ENGER

Water Hammer in Pipes and Relative Phenomena

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WATER HAMMER IN PIPES AND RELATIVE PHENOMENA

BY

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WATER HAMMER IN PIPES AND RELATED PHENOMENA

INTRODUCTION

If one may judge from the number of articles which have lately appeared on the subject of water hammer, there is a growing interest in this branch of hydraulics. The development of water power and the necessity for close governing of hydraulic motors for rapidly changing loads have been the principal reasons for this increased interest.

Many accidents have been caused by the too rapid movement of valves on long pipe lines. When it is remembered that the water in many penstocks weighs several times as much as the heaviest freight trains, the magnitude of the forces required to change the momentum of the water in the pipe is easily understood. The water in the pipe may be likened to a freight train with an engine at the rear continually pushing, and with an engine at the front end with the steam shut off, controlling the speed of the train with the engine brakes alone. It will readily be seen that any change of the braking force (corresponding to a change in the position of the turbine gates on a penstock) will cause acceleration or deceleration of the train, and that considerable time must be consumed in making any change in the velocity.

When the brakes are suddenly applied on the engine, the whole train does not begin to slow down at once, but it is stopped car by car, the last car of the train continuing to move with the original velocity until all of the cars are slowed down. Similarly in the pipe line, the water is stopped layer by layer, the water farthest from the valve flowing for some time after the valve position is changed, before any retardation is felt.

When the gates of a water wheel are suddenly opened, on account of a demand for more power, the quantity of water flowing is not much increased until the water in the penstock has had time to accelerate. The pressure is of course decreased, and since the quantity is increased by only a small amount, the power is diminished and the effect of the gate movement is for a time the opposite of that desired.

When the gates are suddenly partially closed, the quantity of water flowing will not be much decreased until there has been time to change the velocity of all of the water in the penstock. The pressure will however be increased, and therefore the power will be increased. Again the effect of the gate movement is the opposite of what is desired.

The illustrations given show the effect of the kinetic energy of the moving water in the pipe in opposing any change of the velocity of flow, and in this way seriously interfering with speed regulation of water wheels having long penstocks. There is also energy stored in the compressed water and in the distended pipe which has an opposite effect upon speed regulation. Analogous phenomena in high tension electric circuits are known as "transient electric phenomena". It will be the purpose of this thesis to present experimental and theoretical work on transient hydraulic phenomena.

HISTORICAL

E. B. Weston reported some experiments on water hammer in the Transactions of the American Society of Civil Engineers of June, 1885 He made three sets of experiments. The length and size of pipe used was: first series, 111.3 ft. of 6 in., 58.4 ft. of 2 in., 99.3 ft. of 1 1/2 in. and 4 ft. of 1 in.; second series, 111.3 ft. of 6 in., 3 ft of 3 in., 58.4 ft. of 2 in., 96.3 ft. of 1 1/2 in., and 4 ft. of 1 in. third series, 181.6 ft. of 6 in., 65.5 ft. of 4 in., 3.5 ft. of 2 1/2 in., 1.1 ft. of 2 in.,6.6 ft. of 1 1/2 in. and 5.3 ft. of 1 in. The water was supplied from two 24 in. mains. Average static pressure was 75 lb. per sq. in. Orifices of different diameters were screwed on the discharge to secure different discharges. The pressures were measured at anumber of places along the pipe, by means of a Richards steam indicator. The diagrams taken consisted simply of a vertical straight line, from which the maximum pressure could be mrasured.

On account of the short lengths of pipe used, and the number of different diameters, it is difficult to make comparisons with later and more exact experiments. The values seem reasonable, more so than some of the later experiments. The experiments show clearly that as great pressures may be expected at some distance from the valve as at the valve. This fact had to be rediscovered 13 years later before it was given credence. Mr. Weston made no attempt at a mathematical analysis.

Professor I. P. Church in 1890 published an account of his theoretical studies on water hammer, in the Journal of the Franklin Institute. He showed that the maximum pressure caused by closing a valve at the end of a pipe depends upon the manner of closing as

well as upon the time of closing the valve. It has since been shown that the time of closing the valve has no effect if it is less than a certain minimum. His results are applicable however if the time of closing the valve is sufficiently long.

JOSEPH B. RIDER gives an account of his experiments in the Proceedings of the American Water Works Association of April 1891. His experiments were principally with 18 feet of pipe, using an ordinary gauge to measure the pressure. Considering water in the pipe as as an inelastic column whose total kinetic energy is given up during the time of closing the valve, he derived a formula for the maximum pressure due to water hammer. He then applies his formula to determine the amount of pressure generated in a 12 inch pipe line 22000 feet long, in which the water has a velocity of 0.424 feet per second, the water being shut off in 2 seconds. He obtains as the value of the pressure, 3794.65 pounds per square inch. More recent experiments have shown that the pressure in this case would not exceed 25 pounds per square inch, and that the time of closing would be immaterial if made in less than 12 seconds. To produce the pressure obtained by Mr. Rider's formula would require a velocity of 76 feet per second to be extinguished in less than 12 seconds.

PROFESSOR CARPENTER'S experiments are reported in the Transactions of the Am. Soc. of Mech. Eng. 1894 (Vol 15, page 510). The experiments were made on a pipe line consisting of 30 ft. of 2 in., 33 ft. of 2 1/2 in., 150 ft. of 3 in., and 376 ft. of 6 in. pipe. No satisfactory data could be expected from a pipe line containing so many short lengths of different diameter.

Fig. 1 is taken from the peper referred to, and gives the result of Professor Carpenter's experiments. Dotted lines have

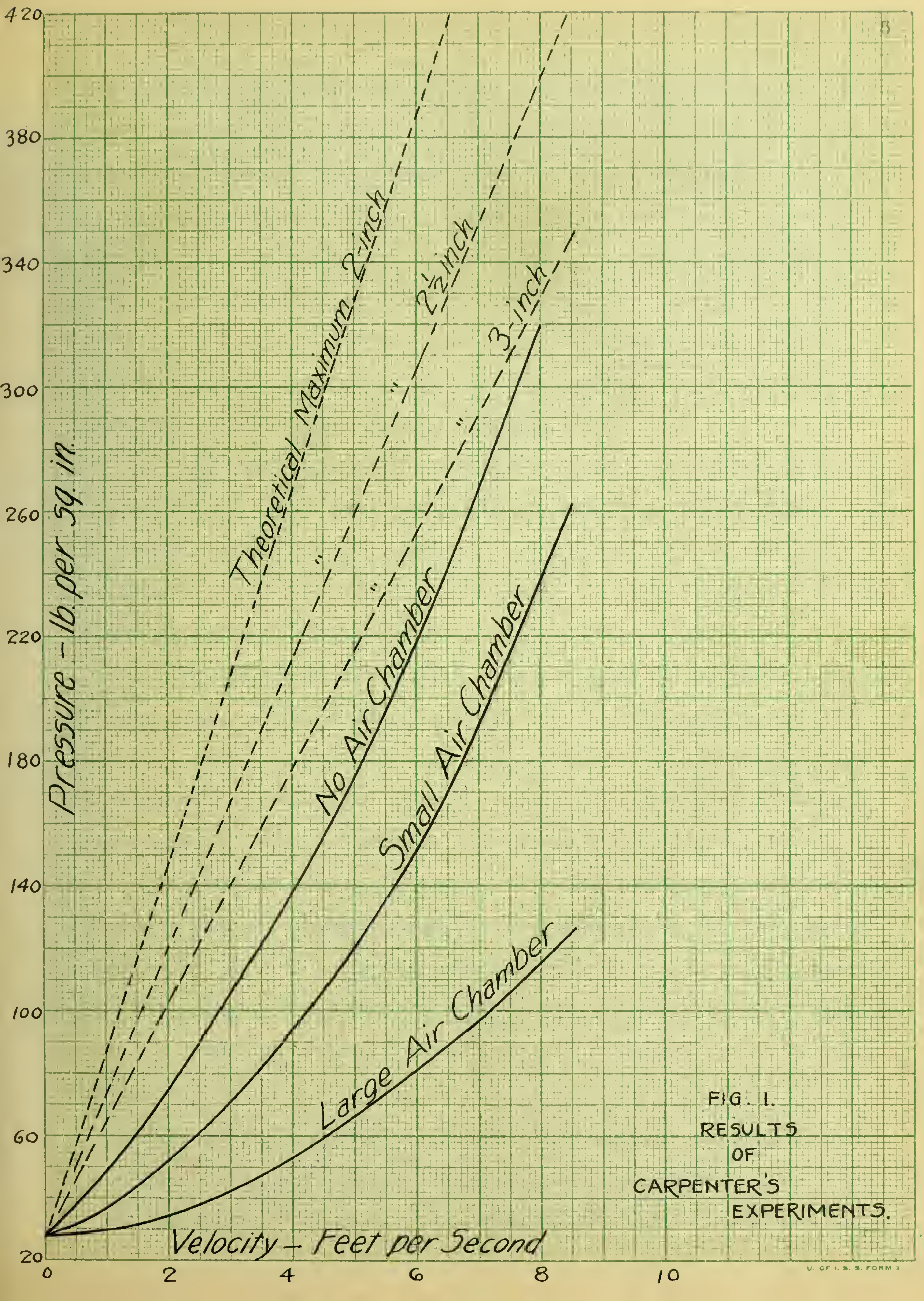


FIG. I.
RESULTS
OF
CARPENTER'S
EXPERIMENTS.

been added showing what the maximum pressure may actually be in long lines of 2 in., 2 1/2 in. and 3 in. pipe. The velocities shown as the abscissas of the diagram are the velocities in the 2in. pipe; the corresponding velocities in the 2 1/2 in. and 3 in. pipes will be in inverse proportion to their areas. It will be noticed that at the higher velocities the pressures increase rapidly. This is because with high velocities the time of the effective closure is less than with low velocities, hence the short 2 in. and 2 1/2 in. pipes have a relatively greater effect.

The experiments of Carpenter, although the best at the time they were made, were very crude and the results inconclusive. The experiments with the air chambers are interesting as indicating the effect they have, but as the quantity of air in the air chambers is not given, the results have no quantitative value.

PROFESSOR N. JOUKOVSKY made experiments in 1897 at Moscow. He used pipes 2 in., 4 in. and 6 in. in diameter with lengths of 2493 ft., 1050 ft. and 1966 ft. respectively. He made numerous experiments, and deduced formulae which were in accord with his experiments. The exhaustive monograph which he presented to the St. Petersburg Academy of Sciences has been translated to German (Stoss in Wasserleitungsrohren, St. Petersburg 1900). An English translation by Miss O. Simin, somewhat modified, is to be found in the Proceedings of the American Water Works Association of 1904.

Joukovsky makes the following conclusions from his experiments:-

1. The shock pressure is transmitted through the pipe with a constant velocity which seems to be independent of the intensity of the shock. This velocity depends upon the elasticity of

the pipe material and upon the ratio of the thickness of the walls to the diameter.

2. The shock pressure is transmitted along the pipe with a constant intensity, and is proportional to the destroyed velocity of flow, and to the speed of propagation of the pressure wave.

3. The periodical vibration of the shock pressure is completely explained by the reflection of the pressure wave from the ends of the pipe.

4. If the column of water continues flowing, such flow has no noticeable effect upon the shock pressure. In this case, the pressure wave is reflected from the open end of the pipe in the same manner as from a reservoir with constant pressure.

5. A dangerous increase in pressure occurs when the pressure wave passes from a pipe into another of smaller diameter with a dead end.

6. The simplest method of protecting pipes from water hammer is found in the use of slow closing valves. The duration of the closure should be proportional to the length of the pipe. Air chambers of adequate size placed near the valve eliminate the shock almost entirely, and do not allow the wave to pass through them, but they must be very large and it is difficult to keep air in them. Safety valves allow only such pressure waves to pass through them as corresponds to the elasticity of the spring used.

7. The diagram of shock pressure enables one to determine the location and extent of air pockets, and the location of leaks, and to study the conditions of the pipe line.

A. H. Gibson, senior demonstrator and assistant lecturer in engineering in the Manchester University, published a book on water hammer in Hydraulic Pipe Lines, in 1908. He made experiments on the effect of the slow closing of the valve at the end of the pipe line. A globe valve seating against the pressure was used, and the closure was made by turning the valve spindle by hand at as uniform a rate as possible. The time of closing was obtained by means of a tuning fork tracing on smoked paper.

Fig. 2 shows Mr. Gibson's theoretical curves and experimental values plotted on the same sheet. It will be noticed that the pressure caused by the gradual closure of a valve does not vary inversely as the time as was stated by Church, Joukovsky and others.

Mr Gibson also made experiments with the valve at the end suddenly opened, and showed that it was possible to cause water hammer by suddenly opening the valve a small amount.

The historical review of the experiments on water hammer which has been given here is by no means complete, but gives merely those which have been most quoted. Experiments have been carried on at the University of Illinois as thesis work by seven men, beginning with Mr. Smith in 1899.

