   = Volume

r   = Pipe radius

t   = time

u   = displacement from equilibrium position x

V   = velocity

x   = distance along pipe from intake or equilibrium position

Y   = vertical distance of section below intake

γ = weight density of liquid

τ = period of wave = $\dfrac{2L}{a}$

θ = inclination of pipe to horizontal

Dimensionless Quantities

h   = H/H_o for graphical treatment

H   = H/H_r for analytical treatment (non-linear study)

v = $\dfrac{V}{V_o}$; $y = \dfrac{Y}{H}$

ϕ = Friction Modulus = $\dfrac{H_{fo}}{H_r}$

ζ = Velocity Modulus = V_o/a

ζv = V/a ,   (Mach. Number)

ζ/ϕ = $\dfrac{2 D_o}{f L}$.

Interlocking equations

$$\frac{\partial V}{\partial x} = -\frac{1}{K_e} \cdot \frac{dP}{dt} = -\frac{1}{K_e}\left[\frac{\partial P}{\partial t} + V\frac{\partial P}{\partial x}\right]$$

$$-\frac{\partial P}{\partial x} - \gamma F V + \gamma \cdot \sin\theta = \frac{\gamma}{g}\cdot\frac{dV}{dt} = \frac{\gamma}{g}\left[\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x}\right]$$

REFERENCES

1. Newton. Mathematical Papers (Whiteside translation)

2. Newton. The Principia (Cajori translation)

3. Euler. "De la Propagation du Son" Mémoires de l'Acad. d. Wiss. Berlin 1759.

4. Lagrange. Miscellanea Physico - Mathematica Turin 1759.

5. Euler. "Principia pro motu sanguinis per arterias determinando" Opera Postuma Tomus Alter XXXIII 1775.

6. Lagrange. Mecanique Analytique 1788.

7. Coulson. Waves (Oliver and Boyd)

8. Monge. "Graphical integration, etc." Ann. des Ing. sortis des Ecoles de Gand 1789.

9. Laplace. Celestial Mechanics (4 Volumes) Bowditch translation.

10. Cauchy. Oeuvres Complétes. Sec. II Tome IV Paris 1890.

11. Young. "Hydraulic investigations" Phil. Trans. Royal Soc. 1808, p. 164.

12. Wertheim. Pogg (Ann. d. Phys. und Chimie) Vol. 77, 1848.

13. Riemann. "Sur la propagation d'Ondes Atmospheriques planes, etc." Mem. de l'Acad. Royale de Gottingen 1860.

14. Riemann. Partielle Differentialgleichungen, Braunschweig 1869.

15. Weber. "Propagation of waves through water or other incompressible fluids in elastic pipes" Akad. d. Wiss. Leipzig Berichte 1865.

16. Weber. (As above) 1850.
    Weber E and W. Wellenlehre (text book) Leipzig 1825.

17. Kundt. Pogg Vol. 153.

18. Dvorak. Pogg Vol. 154, 1875.

19. Marey. "Mouvement des Ondes Liquides pour servir a la Théorie du Pouls" Travaux du Laboratoire de M. Marey, 1875.

20. Resal, H. "Movement of an incompressible fluid in an elastic pipe" Journal de Math. pures et Appliquées, 1876.

21. Rayleigh. Theory of Sound 1877.

22. Korteweg. "Über die Fortpflanzungsgeschwindigkeit des Schalles in Elastischen Rohren" Ann. d. Physik u. Chimie 1878.

23. Lamb. "Motion of Fluids" 1878.

24. Michaud. "Coups de Bélier dans les conduites" Bulletin de la Soc. Vaudoise des Ing. et des Architectes, Lausanne 1878.

25. Gromeka. "Concerning the propagation velocity of water-hammer waves in elastic pipes" Scientific Soc. of Univ. of Kazan, May 1883.

26. Weston, E.B. Trans. ASCE 1885.

27. Church, I.P. Journal of the Franklin Inst. 1890.

28. Church, I.P. Cornell Civil Engineer. 1898.

29. Carpenter, R.C. "Experiments on Water-hammer" Trans. ASME 1893-1894.

30. Frizell, J.P. "Pressures from velocity changes in pipes" Trans. ASCE 1898.

31. Joukowski, N. Paper to Polytechnic Soc. Moscow, Spring of 1898. (English translation by Miss O. Simin. Proc. AWWA, 1904)

32. Allievi.

   (a) "General theory of pressure variation in Pipes" Ann. d. Ing. et Archit. Ital. Dec. 1902.

   (b) "Mouvement de l'eau dans les Tuyaux de Conduite" Revue de Mecanique, Jan. March 1904.

   (c) "Théorie Générale du Mouvement varié, etc." Dunod. Paris 1904.

   (d) "Notes" Atti d. Cole d. Ing. e. Arch. Milan 1913.