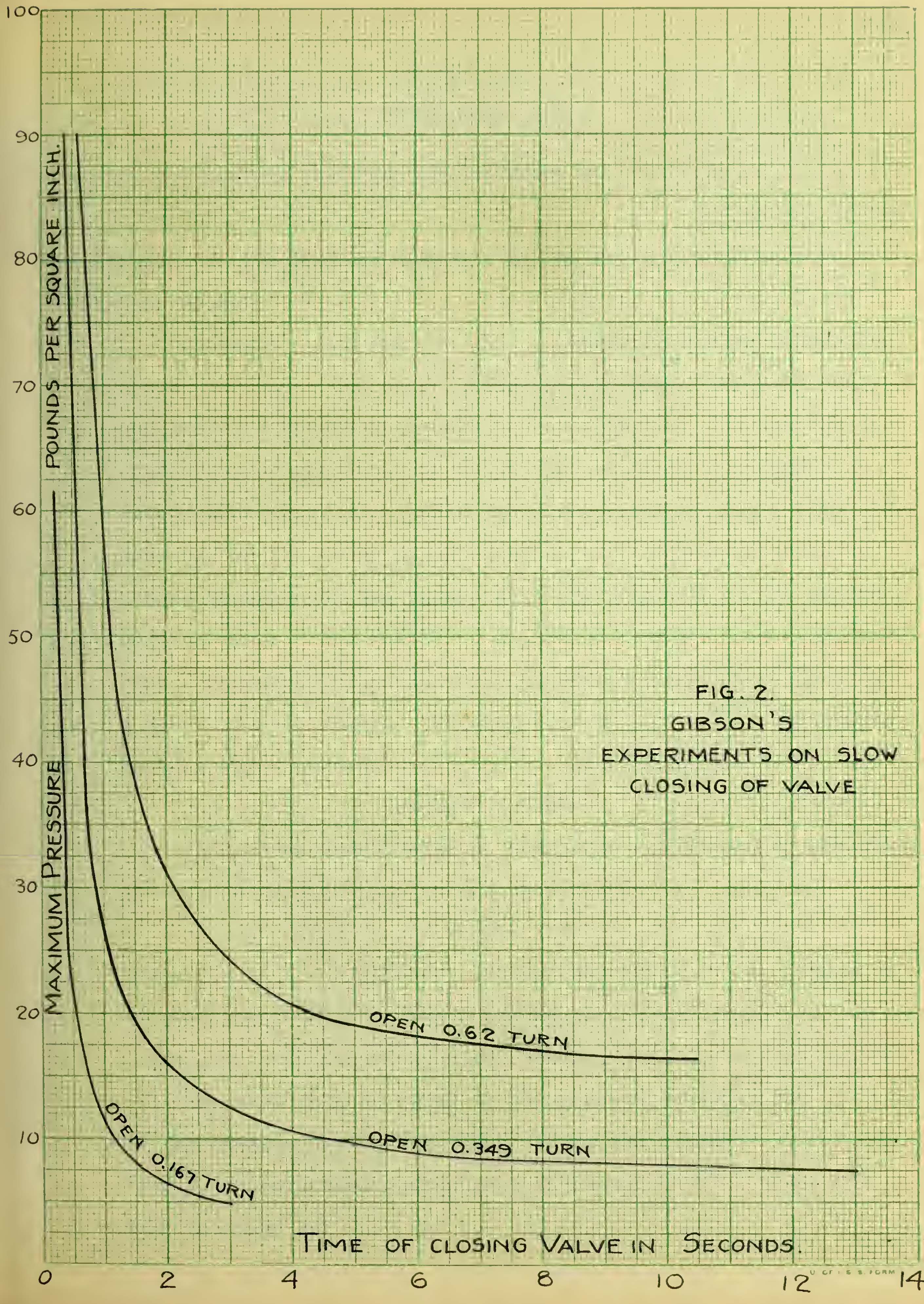


FIG. 2.
GIBSON'S
EXPERIMENTS ON SLOW
CLOSING OF VALVE

PART ONE

THEORETICAL WORK

FORMULA FOR MAXIMUM PRESSURE DUE TO WATER HAMMER. The following derivation of the expression for the maximum pressure due to water hammer is made by Professor Talbot.

Suddenly arresting the flow of water in a pipe causes the water near the valve to be compressed a certain amount, while the water at the free end has no compression. Neglecting the distention at present, let dx be the length of a column of water whose cross-section is one square foot. Let P equal the pressure at the valve on one square inch, generated by the sudden stoppage of the water, then the pressure on one square foot is 144P. The weight of a column of water of one square foot cross-section and dx long, is wdx, w being the weight of a cubic unit of water. Flowing with a velocity v, the energy in this water is equal to $1/2mv^2$. This becomes

$$\text{Energy} = 1/2\frac{wdx}{g}v^2$$

Let r equal the amount this dx length is compressed at the valve. Since r is proportional to the unit stress

$$r = \frac{Sl}{E}$$

where S = P, l = length of column = dx, and E = the modulus of elasticity of water = 300000 pounds per square inch. If dx is expressed in feet, the product, energy, will be in foot pounds.

The work done is equal to the average pressure multiplied by the distance moved through. Since the pressure increases from 0 to 144P, the average pressure equals $\frac{144P}{2}$. The work done is equal to

$$\frac{144P}{2}r = \frac{144P}{2}\frac{Pdx}{E}$$

The work done is also equal to the energy in the water, so equating, we get

$$\frac{144P}{2}\frac{Pdx}{E} = \frac{wdx}{2g}v^2$$

from which

$$P = \sqrt{\frac{wE}{144g}} \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ (1)$$

substituting numerical values for w, E and g, we have

$$P = 63.6v \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ (2)$$

This is the value for the pressure at the valve, generated by sudden stoppage of the water, when the elasticity of the water is not taken into account.

CONSIDERING THE ELASTICITY OF THE METAL IN THE PIPE, the deformation r_2 of the pipe is

$$\frac{dS}{E'}$$

but $\qquad Pd = 2tS \quad$ or $\quad S = \frac{Pd}{2t}$

where P = pressure per unit area, d = diameter of the pipe, t = the thickness of the pipe, and S = tensile stress of the metal in the pipe in pounds per square inch, E' = the modulus of elasticity of the metal = 30 000 000 pounds per square inch.

Then

$$r_2 = \frac{d^2P}{2tE}$$

The total lateral pressure F_2 in the pipe for a length dx

$$F_2 = \frac{Pd}{2}dx$$

The work done in expanding the pipe is

$$\frac{1}{2}F_2r_2 = \frac{1}{2}\cdot\frac{Pd}{2}dx\frac{\pi d^2P}{2tE}$$

which becomes

$$\frac{1}{8}\frac{\pi P^2d^3}{tE'}dx$$

The work done in compressing the water is

$$\frac{1}{2}F_1r_1 = \frac{1}{2}\frac{\pi d^2}{4}\frac{P^2}{E}dx$$

which becomes

$$\frac{1}{8}\frac{\pi P^2d^2}{E}dx$$

The energy of the water is

$$\frac{1}{2}\frac{w}{g}\frac{\pi d^2}{4}dx\cdot v^2$$

Equating energy to work done, we have

$$\frac{w\pi d^2}{8g}dx\cdot v^2 = \frac{1}{8}\frac{\pi P^2d^3}{tE'}dx \quad \frac{1}{8}\frac{\pi d^2P^2}{E}dx$$

reducing

$$P^2 = \frac{1}{1+\frac{Ed}{E't}}\cdot\frac{wEv^2}{g}$$

or

$$P= \frac{1}{\sqrt{1+\frac{Ed}{E't}}}\frac{wE}{g}\cdot v \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; (3)$$

giving the pressure per unit area expressed in homogeneous units.

Expressing P in pounds per square inch and v in feet per second, the above equation reduces to

$$P = \frac{1}{\sqrt{1+\frac{Ed}{E't}}}\sqrt{\frac{wE}{144g}}.v \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; (4)$$

The form of this formula is the same as the one where the elasticity of the ~~water~~ pipe was neglected, the constant

$$\frac{1}{\sqrt{1+\frac{Ed}{E't}}}$$

depending on the size of the pipe and its thickness, being the only difference.

Church, Joukovsky and others by different methods have derived formulas which can be thrown into the same form as formula (4).

FORMULA FOR MAXIMUM PRESSURE DUE TO WATER HAMMER IN TERMS OF THE VELOCITY OF TRANSMISSION OF THE PRESSURE WAVE IN THE WATER IN THE PIPE. The problem of determining the maximum pressure which can be caused by the sudden stopping of the flow can be solved by another method, somewhat easier to comprehend than the proof given in the preceding pages. The follwing notation will be used:

λ = velocity of the pressure wave in the water in the pipe, in ft. per sec.

P = maximum additional water hammer pressure, in lb. per sq. in.

A = area of cross-section of the pipe, in sq. ft.

v = original velocity in the pipe, in ft. per sec.

v_t = velocity of water in the pipe after the valve position has been changed, in ft. per sec.

w = weight of a cubic unit of water (62.5 lb. per cu. ft.).

g = acceleration of gravity (32.2 ft. per sec.).

One second after an instantaneous change of the valve position has been made, it is evident that λ feet of water in the pipe will have had its velocity changed (assuming the pipe long enough). The mass of water retarded is

$$\frac{wA\lambda}{g} \text{ gee-pounds.}$$

The water as was explained before, is stopped layer by layer. The pressure at the valve therefore remains constant during the time the water in the pipe is getting its compression. From the principles of impulse and momentum,

$$144AP = \frac{\lambda A w}{g}(v - v_t)$$

and

$$P = \frac{\lambda w}{144g}(v - v_t) \quad \ldots\ldots\ldots\ldots \quad (5)$$

or if the valve is instantly closed,

$$P = \frac{\lambda w}{144g}v \quad \ldots\ldots\ldots\ldots \quad (6)$$

Equation (5) shows that the pressure due to water hammer varies directly as the extinguished velocity $(v - v_t)$ and with the velocity of transmission of the shock through the water in the pipe. The equation may be rewritten in the convenient form

$$P = h(v - v_t) \quad \ldots\ldots\ldots\ldots \quad (7)$$

The value of the coefficient h can be taken from Table 1 as will be explained in a later paragraph.

FORMULA FOR THE VELOCITY OF TRANSMISSION OF THE PRESSURE WAVE IN THE WATER IN THE PIPE. From equations (4) and (6) we may write

$$\frac{1}{\sqrt{1 + \frac{Ed}{E't}}}\sqrt{\frac{wE}{144g}} = \frac{\lambda w}{144g}$$

$$\lambda = \frac{1}{\sqrt{1 + \frac{Ed}{E't}}}\sqrt{\frac{144gE}{w}} \quad \ldots\ldots\ldots\ldots \quad (8)$$

Substituting g = 32.2 ft. per sec., E = 300,000 lb. per sq. in. and w = 62.5 lb., the expression becomes

$$\lambda = \frac{4710}{\sqrt{1 + \frac{Ed}{E't}}} \quad \ldots\ldots\ldots\ldots \quad (9)$$

This expression shows that the velocity of transmission of the pressure wave diminishes somewhat as the ratio of $\frac{t}{d}$ diminishes. The ratio of $\frac{t}{d}$ for riveted steel pipe usually diminishes as the diameter increases. For large sizes of riveted pipe the pressure wave travels at a comparatively slow rate. The numerator, 4710, of equation (9) is the velocity of sound in free water, the denominator gives the reduction necessary to take account the effect of the yielding of the pipe.

TABLE OF COEFFICIENTS FOR USE IN THE FORMULA FOR MAXIMUM PRESSURE DUE TO WATER HAMMER. Equation (4) for the maximum pressure generated by suddenly stopping the flow in a pipe, may be written in the form

$$P = hv$$

Equation (7) is

$$P = h(v - v_t)$$

The last equation being the more general. The value of h as determined from equation is

$$h = \frac{1}{\sqrt{1 + \frac{Ed}{E't}}} \sqrt{\frac{wE}{144g}} \quad \ldots\ldots\ldots\ldots (10)$$

The values of h for all ordinary sizes of cast-iron and wrought iron pipe have been calculated from equation (10) and are given in Table 1. As an illustration of the use of Table 1 the following example is given.

Example. What maximum pressure can be developed by suddenly closing a valve at the end of a 36-in. cast-iron pipe 1-in. thick, if the velocity in the pipe is 4 feet per second? From Table 1 for a cast-iron pipe 1-in. thick and 36-in. in diameter

TABLE 1. VALUES OF h IN P=hv.

DIAM. OF PIPE	WROUGHT IRON PIPE			CAST IRON PIPE — THICKNESS IN INCHES.																			
	Str'd	Extra	Double	7/16	1/2	9/16	5/8	11/16	3/4	13/16	7/8	15/16	1	1 1/8	1 1/4	1 3/8	1 1/2	1 5/8	1 3/4	1 7/8	2	2½	2¼
1/4	62.4	62.8																					
3/8	62.0	62.5	63.3																				
1/2	61.8	62.4	63.3																				
3/4	61.4	62.2	63.2																				
1	61.2	62.0	63.1																				
1 1/4	60.7	61.7	62.8																				
1 1/2	60.3	61.3	62.7																				
2	59.7	61.2	62.6																				
2 1/2	60.1	61.0	62.6																				
3	59.6	60.7	62.5																				
3 1/2	59.2	60.4	62.3																				
4	58.8	60.3	62.2	58.3	59.0	60.2																	
5	58.2	59.8	62.0																				
6	57.7	59.7	61.7	57.5	57.2	57.8	58.3																
7	57.3	59.8	61.5																				
8	57.0	60.0	61.3	53.9	55.3	56.2	56.6																
10	56.3				53.7	54.5	55.3	56.0	57.5														
12	55.3				52.2	53.1	54.7	54.7	55.4	55.8	55.9												
14	54.2					51.9	52.8	53.6	54.3	54.9	55.4	55.8	56.2										
16							51.7	52.4	53.1	53.9	54.3	54.9	55.3										
18							50.6	51.7	52.3	53.1	53.6	54.2	54.5	55.7									
20								50.6	51.4	52.2	52.6	53.1	53.7	54.7	55.3								
22								49.7	50.4	51.2	51.8	52.4	53.0	53.8	54.7								
24								48.7	49.7	50.4	51.1	51.7	52.3	53.1	54.2	54.7							
27								47.7	48.5	49.3	50.0	50.8	51.2	52.2	53.1	53.9	54.5						
30									47.4	48.2	48.8	49.8	50.3	51.4	52.2	53.0	53.7						
33											48.1	48.8	49.3	50.4	51.4	52.3	53.0	53.5	54.2				
36											47.2	47.8	48.4	49.7	50.8	51.6	52.2	52.9	53.6				
42													46.8	48.1	49.2	50.2	50.8	51.7	52.2	52.9	53.4		
48													45.4	46.7	47.7	48.8	49.7	50.4	51.3	51.7	52.3	52.8	53.3
60															45.4	46.5	47.3	48.2	48.9	49.6	50.2	50.8	51.0

h = 48.4. The pressure will then be

$$P = 48.4 \times 4 = 193.6 \text{ lb. per sq. in.}$$

TABLE GIVING VELOCITY OF TRANSMISSION OF THE PRESSURE IN THE PIPE FOR VARIOUS VALUES OF h. Writing equations (8) and (10)

$$\lambda = \frac{1}{\sqrt{1 + \frac{Ed}{E't}}} \sqrt{\frac{144gE}{w}} \quad \ldots \ldots \ldots \ldots \ldots (8)$$

$$h = \frac{1}{\sqrt{1 + \frac{Ed}{E't}}} \sqrt{\frac{wE}{144g}} \quad \ldots \ldots \ldots \ldots \ldots (10)$$

and combining

$$\lambda = \frac{144gh}{w} = 74.2h \quad \ldots \ldots \ldots \ldots \ldots (11)$$

Table 2 has been prepared from equation (11) for various values of h.

To use this table the value of h must first be determined. This is most easily done by use of Table 1. If, for example, it is desired to determine the velocity of the transmission of the pressure in a 30-in. pipe 1 3/8-in. thick, the value of h determined from Table 1 is 53.0. From Table 2 the value of λ is found to be 3930ft. per sec.

TABLE 2.

THE VELOCITY OF TRANSMISSION OF THE PRESSURE

WAVE

Value of "h"	.0	.2	.4	.6	.8
45	3339	3354	3369	3385	3399
46	3413	3428	3443	3458	3473
47	3487	3502	3517	3532	3547
48	3562	3577	3592	3607	3622
49	3636	3651	3666	3681	3696
50	3710	3725	3740	3755	3770
51	3784	3799	3814	3829	3844
52	3858	3873	3888	3903	3918
53	3933	3948	3963	3978	3993
54	4007	4022	4037	4052	4067
55	4081	4096	4111	4126	4141
56	4156	4171	4186	4201	4216
57	4230	4245	4260	4275	4290
58	4304	4319	4334	4349	4364
59	4378	4393	4408	4423	4438
60	4452	4467	4482	4497	4512
61	4526	4541	4556	4571	4586
62	4601	4616	4631	4646	4661
63	4675	4690	4705	----	----

RECURRENCE OF PRESSURE PULSATIONS.

MAGNITUDE OF THE PRESSURE. After the valve has been suddenly closed the pressure will have its maximum value at the valve for a time $\frac{2l}{\lambda}$ seconds, l being the length of the pipe line in feet, and λ the velocity of transmission of the pressure wave in feet per second. When all of the water in the pipe has been brought to rest it will all be compressed to hv lb. per sq. in. above static pressure. The column of water will now begin to expand causing a flow in the opposite direction to the original flow. The pressure tends to become as much below static pressure as the maximum pressure was above. Two cases will be considered.

First, when the pressure due to water hammer is less than the sum of the static and atmospheric pressures. In this case the pressure will be $p_a + p + P$ lb. per sq. in. for $\frac{2l}{\lambda}$ sec. after closing the valve, or the time it takes the pressure wave to run to the open end plus the time it takes the wave of re-expansion, or release of pressure, to run from the open end to the valve. When the wave of re-expansion gets to the valve the pressure will drop to $p_a + p - P$ and this value of the pressure will continue until $\frac{2l}{\lambda}$ sec. have elapsed, when another pressure wave comes to the valve. The magnitude of this pressure would be the same as that of the first wave if it were not for friction. The waves in this case come at regular intervals; the time between the waves will be $\frac{4l}{\lambda}$ seconds. The waves will continue until the energy is dissipated in friction.

Second, when the pressure due to water hammer is greater than the absolute static pressure, the first pressure wave will press against the valve for a time $\frac{2l}{\lambda}$ as explained for the first

case, and then when the column of water has all been compressed, it will begin to expand and flow back into the source. As the water expands the pressure drops from $p_a + p + P$ to static pressure $p_a + p$. This wave travels to the valve with the velocity λ feet per second. When the wave of static pressure gets to the valve it is reflected as a pressure $p_a + p - P$, or if as we have assumed, P is greater than $p_a + p$, the pressure will be reduced to zero, or a value as near to zero as is consistent with the vapor pressure and the pressure of the air in the water. The column of water in the pipe will therefore separate in a number of places, particularly at all air pockets, and the column will expand until all of the water in the pipe has a pressure of nearly zero. The velocity at this instant would have the same value as the original velocity in the opposite direction, if it were not for friction and imperfect restitution. The velocity u_1 of the rebounding column of water is

$$u_1 = ev_1$$

e being the coefficient of restitution.

The water continues to flow with this velocity for the time it takes the wave to make a round trip from the source to the valve. As soon as the wave of re-expansion reaches the valve, the column commences to slow down under the influence of the atmospheric pressure, the pressure at the source, and friction. Denoting these forces

$$p_a,\ p \text{ and } cw^2 \text{ lb. per sq. in.}$$

respectively, the retardation may be written (from F = ma),

$$a_1 = \frac{p_a + p + cw^2}{62.5 l} \quad \ldots\ldots\ldots\ldots (12)$$

w is the velocity in the pipe at any instant, varying from u_1 to 0, and l is the length of the pipe in feet. The average acceleration will then be

$$a_1 = \frac{p_a + p + \frac{ce^2v^2}{3}}{(62.5)l} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

Assuming that the average value of cw^2 as w varies from u_1 to 0 can be expressed by $\frac{cu_1^2}{3}$

The column moves

$$s = \frac{u_1^2}{2a_1} = \frac{e^2v^2}{2a_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

in coming to rest. Then it is forced back with an average acceleration,

$$a_2 = \frac{p_a + p - \frac{cu_2^2}{3}}{(62.5)l} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

u_2 being the average velocity of the water in the pipe when the column of water again strikes the valve. This velocity may be written

$$u_2 = \sqrt{2a_2s}$$

Substituting the value of s from equation (14)

$$u_2 = \sqrt{\frac{a_2}{a_1}}ev. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

Substituting in equation (15)

$$a_2 = \frac{p_a + p - \frac{c\ a_2e^2v^2}{3a_1}}{(62.5)\,l}$$

Solving

$$a_2 = \frac{p_a + p}{(62.5)l + \frac{ce^2v^2}{3a_1}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

Substituting in equation (16) and simplifying,

$$u_2 = ev\sqrt{\frac{1}{1 + \frac{2ce^2v^2}{3(p_a + p)}}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

Multiply both sides by h and substituting $P_1 = hv_1$, and $P_2 = hv_2$

$$P_2 = eP_1 \sqrt{\frac{1}{1 + \frac{2ce^2v^2}{3(p_a + p)}}} \quad \ldots\ \ldots\ldots\ \ldots\ldots(19)$$

For the n-th wave the pressure is

$$P_n = eP_{n-1} \sqrt{\frac{1}{1 + \frac{2ce^2 \frac{P^2_{n-1}}{h^2}}{3(p_a + p)}}} \quad \ldots\ldots\ldots\ldots\ldots(20)$$

This is the general expression for the n-th wave It will be noted that the only variable under the radical is

$$\frac{P^2_{n-1}}{h^2}$$

and that quite large changes in this term produce relatively small changes in the value of the radical. We may therefore write equation (20) approximately,

$$P_n = mP_{n-1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(21)$$

m being a constant. Experiments made by the writer, to be described under Experimental Work, confirm equation (21). The value of m determined experimentally with 730 feet of 2 inch pipe was 0.83. This indicates that the value of the coefficient of restitution for this pipe line was about 0.95.

TIME BETWEEN PRESSURE PULSATIONS. Two cases will be considered.

First, when the maximum water hammer pressure is less than the sum of the static and atmospheric pressures. When the valve is suddenly closed the pressure hv will act on the valve for a time of $\frac{2\,l}{\lambda}$ seconds. Then a pressure wave of $p_a + p + P$ lb. per sq. in. acts on the valve for another period of $\frac{2\,l}{\lambda}$ seconds, at which time the second pulsation of pressure begins. The whole cycle lasts $\frac{4\,l}{\lambda}$ seconds. In this case the water in the pipe has a regular period of vibration. This is the case reported by Joukovsky, Gibson and other experimenters.

Second, when the maximum water hammer pressure is more than the sum of the static and atmospheric pressures. The time between the first and second pressure waves will be the time necessary for the column of water in the pipe to be compressed, plus the time the water in the pipe is expanding and flowing at a uniform rate toward the source, plus the time required by the forces (static pressure, atmospheric pressure and pipe friction) to bring the moving water to rest, plus the time required to force the water back to the valve to cause the second pulsation of pressure. The total time is,

$$t_1 = \frac{l}{\lambda} + \frac{2\,l}{\lambda} + \frac{ev}{a_1} + \frac{u_2}{a_2}$$

Sustituting for u_2 the value given in equation (16)

$$t_1 = \frac{3\,l}{\lambda} + \frac{ev}{a_1}\left(1 + \sqrt{\frac{a_1}{a_2}}\right)$$

$$= \frac{3\,l}{\lambda} + \frac{eP_1}{ha_1}\left(1 + \sqrt{\frac{a_1}{a_2}}\right) \qquad \ldots\ldots\ldots\ldots\ldots\ldots (22)$$

in which a_1 and a_2 are given by equations (13) and (15).

$$t_n = \frac{3\,l}{\lambda} + \frac{eP_{(n-1)}}{ha'_{(n-1)}}\left(1 + \sqrt{\frac{a'_{n-1}}{a''_{n-1}}}\right) \qquad \ldots\ldots\ldots\ldots (23)$$

t_n is the time between the (n-1)th and the (n-1)th pressure wave, $P_{(n-1)}$ is the pressure of the n-th wave, $a'_{(n-1)}$ is the acceleration of the column of water rebounding after the (n-1)th pressure wave, and $a''_{(n-1)}$ is the acceleration of the column of water flowing back to produce the n-th pressure wave.

In most cases the quantities a'_{n-1} and a''_{n-1} have a nearly constant ratio. Equation (23) may therefore be written in the simpler form,

$$t_n = m' \frac{P_{(n-1)}}{p_a + p} + \frac{3\,l}{\lambda} \quad \ldots\ldots\ldots\ldots\ldots\ldots (24)$$

m' being a constant for any given pipe line. This equation as well as equation (23) is to be used only when P_{n-1} is greater than the sum of the atmospheric and static pressures. In the experiments made by the author the formula,

$$t_n = 0.17 \frac{P_{(n-1)}}{p_a + p} + 0.517$$

seemed to fit very well. These experiments will be fully described in another part of this thesis.

So far as known to the writer, no mention is made in any of the literature on the subject of water hammer, of the second case just considered. Joukovsky makes the statement that the time between the successive pressure waves is $\frac{4\,l}{\lambda}$.

LEAKS

LOCATING A LEAK IN A PIPE LINE BY MEANS OF THE WATER HAMMER DIAGRAM.

A leak in a pipe line is indicated by a sudden drop of pressure on the water hammer diagram. Fig. 3 represents a water hammer diagram taken at the end of a pipe line. The drop at B is due to the reduction of pressure at the leak. The distance AB' represents the time it takes the pressure wave to travel to the leak and back to the indicator. The distance to the leak can therefore be determined by measuring AB' and substituting in

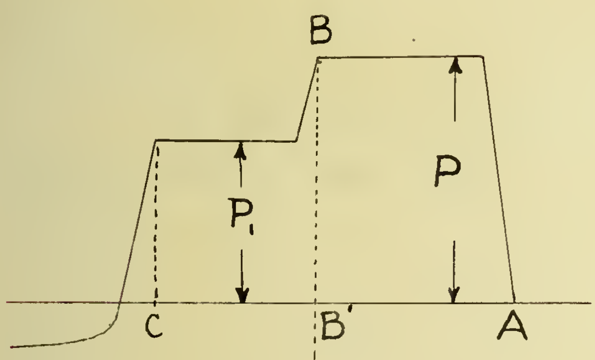


Fig. 3.

$$s = \frac{AB'}{2r}\lambda \qquad (25)$$

s being the distance to the leak in feet, r is the velocity of the paper in inches per second, AB' is measured in inches, and λ is the velocity of the pressure wave in the water in the pipe. The value of λ can be determined from Table 2, as explained previously.

If the distance from the valve to the reservoir or large supply main is known, the distance to the leak may be most easily found by the formula,

$$s = \frac{AB'}{AC}D \qquad (26)$$

D being the known distance to the reservoir or supply main. This is the most satisfactory and accurate method of locating the leak. It is unnecessary when using this method to determine the value of the velocity of transmission of the pressure wave λ, the value of which may be affected by the air content of the water.