   (e) English translation by E.E. Halmos ASME 1925.

33. Warren. "Penstock and Surge Tank Problems" Trans. ASCE 1915.

34. Constantinescu. Ann. des Mines de Roumanie, Dec. 1919 - Jan. 1920. (abstract in Le Genie Civil Vol. 76, No. 15)

35. Gibson, N.R.   "Pressures in Penstocks caused by gradual
                    closing of turbine gates" Trans. ASCE 1920.

36. Strowger and Kerr.   "Speed Changes of Hydraulic Turbines
                    for sudden changes of load" ASME June 1926.

37. Löwy.   "Druckschwankungen in Druckrohrleitungen"
                    Springer 1928.

38. Schnyder.   "Druckstösse in Pumpensteigleitungen"
                    Schweizerische Bauzeitung. Nov, Dec. 1929.

39. Bergeron.   "Variations de regime dans les conduites d'eau"
                    Comptes Rendus des Travaux de la Soc. Hydrotech.
                    de France 1931.

40. Schnyder.   "Uber druckstösse in Rohrleitungen" Wasserkraft u.
                    Wasserwirtschaft. Heft 5, 1932.

41. "Symposium on Water-Hammer" ASME and ASCE. Chicago, June 1933
                    (also a Supplement to the Symposium).

42. Angus.   "Simple Graphical Solutions for Pressure Rise in
                    Pipe and Pump Discharge Lines" Journal EIC
                    Feb. 1935.

43. Bergeron.   "Méthode Graph. Générale de Calcul des Propagations
                    d'ondes planes" Soc. des Ing. Civils de France
                    1937.

    Bergeron.   "Méthode Graph. de Resolution des Problemes de
                    Propagation d'ondes planes" 5th Inter. Cong. of
                    App. Mechanics 1938.

References 44 to 51 all at ASME Meeting Dec. 1937.

44. Allievi.   Air Chambers for Discharge Pipes.

45. Angus.   Air Chambers and Valves in relation to water-hammer.

46. De Juhasz.   Hydraulic Phenomena in Fuel-injection Systems
                    for Diesel Engines.

47. Knapp, F.   Operation of emergency shut-off valves in pipe lines.

48. Knapp, R.T.   Complete Characteristics of Centrifugal Pumps and
                    their use in the prediction of transient behavior.

49. Schnyder.   Comparison between calculated and test results on
                    water-hammer in Pumping Plants.

50. Strowger.   Relation of Relief valves and Turbine characteristics
                    in the determination of water-hammer.

51. Wood. "The application of Heaviside's operational calculus to the solution of problems in Water-Hammer"

52. Angus. "Water-Hammer Pressures in Compound and Branches Pipes" Proc. ASCE 1938.

53. Boulder Canyon Project. "Water-Hammer Analysis" Final Reports Part V Bull. 5, 1940.

54. Rich. "Water-Hammer Analysis by the Laplace-Mellin Transformation" Trans. ASME 1944-5.

55. Wood. "Graphical Solutions of Partial Differential Equations with Engineering Applications" Tech. Paper No. 4, EIC 1951.

56. Wood. "Graphical treatment of non-linear friction in water-hammer and surge tank study" Trans EIC Jan. 1958.

57. Wood. Computer solutions of surges in a simple inclined pipe for cases of high friction losses and low wave velocity" Trans EIC 1966.

58. Wood. "General water-hammer analysis for simple pipe lines" C.E. Research Report No. 61, Dept. of Civil Engineering, Queen's University 1969.

59. Streeter and Lai. "Water-hammer Analysis including fluid friction" Proc. May 1962 and Trans. 1963. ASCE.

60. Streeter. "Computer Solution of Surge Problems" Proc. Inst. Mech. Engrs. 1965-1966.

61. Streeter. "Water-Hammer Analysis of Pipe Lines" Proc. ASCE Journal of Hyd. Division, July 1964.

62. Lupton. "Graphical Analysis of Pressure Surges in Pumping Systems" Journal Inst. of Water Engrs. 1953.

63. Richards. "Water Column Separation in Pump Discharge Lines" Trans. ASME 1956.

64. Kittredge. "Hydraulic Transients in Centrifugal Pump Systems" Trans. ASME 1956.

65. Duc. "Water Column Separation" Sulzer Tech. Review, V 41, 1959.

66. Sharp. "Rupture of the Water Column" Proc. 2nd Australasian Conf. on Hydr. and Fluid Mechanics, Auckland, N.Z. 1966.

67. Signor and Ashrae. Detroit Edison Co. 1960.

68. Parmakian. "Water-hammer Analysis" Dover 1963.

69. Rich. "Hydraulic Transients" Dover 1963.

70. Streeter and Wylie. "Hydraulic Transients"
       McGraw-Hill 1967.

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