DETERMINATION OF THE QUANTITY FLOWING FROM A LEAK BY MEANS OF THE WATER HAMMER DIAGRAM. Fig. 4 represents the conditions in the pipe a short time after the pressuro wave has passed the leak. Let v be the velocity of the flow in the pipe between the leak and the valve, before the valve is closed. Let $v + w$ be the velocity of the flow in the pipe between the source and the leak. Then if A is the area of the cross-section of the pipe in square feet, and if q is the quantity flowing from the leak,

$$q = Av$$

If the valve is closed very quickly, a diagram similar to the one shown in Fig. 1 can be obtained. Let P be the water hammer pressure at the valve caused by extinguishing a velocity of v ft. per sec., and let P_1 be the pressure shown on the diagram after the drop in the pressure diagram is passed. Represent the actual pressure in the pipe at the leak just after the pressure wave passes the leak, by P'. When the valve is closed a pressure wave

$$P = hv$$

travels from the valve towards the leak. The pressure $P = hv$ is added to the original pressure in the pipe, and therefore when the leak is almost reached by the pressure wave, the pressure will be $P + p$, p being the original pressure at this point. When this wave of pressure gets to the leak, the discharge is increased, causing a drop of pressure, because the extinguished velocity is less than v. Represent the pressure at the leak just after the wave gets to it as $P' + p$. Since the pressure between the leak and the supply is increased P' lb. per sq. in. the velcity extinguished to cause this pressure must be

$$\frac{P'}{h} \text{ ft. per sec.}$$

There is still a velocity of

$$v + w - \frac{P'}{h} \text{ ft. per sec.}$$

from the supply to the leak. There is a re-expansion of the water between the leak and the valve, which causes a velocity toward the leak of

$$\frac{P - P'}{h} \text{ ft. per sec.}$$

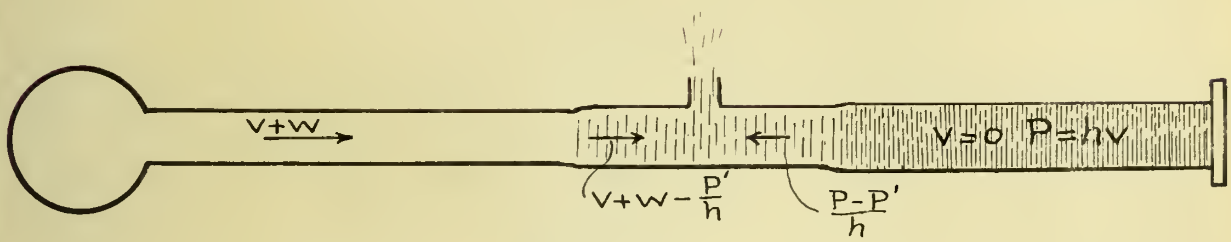


Fig. 4.

At the instant the pressure wave P' gets to the valve, all of the water in the pipe between the valve and the leak will have a pressure P' and a velocity of $\frac{P-P'}{h}$ ft. per sec. A further re-expansion will now take place, causing a wave of reduced pressure to run toward the leak. The amount of the reduction will be the same as the reduction $P - P'$. The pressure at the valve at this time is,

$$P_1 = P - 2(P - P')$$

or, solving for P'

$$P' = \frac{P - P_1}{2}$$

The pressure P_1 is the one shown on the water hammer diagram. The conditions just after the wave of re-expansion gets to the valve are shown on Fig. 5.

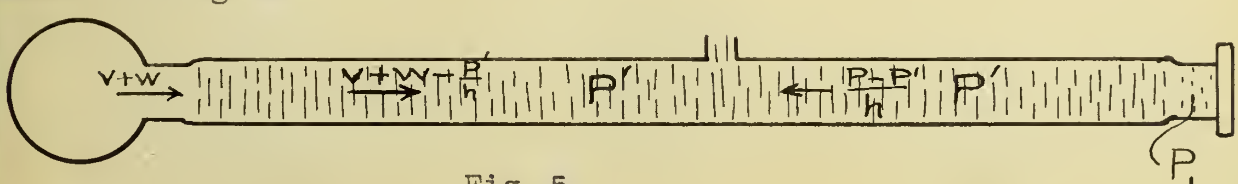


Fig. 5.

The total quantity flowing from the leak just after the pressure wave gets to it , is

$$Q = A\left(\frac{P}{h} + w - \frac{P'}{h} + \frac{P - P'}{h}\right)$$

$$= Aw + 2A\frac{P - P'}{h}$$

$$= q + 2A\frac{P - \frac{P + P_1}{2}}{h}$$

$$= q + \frac{\frac{P - P_1}{2}}{h}2A$$

$$Q = q + A\frac{(P - P_1)}{h}$$

But

$$Q : q :: \sqrt{H} : \sqrt{h}$$

$$\frac{Q}{q} = \sqrt{\frac{\frac{P + P_1}{2} + p}{p}}$$

$$Q = q\sqrt{\frac{P + P_1 + 2p}{2p}} = q + A\frac{P - P_1}{h}$$

Solving for q

$$q = \frac{A\frac{P - P_1}{h}}{\sqrt{\frac{P + P_1}{2p} + 1} - 1} \quad \ldots\ldots\ldots\ldots \quad (27)$$

which is the general formula for the quantity of flow from a leak in a pipe line as determined from measurements of the water hammer diagram.

DIAGRAM FOR DETERMINING THE QUANTITY OF WATER FLOWING FROM A LEAK BY MEANS OF MEASUREMENTS TAKEN FROM THE WATER HAMMER DIAGRAM.

Fig. (6) has been prepared from eq. (27) for a 12-in. pipe, with a value of h = 55, showing the value of the discharge from the leak for given values of $\frac{P+P_1}{2p}$ and $P-P_1$. For example, if the pressure at the leak before the valve is closed is p = 20 lb. per sq. in., the maximum water hammer pressure is P = 300 lb. per sq. in., and the pressure P_1 = 200 lb. per sq. in., then $\frac{P+P_1}{2p} = 12.5$ and $P-P_1 = 100$. Using these values as ordinate and abscissa, the discharge is found to be about 0.53 cu. ft. per sec.

If a pipe has a different diameter than 12 inches, or has a different value of h than 55, the result obtained from the diagram should be multiplied by $55\frac{D^2}{h}$. D is the diameter of the pipe in feet and h is the value of the constant for the pipe under consideration. If the pipe in the preceding problem had been had been a 24-in. pipe instead of a 12-in. pipe, the quantity flowing from the leak would be $\frac{55}{55} \times 4 \times 0.53 = 2.12$ cu. ft. per sec. This was on the assumption that the value of h is 55.

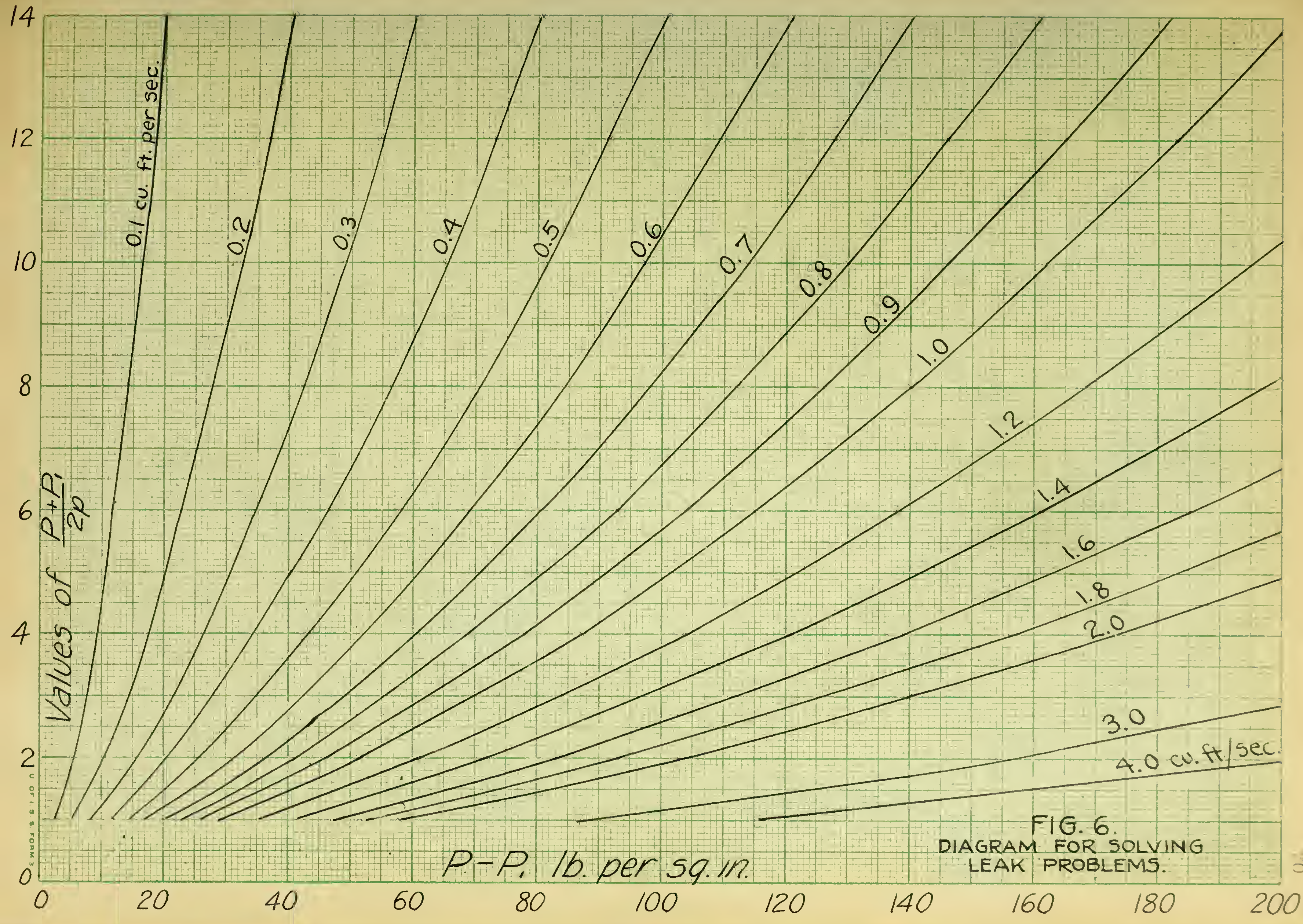


FIG. 6.
DIAGRAM FOR SOLVING LEAK PROBLEMS.

AIR CHAMBERS

JOUKOVSKY'S DISCUSSION. The following discussion is taken from the translation of Joukovsky's monograph in the Proceedings American Water Works Association, 1904.

Fig. 7 and Fig. 8 show curves obtained with a small air chamber (about 60 cu. in.), the velocity in the pipe being 4.4 feet per second. The curve in Fig. 7 was obtained between the valve and the air chamber, while that in Fig. 8 was obtained between the air chamber and the source. We see, from the curves, that an air chamber of that small size caused no lowering of the first stage of the curve obtained between the gate and the air chamber, which showed a pressure of 17.3 atmospheres, which is very nearly equal to the theoretical pressure of 17.6 atmospheres. As to the second stage of this curve, the air chamber even increased the pressure to almost 1.3 times the pressure in the first stage. The pressures in the third and following stages are notably diminished. The curve in Fig. 8, obtained between the air chamber and the origin, shows a slightly reduced pressure (14.6 atmospheres); and the stages of the curve become rounded and rapidly lower.

Quite different results were obtained when the size of air chamber was materially increased. Fig. 9 shows a curve obtained between the air chamber and the valve, with the use of an air chamber of 548 cubic inches, or 9 times that used in the experiments just described. The velocity of flow was 1.8 feet per second. This curve is very much like those usually obtained without the use of air chambers; the pressure wave is reflected from the air chamber as from the origin. The pressure here is 7.1 atmos-

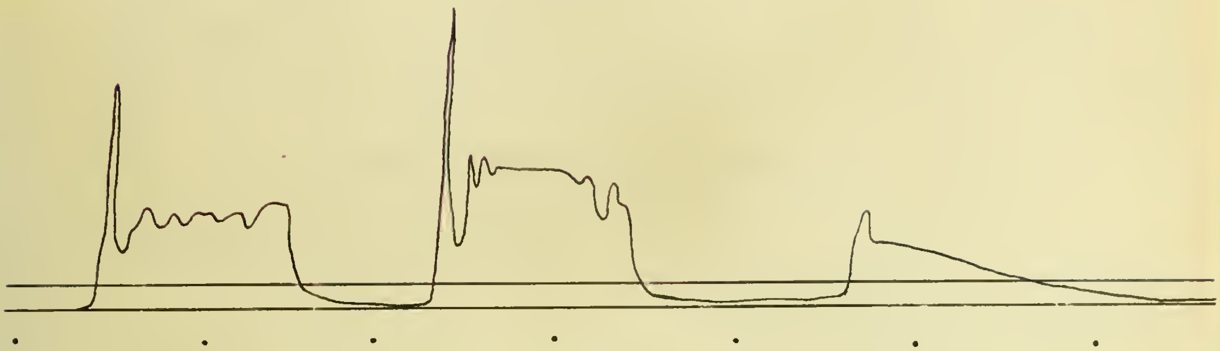


Fig. 7.

Diagram taken between the valve and the air chamber. (Small air chamber).

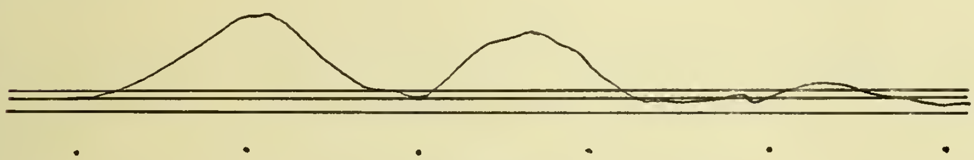


Fig. 8.

Diagram taken between the air chamber and the origin. (Small air chamber).

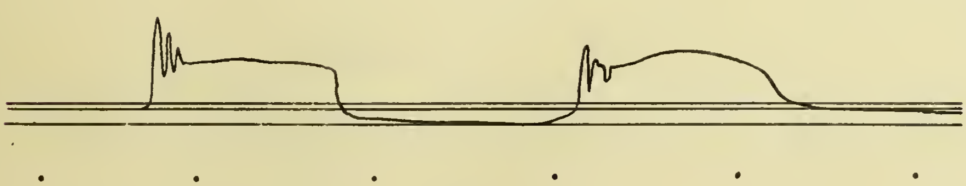


Fig. 9.

Diagram taken between the valve and a large air chamber.

pheres, which corresponds closely with the theoretical pressure, P = 7.2 atmospheres. The diagram recorded in this case between the origin and the air chamber is a straight line, coinciding with the line of the original hydrodynamic pressure, showing that an air chamber of this size does not allow water hammer of the given intensity to pass through it.

EXPLANATION OF PRINCIPLES. Suppose a water pipe AB, on which an air chamber is placed (see Fig. 10). Upon sudden closure of the valve B, the water near the valve is compressed to the pressure P = hv, and after some moments, the pressure wave moving with a velocity λ, will arrive at the air chamber, C. At that moment the pressure at C, as well as in the pipe between the air chamber and the origin, is still the hydrodynamic pressure, p_1. Therefore the compressed water finds an outlet into the air chamber, which enters with the original velocity v (which is now in the inverse direction).

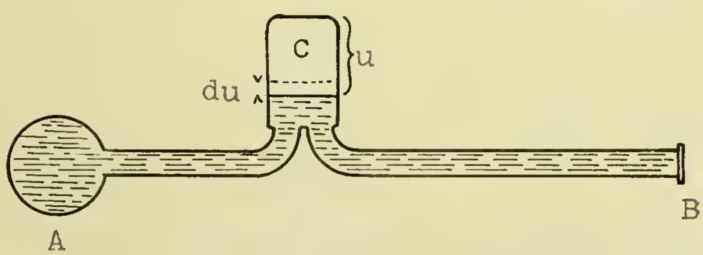


Fig. 10.

Hence the presence of the air chamber causes the pressure wave to be reflected not from the main, A, but from the air chamber, C.

We see also that the air chamber causes no reduction of the shock pressure in the section, BC, of the pipe between the valve and the air chamber. The experiments with air chambers have even

shown an increase of pressure in this section at the beginning of the second cycle, this increase (which in one case reached 50%) being due to the compression of the air in the chamber and its subsequent reaction, which throws the water to the gate, where a subnormal pressure already exists.

Let us now pass to the effect of the air chamber upon the section of the pipe, CA, between the air chamber and the origin. The water here continues to flow in the original direction, AC, and with the original velocity, v, until the pressure wave arrives at the air chamber, C. From this moment the pressure in the chamber increases because of the water coming into it from both sides (from BC and from AC). But the increase of pressure in the chamber and the compression of the air there, gradually stop the entrance of water into it, and affect the part, AC (and that part only) in the same way as if the flow from the pipe AC, were stopped by a slow closure of a valve placed on the pipe at C.

Conclusion: It has already been explained that the slow closure of a valve, lasting longer than the double run of the pressure wave through the length of the pipe, prevents the exertion of the maximum shock pressure.

It is evident that the flowing of water into the air chamber will last longer with a large than with a small volume of air; consequently, the stoppage of the column of water, AC, will be slower, and the shock pressure less, in the first case. Therefore the air chamber must be of large dimensions. As already stated small air chambers do not accomplish their purpose, being even harmful, increasing the pressure in the section, BC, between the valve and the air chamber.

We see then, that the air chamber reflects the pressure wave, allowing to pass through it only that part of the shock pressure which corresponds to the size of the air chamber; in other words, only that pressure is exerted in the pipe, AC (between the air chamber and the origin), to which the air in the chamber has been raised.

FORMULA FOR THE REQUIRED VOLUME OF AIR CHAMBER. The formula for the determination of the size of air chamber which will allow to pass through it only a shock pressure of given intensity, is deduced as follows:

Let u = variable volume of air in the air chamber.

v = original velocity of flow in the pipe.

d = diameter of the pipe.

P = hv = maximum additional shock pressure due to the checking of the velocity, v.

P_a = maximum additional pressure which will appear in the air chamber and will be carried along the pipe beyond the air chamber.

P_u = variable additional pressure in the air chamber, corresponding to the variable volume, u.

k = a constant numerical value, characterizing the thermodynamic properties of the gas contained in the air chamber. (In the case of air, k = 1.4).

t = duration of the round trip of the pressure wave (with velocity λ) from the air chamber to the nearest end of the pipe (whether valve or origin) and back.

p_o = the original hydrostatic pressure in the air chamber.

u_o = volume of air in the air chamber corresponding to to the original hydrostatic pressure, p_o.

p_1 = original hydrodynamic pressure in the air chamber.

u_1 = volume of air in the air chamber corresponding to the original hydrodynamic pressure, p_1.

At the moment when the pressure wave reaches the air chamber (which still has the original hydrodynamic pressure, p_1), the water will enter the chamber from both sides with the original velocity, v. It is evident that the volume of water, flowing into the chamber, must be equal to the reduction of the volume of air due to its compression. This compression and the resulting increase of pressure in the chamber gradually reduce the velocity of the water flowing into it. When the additional pressure in the chamber becomes = P_a, that is to say, when the total pressure there is = $p_1 + P_a$, the flow of water into the chamber will cease.

From the theory of water hammer, we know that P stands in direct proportion to the reduction of velocity of flow in the pipe($P = hv$). Consequently, if some additional pressure, P_u, has appeared in the air chamber, we know that the velocity of flow in the pipe has been reduced by $\frac{P_o}{h}$ and is equal to $(v - \frac{P_u}{h})$. This is the velocity with which water will flow from both sides into the air chamber when the additional pressure there is P_u.

We may thus obtain the equation: The reduction of air volume in the chamber (-du) during each small interval of time (dt) is equal to the volume of the water, which during this interval of time (dt) has entered the air chamber from both sides.

The velocity of the entering water is, as has been shown. $(v - \frac{P}{h}u)$.

The area of the cross-section of the pipe is $\frac{\pi d^2}{4}$.

Hence, in a unit time, each of the pipes will bring into the air chamber $\frac{\pi d^2}{4}\left(v - \frac{P}{h}u\right)$ cubic units of water.

During the interval dt, each pipe will bring in

$$\frac{\pi d^2}{4}\left(v - \frac{P}{h}u\right)dt$$

of water. Two pipes will let in twice this amount of water, and this amount will be equal to the reduction of volume of air in the air chamber. Consequently,

$$-du = \frac{\pi d^2}{2}\left(v - \frac{P}{h}u\right)dt$$

This is the differential equation of air compression in the chamber by the water entering it.

The flowing of water into the air chamber and the compression of air therein will continue until the state of compression of water in the air chamber (in the form of a wave, moving with velocity λ) arrives at the end of the pipe nearest to the chamber, and will return thence to the air chamber with reduced pressure. In other words, it will continue until the water begins to flow from the air chamber, which will result in a lowering of pressure. In order to find the amount of additional pressure, which will originate in the air chamber and will be transmitted along the pipe beyond the air chamber, we must integrate the above equation for the time, t.

The process of the compression of air in the chamber being rapid, we take it to be adiabatic, which is expressed by the physical law

$$u^k p = \text{constant}.$$

Beginning with the equation on preceding page

$$-du = \frac{\pi d^2}{2}\left(v - \frac{P_u}{h}\right)dt \; . \; . \; . \; . \; . \; . \; . \; (28)$$

and, substituting $v = \frac{P}{h}$, (according to the equation $P = hv$) we have

$$-du = \frac{\pi d^2}{2}\left(\frac{P - P_u}{h}\right)dt \; . \; . \; . \; . \; . \; . \; . \; . \; (28')$$

From the equation of the adiabatic compression we have:

$$u_1^k p_1 = u^k(p_1 + P_u) = \text{constant} \; . \; . \; . \; (29)$$

$$u^k = \frac{u_1^k p_1}{p_1 + P_u} \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; (29')$$

Differentiating this,

$$ku^{k-1}du = \frac{-u_1^k p_1 dP_u}{(p + P_u)^2}$$

consequently

$$du = -\frac{u_1^k p_1 dP_u}{ku^{k-1}(p_1 + P_u)^2} \; . \; . \; . \; . \; . \; . \; . \; . \; (30)$$

From equation (29')

$$u^{k-1} = \frac{u_1^k p_1}{u(p_1 + P_u)} \; ;$$

and that

$$u = \sqrt[k]{\frac{u_1^k p_1}{p_1 + P_u}}$$

consequently

$$u^{k-1} = \frac{u_1^k p_1}{\sqrt[k]{\frac{u_1 p_1}{p + P_u}} \cdot (p_1 + P_u)}$$

Substituting for u^{k-1}, in equation (30), the expression becomes

$$du = -\frac{1}{k}\,\frac{u_1 p_1^{\frac{1}{k}} dP_u}{p_1 + P_u^{\frac{k-1}{k}}} \; . \; . \; . \; . \; . \; . \; . \; . \; (31)$$

Substituting for du, in equation (28), the expression (31), we obtain;

$$\frac{1}{k}\cdot\frac{u_1 p_1^{\frac{1}{k}} dP_u}{(p_1 + P_u)^{\frac{k+1}{k}}} = \frac{\pi d^2}{2}\left(\frac{P - P_u}{h}\right)dt \ ;$$

or

$$k\cdot\frac{\pi d^2}{2}\cdot\frac{dt}{p_1^{\frac{1}{k}} h} = u_1\frac{dP_u}{(p_1 + P_u)^{\frac{k+1}{k}}(P + P_u)}$$

For convenience, let

$$z = \frac{p_1 + P_u}{p_1 + P} = \frac{\text{pressure in air chamber}}{\text{total max. pres. in pres. wave}} \quad . \ . \ (32)$$

Then

$$P_u = z(p_1 + P) - p_1 \ ; \quad dP_u = (p_1 + P)dz.$$

Substituting h by $\frac{P}{v}$ and using the adopted designations, we write:

$$\frac{k\pi d^2}{2}\cdot\frac{vdt}{p_1^{\frac{1}{k}}P} = u_1\frac{(p_1 + P)dz}{(p_1 + P_u)^{\frac{k+1}{k}}\left[P - z(p_1 + P) + p_1\right]} \ ,$$

$$\frac{k\pi d^2}{2}\ \frac{p_1(p_1 + P)^{\frac{k+1}{k}}}{p_1^{1+\frac{1}{k}}P}\,vdt = u_1\frac{dz}{\left(\frac{p_1 + P_u}{p_1 + P}\right)^{\frac{k+1}{k}}\frac{(p_1 + P)(1 - z)}{p_1 + P}} \ ;$$

$$\frac{k\pi d^2}{2}\left(\frac{p_1 + P}{p_1}\right)^{\frac{k+1}{k}}\frac{p_1}{P}vdt = u_1\frac{dz}{z^{\frac{k+1}{k}}(1 - z)} \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ (33)$$

Let us now integrate this equation for the time, t, during which the air in the air chamber is compressed and the additional pressure, P_u, increased from 0 to P_a. From our designation for z, it follows that the corresponding limits of integration will be

$$z_o = \frac{p_1}{p_1 + P} \quad \text{and} \quad z_t = \frac{p_1 + P_a}{p_1 + P}$$

The integration gives us;

$$\frac{k\pi d^2}{2}\left(\frac{p_1 + P}{p_1}\right)\frac{p}{P}vt = u_1\int_{z_o}^{z_t}\frac{dz}{z^{\frac{k+1}{k}}(1 - z)} \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ (34)$$

For brevity, let

$$\psi(z) = \int_{z_o}^{z_t}\frac{dz}{z^{\frac{k+1}{k}}(1 - z)} \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ (35)$$

For convenience, we will express u_1 in terms of the volume of air in the air chamber, u_o, corresponding to the static pressure, p_o.

According to the law of Mariotte,

$$u_1 = \frac{p_o u_o}{p_1}$$

Substituting the expressions given, we obtain from (34)

$$u_o = \frac{k\pi d^2}{2\psi(z)}\left(\frac{p_1 + P}{p_1}\right)\frac{p_1^2}{Pp_o}vt \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . (36)$$

This is the exact analytical expression showing the relation between the volume of the air chamber, u_o, and the intensity of the additional shock pressure, P_a, which the air chamber will allow to pass through it, and which is transmitted through the pipe beyond the chamber.

This formula owing to the complexity of the function $\psi(z)$, is inconvenient for practical use. But for the special case mentioned below it is possible to obtain, from this general and exact formula, a convenient and sufficiently approximate one. When the additional shock pressure, P_a, which is allowed to pass through the air chamber, is small, the difference between the limits of integration

$$z_o = \frac{p_1}{p_1 + P} \quad \text{and} \quad z_t = \frac{p_1 + P_a}{p_1 + P}$$

is also small, and we may write, approximately:

$$\psi(z) = \int_{z_o}^{z_t} \frac{dz}{z^{\frac{k+1}{k}}(1 - z)} = \frac{z_t - z_o}{z_o^{\frac{k+1}{k}}(1 - z_o)} \; ;$$

or

$$\psi(z) = \frac{\dfrac{p_1 + P_a}{p_1 + P} - \dfrac{p_1}{p_1 + P}}{\left(\dfrac{p_1}{p_1 + P}\right)^{\frac{k+1}{k}}\left(1 - \dfrac{p_1}{p_1 + P}\right)}$$

Simplifying, we have

$$\psi(z) = \left(\frac{p_1 + P}{p_1}\right)^{\frac{k+1}{k}} \frac{P_a}{P} \quad \ldots\ldots\ldots\ldots\ldots (37)$$

Substituting this approximate value of $\psi(z)$ in the formula (36), we obtain:

$$u_o = \frac{k\pi d^2}{2} vt \frac{p_1^2}{p_o P_a} \quad \ldots\ldots\ldots\ldots\ldots (38)$$

Here u_o is the minimum volume which the air chamber must have, in order that the shock pressure, P_a, in that portion of the pipe lying between it and the origin, shall not exceed the allowed intensity.

But the formula (38) is sufficiently approximate only when the allowed P_a is small; that is to say, when the volume of the air chamber is great. With small air chambers, great shock pressures pass through them, and the exact formula must be used.

For direct use of formula (38), tables of the function ψ should be calculated; but the formula may be used to calculate two limits, between which the true value of ψ is included.

According to formula (32), $z < 1$.

Therefore, by formula (35), where as a matter of fact $k = 1.4$;

$$\text{if } k = 1, \quad \psi_1 > \psi;$$
$$\text{if } k = 2 \quad \psi_2 < \psi.$$

The corresponding actual value of ψ lies between ψ_1 and ψ_2, or between

$$\left.\begin{aligned} \psi_1 &= \frac{1}{z_o} - \frac{1}{z_t} + \log\left(\frac{1}{z_o} - 1\right) - \log\left(\frac{1}{z_t} - 1\right) \\ \psi_2 &= 2\left(\frac{1}{y_o} - \frac{1}{y_t}\right) + \log\left[\frac{\frac{1}{y_o} - 1}{\frac{1}{y_o} + 1}\right] - \log\left[\frac{\frac{1}{y_t} - 1}{\frac{1}{y_t} + 1}\right] \end{aligned}\right\} \ldots\ldots (39)$$

Here log = Naperian logarithm, and y = z. Thus

$$y_o = \sqrt{z_o} \text{ and } y_t = \sqrt{z_t}$$

The volumes given by the formula should be divided by two if the air chamber is near the valve. This follows from what has been said before, the water in this case entering the air chamber from one side only.

Joukovsky's analysis has been given in full because it is often quoted as a correct solution of the air chamber problem. Joukovsky made a few experiments which checked his formula. A little study of the general formula (36) and of the approximate formula (38) will show that there are some inconsistencies. For example, when the air chamber is placed at the valve, the value of t being zero, the formulas will give zero volume as required to keep the pressure below any assumed value of maximum allowable pressure. It is stated in the last paragraph of the quotation, that the volume in this case should be divided by two. No reason is given, and no possible interpretation of the formula will give any such result. According to the formula if an air chamber is located, say 1000 feet from the valve, it makes no difference whether the pipe line is 2000 feet or 200000 feet long. This is evidently wrong.

Joukovsky's analysis rests on the equation

$$-du = \frac{\pi d^2}{2}\left(v - \frac{P_u}{h}\right)dt$$

which is true only for the time $t = \frac{2\,l}{\lambda}$ seconds after the valve closed, in case the air chamber is located at the valve. The length of the pipe line is l. After that time has elapsed the expression $\left(v - \frac{P_u}{h}\right)$ no longer expresses the change of the velocity in the pipe. The formulas derived on this assumption can therefore apply only to

air chambers of very small size. No such restriction is made by Joukovsky. The following analysis is thought to be free from the objections offered to the analysis of Joukovsky.

FORMULA FOR THE REQUIRED VOLUME OF AIR CHAMBER.

STATEMENT OF PRINCIPLES. The energy which will be stored in the air chamber when the valve at the end of the pipe is suddenly closed is equal to:

(1) the kinetic energy of the water flowing in the pipe,

minus (2) the energy stored by the compressed water and by the distended pipe,

plus (3), the work done by the flow head after the valve closes,

minus (4), the work done by pipe friction,

minus (5), the work done in lifting the water into the air chamber against gravity.

There is no difficulty of expressing any of these except (4). This term, however, is very difficult to express. When the air chamber is large enough to have practical value in reducing the pressure, (2) will ordinarily be small. The term (5) depends upon the size and shape of the air chamber, and will usually be small compared with the energy of the moving water.

In the following discussion the terms (4) and (5) will be omitted. The effect of omitting these terms will be to make the energy which must be stored by the air chamber greater than the true value. The volume of air required to keep the pressure below a given value will therefore be on the safe side. There will be two cases to consider; isothermal compression of the air in the air chamber, and adiabatic compression. In a large air chamber the isothermal law would perhaps be the nearest correct, while with a small air

chamber the compression takes place so rapidly that there is not time for heat to be transferred to the walls of the air chamber.

SIZE OF AIR CHAMBER REQUIRED. (ISOTHERMAL COMPRESSION) The following notation will be used:

p_1 = original dynamic pressure in the air chamber (lb per sq in)

u_1 = the volume of air in the air chamber corresponding to the pressure p_1 (cu. ft.)

p_2 = the maximum pressure in the air chamber (lb. per sq. in.)

u_2 = the volume in the air chamber when the pressure is p_2 (cu. ft.)

p_s = static pressure (lb. per sq. in.)

d = diameter of pipe (ft.)

l = length of the pipe line (ft.)

h = the constant in the formula P = hv (See Table 1.)

When the valve is closed, the pressure in the air chamber increases from p_1 to p_2 lb. per sq. in. From the water hammer formula, the velocity necessary to raise the pressure $p_2 - p_1$ lb. per sq. in., if there is no air chamber in the pipe line, is

$$v_e = \frac{p_2 - p_1}{h} \text{ ft. per sec.}$$

In this case all of the energy of the moving water is stored in the compressed water and in the distended pipe. We may therefore say that when the water has its pressure increased $p_2 - p_1$ lb. per sq. in., the energy is equivalent to the kinetic energy of the water in the pipe moving with a velocity v_e, as given by the equation above.

The energy stored in the air chamber, in foot pounds, is given by the well known formula

$$144 p_1 u_1 \log_e \frac{p_2}{p_1}$$

The kinetic energy of the water in the pipe, in foot pounds, is

$$\frac{w\pi d^2 l}{8g}v^2$$

The energy stored in the compressed water and distended pipe, is

$$\frac{w\pi d^2 l}{8g}\left(\frac{p_2 - p_1}{h}\right)^2$$

since all of the kinetic energy of the water in the pipe moving with a velocity of $\frac{p_2 - p_1}{h}$ feet per second is required to produce the increase in pressure from p_1 to p_2. The work done by the pressure head, in excess of the work done by friction, is

$$144m(p_s - p_1)(u_1 - u_2)$$

m being a factor depending upon the relation of the work done by the pressure head to the work done by the friction after the valve is closed. Equating,

$$144p_1 u_1 \log_e \frac{p_2}{p_1} = \frac{w\pi d^2 l}{8g}\left(v^2 - \frac{p_2 - p_1}{h}\right) + 144m(p_s - p_1)(u_1 - u_2)$$

Solving for u_1,

$$u_1 = \frac{.00528d^2 l\left[v^2 - \left(\frac{p_2 - p_1}{h}\right)^2\right]}{2.3p_1 \log_{10}\frac{p_2}{p_1} - m(p_s - p_1)\left(1 - \frac{p_1}{p_2}\right)} \quad \ldots\ldots\ldots\ldots\ldots(41)$$

The value of m will be between 0 and 1. For rough estimates it may be assumed to be 0.5. If a closer approximation is needed, it will be necessary to consider the energy relations for small increments of the velocity during the time the water is coming to rest.

SIZE OF AIR CHAMBER REQUIRED. (ADIABATIC COMPRESSION) The same notation will be used that was used in the last derivation. The only change that needs to be made is to substitute the expression for the energy stored in the air chamber during an adiabatic compression for the expression used in the previous proof. The expression then becomes,

$$\frac{144 p_1 u_1}{1 - k}\left[1 - \left(\frac{p_1}{p_2}\right)^{\frac{1-k}{k}}\right] = \frac{w\pi d^2 l}{8g}\left[v^2 - \left(\frac{p_2 - p_1}{h}\right)^2\right] - 144m(p_s - p_1)(u_1 - u_2)$$

Solving for u_1,

$$u_1 = \frac{.00528 d^2 l \left[v^2 - \left(\frac{p_2 - p_1}{h}\right)^2\right]}{\frac{p_1}{1 - k}\left[1 - \left(\frac{p_1}{p_2}\right)^{\frac{1-k}{k}}\right] - m(p_s - p_1)\left(1 - \frac{p_1}{p_2}\right)} \quad \ldots\ldots\ldots\ldots\ldots(42)$$

This equation is the same as eqation 41 except for one term in the denominator. It must be remembered that no account has been taken of the work required to raise the water into the air chamber.

For air the value of k is 1.4. Substituting, the expression becomes,

$$u_1 = \frac{.00528 d^2 l \left[v^2 - \left(\frac{p_2 - p_1}{h}\right)^2\right]}{2.5 p_1\left[\left(\frac{p_2}{p_1}\right)^{.286} - 1\right] - m(p_s - p_1)\left(1 - \frac{p_1}{p_2}\right)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(43)$$

THE VELOCITY OF FLOW AND THE PRESSURE AT THE VALVE DURING THE TIME THE VALVE IS CLOSING

GRADUAL CLOSURE. By a gradual closure is meant a closure taking place so slowly that no dynamic pressures are generated. The conditions of flow at any time is the same as for steady flow.

Assume a pipe line l feet long, and d feet in diameter. Let the flow head be represented by H, and let $m\frac{v^2}{2g}$, $f\frac{l}{d}\frac{v^2}{2g}$, $e\frac{v^2}{2g}$ and $\frac{v^2}{2g}$ represent the losses due to entrance, pipe friction, elbows, and velocity head, respectively. Let q represent the coefficient of valve loss at any valve opening. Then under steady flow conditions,

$$H = \left(m+e+f\frac{l}{d}+1\right)\frac{v^2}{2g}$$

For simplicity write

$$m+e+f\frac{l}{d}+1 = a$$

Then

$$H = (a + q)\frac{v^2}{2g} \quad \ldots\ldots (44)$$

The pressure at the valve (assuming that the valve is at the end of the pipe line), is

$$H_v = q\frac{v^2}{2g} \quad \ldots\ldots (45)$$

From (44) and (45)

$$H_v : H :: q : a+q$$

$$H_v = \frac{q}{a + q}H \quad \ldots\ldots (46)$$

Solving equation (44) for v,

$$v = \sqrt{\frac{2gH}{a + q}} \quad \ldots\ldots (47)$$

From equations (46) and (47) it will be seen that the effect of a partial closure of the valve at the end of a pipe line, depends upon the value of a (the resistance of the pipe line), as well as upon the value of the valve coefficient q. As an illustration of this consider the case of a 24-in. gate valve at the end of pipe lines having different lengths (or different resistances). Kuichling found that the coefficient of resistance of a 24-in. gate valve 82 per cent closed, was q = 41.2. Using equation (47) it is found that when

a = 137, the velocity in pipe = 1/4 of the velocity with valve open;
a = 687 " " " " = 1/2 " " " " " "
a = 2650 " " " " = 3/4 " " " " " "
a = 8780 " " " " = 9/10 " " " " " "

The effect upon the pressure at the valve can be obtained from equation (46). It is found that when

a = 137 the pressure at the valve = 23% of static pressure on valve,
a = 687 " " " " " = 5.6%" " " " "
a =2650 " " " " " = 1.5% " " " " "
a = 8780 " " " " " = 0.5% " " " " "

From this it is easily seen that in the last case (a = 8780 it makes no difference how slowly the first 82 per cent of the closure is made, there is still a velocity of 9/10 of the original velocity to be extinguished before the valve is fully closed. In order to avoid water hammer it would be necessary to make the last 18 per cent of the closure in a very long time.

Figure 11 shows how the velocity of the water in the pipe changes during a very gradual closure. Several values of a are given

Figure 12 shows how the pressure at the valve increases during a very slow closure of a gate valve at the end of pipes of different lengths.

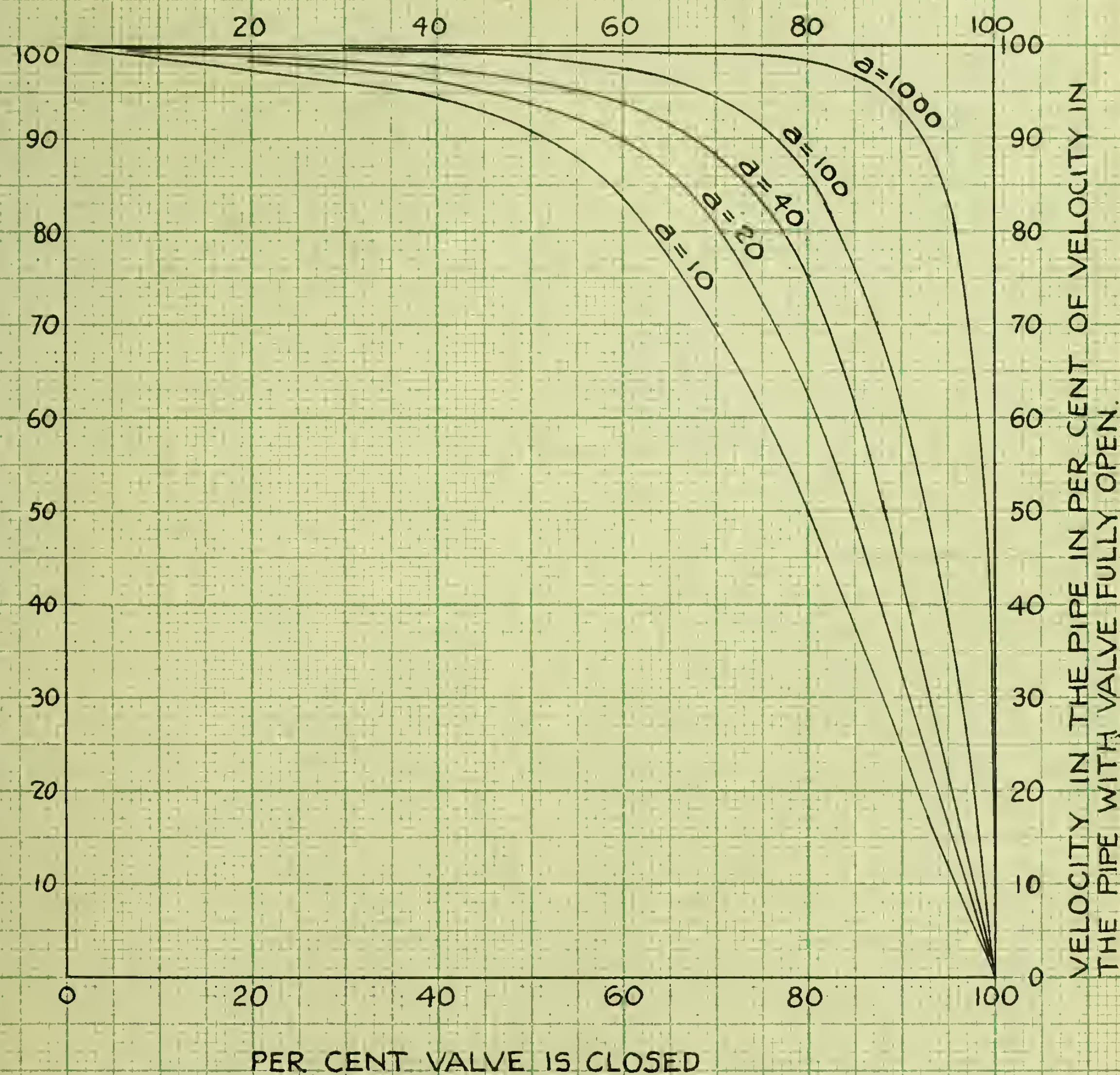


FIG. 11.

CURVES
SHOWING THE PROPORTIONAL VELOCITY, FOR ANY VALVE POSITION, DURING A VERY GRADUAL CLOSURE.

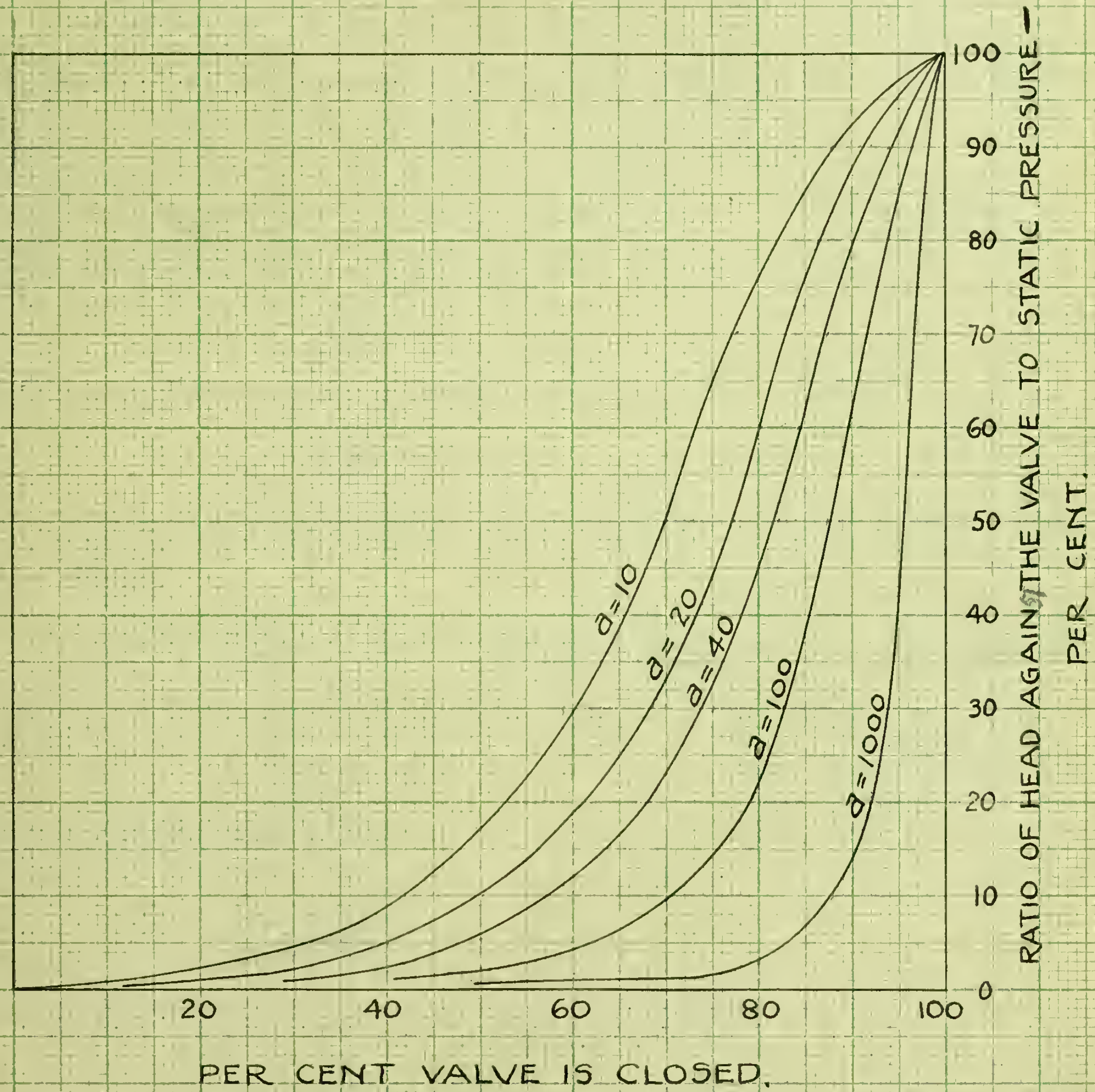


FIG. 12.

CURVES
SHOWING HOW THE PRESSURE AT THE VALVE DURING A VERY GRADUAL CLOSURE OF A VALVE AT THE END OF PIPE LINES HAVING VARIOUS VALUES OF a

THE PRESSURE DEVELOPED IN A PIPE LINE DURING THE TIME THE VALVE IS CLOSING

SUDDEN CLOSURE. If the closure takes place very rapidly, the value of q changes rapidly, causing the pressure at the valve to increase, which causes more water to flow through the partially closed valve than would be the case with a very slow closure. If a partial closure of the valve is made in less time than is required by the pressure wave to run to the open end and back, it may be considered a sudden closure. An expression for the velocity in the pipe after such a sudden closure has been made will now be obtained.

The value of q will have changed from q_1 to q_2, and the head on the valve,(at the end of the pipe line) is now,

$$P = q_2\frac{v^2}{2g} \qquad (48)$$

When the flow has had time to become steady, the pressure will be,

$$P_s = q_2\frac{v_s^2}{2g} \qquad (49)$$

v_s can be computed from equation (47), when q_2 and the other conditions are known.

If the original velocity in the pipe was v_o, the dynamic pressure generated during the sudden partial closure, is

$$P = h(v_o - v) \qquad (50)$$

From equations (48) and (49)

$$v = v_s\sqrt{\frac{P}{P_s}}$$

Substituting for P its value from equation (50)

$$v^2 = \frac{hv_s^2}{P_s}(v_o - v)$$

Solving for v,

$$v = -\frac{hv_s^2}{P_s} \pm \sqrt{\frac{hv_s^2v_o}{P_s} + \frac{h^2v_s^4}{P_s^2}} \qquad (51)$$

When v is known, the value of P can be found by substituting in equation (50)

If the values of q are known for any valve, the velocity in the pipe and the pressure at the valve for any valve position during a sudden closure can be easily calculated. Figure 13 shows the velocity and pressure for any position of the valve during a sudden closure of a valve at the end of a 2-in. pipe. An examination of this figure shows how short the effective time of closure is when the valve is closed rapidly. The effective portion of the closure is the last 10 per cent of the valve travel. For example, if a valve is closed at a uniform rate in 1/10 of a second, the effective portion of the closure will be made in about 1/100 of a second.

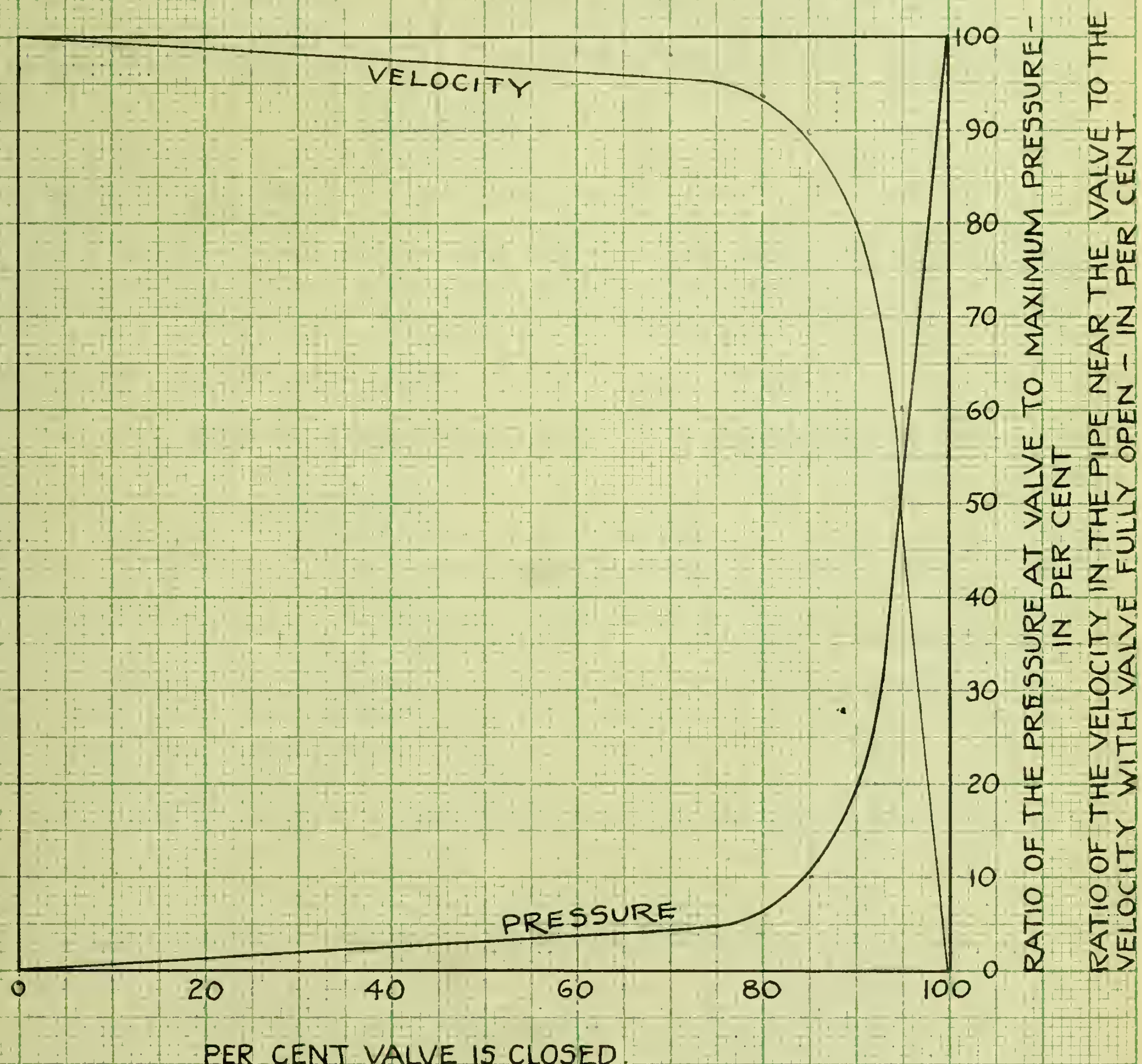


FIG. 13.

CURVES
SHOWING THE PROPORTIONAL PRESSURE AND VELOCITY, FOR ANY VALVE POSITION, DURING A SUDDEN CLOSURE OF VALVE.

THE MAXIMUM PRESSURE DEVELOPED BY THE SLOW UNIFORM CLOSURE OF A CYLINDER VALVE AT THE END OF A PIPE LINE

In a preceding article, equations were developed for the velocities and the pressures in the pipe during the time the valve is closing, when the closing takes place so slowly that no dynamic pressures are generated. The same thing was also worked out for sudden closures of a valve. In this article will be discussed the maximum pressure developed by the uniform closure of a gate valve.

Nomenclature

T = total time required in closing the valve (sec.).

t = time required to close the valve from any valve position.

c_d = the coefficient of discharge of the valve opening.

l = length of the pipe line (ft.).

v_o = The velocity of the water in the pipe before the valve is closed (ft. per sec.).

v = the velocity of the water in the pipe at any time t seconds before the valve is fully closed (ft. per sq. in.).

P = the pressure at the valve any time t seconds before the valve is closed, (lb. per sq. in.).

a = area of the valve opening before the closure is made, (sq. ft.).

A = the area of the cross-section of the pipe, (sq. ft.).

In order to make possible an analytical solution of this problem, the effect of friction will be neglected. This assumption will not be much in error in the case of penstocks for water power plants, since the total friction loss is not great in proportion to the head. The values of the maximum pressures will be a little too

large as a result of omitting friction.

The valve opening at any time during a uniform closure of a cylinder valve, is

$$\frac{t}{T}a \quad \text{square feet.}$$

The discharge through the valve opening at this time, is

$$c_d\frac{t}{T}a\sqrt{4.6gP} \quad \text{cubic feet per second.}$$

The discharge in the pipe is

$$Av \quad \text{cubic feet per second.}$$

Equating, and solving for P,

$$P = \frac{A^2T^2}{4.6gc_d^2a^2}\left(\frac{v^2}{t^2}\right) = j\frac{v^2}{t^2} \quad \ldots\ldots\ldots\ldots(52)$$

From the formula for impulse and momentum,

$$(P - p)Adt = \frac{wAl}{144g}dv \quad \ldots\ldots\ldots\ldots\ldots\ldots(53)$$

Let

$$m = \frac{wl}{144g}$$

Substituting in eq. (53),

$$(jv^2 - pt^2)dt = mt^2dv$$

To integrate, let $v = xt$. Then $v_o = x_oT$.

The equation can then be reduced to:

$$\frac{1}{m}\int_t^T \frac{dt}{t} = -\int_x^{x_o}\frac{dx}{p + mx - jx^2}$$

Integrating,

$$\frac{1}{m}\log_e\frac{T}{t} = -\frac{1}{n}\log_e\left[\frac{r + 2jx_o}{q - 2jx_o}\cdot\frac{q - 2jx}{r + 2jx}\right]$$

Where

$$r = \sqrt{m^2 + 4pj} - m$$

$$q = \sqrt{m^2 + 4pj} + m$$

$$n = \sqrt{m^2 + 4pj}$$

Substituting $x = \frac{v}{t}$, and solving for P, there is obtained,

$$P = j\left[\frac{qT^{\frac{n}{m}} - \dfrac{q - 2j\frac{v_o}{T}}{r + 2j\frac{v_o}{T}}rt^{\frac{n}{m}}}{2j\left(\dfrac{q - 2j\frac{v_o}{T}}{r + 2j\frac{v_o}{T}}t^{\frac{n}{m}} + T^{\frac{n}{m}}\right)}\right]^2 \qquad (54)$$

The maximum value of P occurs when the valve just reaches its seat, or, when $t = 0$. The value of P at this time is,

$$P = \frac{q^2}{4j}$$

Substituting the values of q and j, and simplifying, there is found,

$$P = p + \frac{w}{144g}\left[\left(\frac{c_d a l}{AT}\right)^2 + \left(\frac{c_d a l}{AT}\right)\sqrt{\left(\frac{c_d a l}{AT}\right)^2 + 4.6gp}\right] \qquad (55)$$

This is the expression for the maximum pressure due to a slow closure of a valve. A similar expression was derived by A. H. Gibson in a different manner.

The pressure at any part of the valve travel can be obtained from eq. (54)

Since friction was omitted in the derivation of equation (55), it is applicable only when the ratio of a:A is very small. Only the effective portion, or say the last 10 or 15 per cent, of the valve movement should be considered.

RELIEF VALVES

Relief valves are much used to prevent dangerous water hammer pressures from being generated in pipe lines and penstocks. When they are made of sufficient size, complete protection can be obtained. However, the selection of relief valves has been left to the judgment of the designer, with the result that inadequate sizes have been frequently used. The following theoretical discussion is offered as a help in the selection of relief valves.

Nomenclature.

a = the area of the opening of the relief valve (sq. ft.).

A = " " " " cross-section of the pipe (sq. ft.).

p = static pressure (lb. per sq. in.).

P_r = the pressure for which the relief valve is set (lb. per sq. in.)

c_d = the coefficient of discharge of the relief valve.

v = the velocity of the water in the pipe before the valve is closed (ft. per sec.)

h = the water hammer coefficient.

It will be assumed that the relief valve is at the end of the pipe line. If the relief valve is not at the end of the pipe, full water hammer pressure will be had between the valve and the relief valve.

When the valve is suddenly closed the relief valve will open as soon as the pressure becomes equal to P_r pounds per square inch. In order to produce a pressure of $P_r - p$ pounds per square inch above static pressure, a velocity of

$$\frac{P_r - p}{h} \text{ ft. per sec.}$$

must be extinguished. The velocity in the pipe at this time must be

$$v - \frac{P_r - p}{h} \text{ ft. per sec.}$$

The quantity of water flowing past any section of the pipe, is

$$A(v - \frac{P_r - p}{h}) \text{ cu. ft. per sec.}$$

The flow in the pipe after the valve is shut must all go through the relief valve. The flow through the relief valve can be written

$$c_d a \sqrt{2g \times 2.3P_r}$$

Equating the two expressions for the quantity, and solving for a, we obtain:

$$a = \frac{.082(v - \frac{P_r - p}{h})}{c_d\sqrt{P_r}} A \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; (56)$$

In the above formula, the coefficient c_d is hard to estimate. Not until experiments have been made on the principal types of relief valves, will it be possible to solve the problem accurately. Some experiments by the author, to be more fully described under Experimental Work, indicate that the value of c_d based on the nominal area of the opening, is in the neighborhood of 0.25.

When the relief valve is placed in the pipe line, at some distance from the valve, the water will come from both directions, with the original velocity, v, to the relief valve. In order to discharge twice the amount of water, twice the area of relief valve must be provided. A relief valve in this position will protect only that portion of the pipe between the relief valve and the supply.

THE QUANTITY OF WATER DISCHARGED BY A RELIEF VALVE IN ITS OPERATION.

The column of water in the pipe, moving with a velocity of v feet per second, is brought to rest by the pressure generated by the relief valve. If the pipe is <u>l</u> feet long, and if the cross-sectional area of the pipe is A square feet, from the principles of impulse and momentum,

$$A144\int(P_r - p)dt = \int\frac{wAl}{g}dv.$$

The relief valve will close when the pressure falls a trifle below P_r pounds per square inch. At this time the column of water will have absorbed energy, due to its resilience, to an amount corresponding to the kinetic energy of the water in the pipe when the velocity is

$$\frac{P_r - p}{h}\text{feet per second.}$$

The lower limit of velocity in the above integration, will be this amount. Integrating, we obtain,

$$A\,144(P_r - p)t = \frac{wAl}{g}\left[v - \frac{P_r - p}{h}\right]$$

Solving for <u>t</u>

$$t = \frac{wAl}{144g}\left[\frac{v}{P_r - p} - \frac{1}{h}\right] \text{ seconds.} \qquad ....(57)$$

The pressure at the relief valve during all this time has been P_r pounds per square inch. The negative work done by this force added to the positive work done by the static pressure, will be equal to the original kinetic energy minus the energy stored in the compressed water and in the distended pipe, plus [illegible] has [illegible]

$$144(P_r - p)Q = \frac{wAl}{2g}\left[v^2 - \left(\frac{P_r - p}{h}\right)^2\right] - 2.3wQP_r$$

Solving for Q, we obtain,

$$Q = \frac{wAl}{2g\left[144(P_r - p) + 2.3\rho\right]}\left[v^2 - \left(\frac{P_r - p}{h}\right)^2\right] \text{......(58)}$$

Q is the quantity flowing from the relief valve in cubic feet. It is usual to carry away the water discharged from the relief valve through pipes. In order to proportion such pipes, a knowledge of the maximum rate of discharge is needed. The maximum rate evidently occurs just after the relief valve begins to act. At this time the total flow in the pipe, diminished by the compression of the water, flows from the relief valve. The rate is therefore,

$$A\left(v - \frac{P_r - p}{h}\right) \text{ cubic feet per second. } (59)$$

EXAMPLE. A penstock, 6 feet in diameter and 4000 feet long, has a static pressure of 100 pounds per square inch. The coefficient h, in the water hammer formula, for this pipe is 45. The velocity in the pipe is 10 feet per second. The turbine gates can be closed in less than 4 seconds. (a) What total area of relief valve will be required in order that the pressure may not exceed 190 pounds per square inch? (b) For how long a time will the relief valve act? (c) How much water will be discharged? (d) What will be the maximum rate of discharge?

(a) Assuming $c_d = 0.25$, based on the nominal area of the relief valve, and substituting in the equation for the size, we obtain

$$a = \frac{0.82\left(10 - \frac{90}{45}\right)}{25\sqrt{180}}(9\pi) = 5.51 \text{ sq. ft.}$$

Twenty-eight 6-in. relief valves would be required to give this area.

(b) The time is,

$$t = \frac{62.5 \times \text{[illegible]} \times 4000}{144 \times 32.2}\left[\frac{10}{90} - \frac{1}{45}\right] = \text{~~156~~ } 4.8 \text{ seconds.}$$

(c) The quantity of water discharged will be,

$$Q = \frac{62.5(9\pi)4000}{[illegible]\times 32.2\times 90}(100 - 4) = 2616 \text{ cu. ft.}$$

(d) The maximum rate of discharge will be,

$$9\pi\ (10 - 2) = 226 \text{ cu. ft. per sec.}$$

a quantity only 20 per cent less than the original flow in the penstock. If the water is to be piped away from the relief valves, pipes of sufficient size must be used in order not to make too much back pressure againt the valves.

The expression for the time the relief valve will act as given by eq. (57), is not an exact one. The influence of friction has not been taken into account. The expression is very simple, however, and when the friction is only a small proportion of the head, as in the case of a penstock, the results obtained by its use will be sufficiently near to the truth.

The expression for the time the relief valve will act, when friction is taken into account, is given in the following discussion.

TIME RELIEF VALVE DISCHARGES TAKING ACCOUNT OF FRICTION. The pressure at the valve remains a constant value P, during the time the relief valve is open. The forces acting on the moving column of water in the pipe are: (a) static pressure, (b) friction in the pipe, and (c) the relief valve pressure. From the principles of impulse and momentum,

$$A(P - p + bv^2)dt = \frac{wAl}{144g}dv \quad \ldots\ldots\ldots\ldots\ldots (60)$$

P = the pressure for which the relief valve is set, (lb. per sq. in.).

p = the static pressure at the valve when there is no flow in the pipe.

b = the coefficient which multiplied by v^2 gives the total loss in the pipe line in lb. per sq. in. The losses considered are: entrance, pipe friction, elbow losses, etc. If the pressure at the valve during the time the valve is open is, p_v, then

$$b = \frac{p - p_v}{v^2}$$

l = the length of the pipe line in feet.

T = the total time the valve is open in seconds.

Equation (60) may then be written,

$$\int_0^T dt = 0.0135\ l \int_0^v \frac{dv}{P - p + bv^2}$$

Then

$$T = 0.0135 \frac{l}{\sqrt{(P - p)b}} \tan^{-1} v\sqrt{\frac{b}{P - p}} \quad \text{..............} (61)$$

Example. A water column at the end of a 12 inch supply line, 2000 feet long, is discharging at the rate of 4000 gal. per min. The static pressure at the water column is 40 lb. per sq. in., and the pressure during the time the column is discharging is 10 lb. per sq. in. When the water column is suddenly shut off, a large relief valve keeps the pressure from exceeding 80 lb. per sq. in. How long will the relief valve discharge when the water column is suddenly closed?

A discharge of 4000 gal. per min. in a 12 inch pipe causes a velocity of 11.3 ft. per sec. The value of b is,

$$b = \frac{80 - 10}{(11.3)^2} = 0.546$$

Substituting in eq. (61)

$$T = 0.0135 \frac{2000}{40\ 0.546} \tan^{-1} 11.3 \frac{0.546}{40}$$

$$= 5.8 \tan^{-1} 1.32 = 5.46 \text{ seconds.}$$

Equation (61) does not take account of the compression of the water in the pipe and the distention of the pipe. The effect of this correction is ordinarily not great, unless the relief valve is set for very high pressures.

If it is desired to take account of this factor, it can be done by integrating the eq. (60) between the limits, v and $\frac{P-p}{h}$, as was explained in the article on relief valves. The expression then becomes,

$$T = 0.0135 \frac{1}{\sqrt{(P - p)b}} \left[\tan^{-1} v \sqrt{\frac{b}{P - p}} - \tan^{-1} \frac{P - p}{h} \sqrt{\frac{b}{P - p}} \right] \qquad \ldots\ldots.(62)$$

Applying this equation to the example, assuming the value of h = 55, we obtain

$$T = 5.43 \text{ seconds}$$

instead of 5.46 seconds by the less exact formula.

THE TIME REQUIRED TO MAKE A GIVEN CHANGE IN THE DISCHARGE OF A PENSTOCK

When long penstocks are used for water power plants, much trouble is found with the speed regulation on account of the inertia of the water in the pipe. When a sudden load is thrown on, the speed decreases, causing the governor to open the gates. Practically the same quantity of water flows after the gate is opened as before, for a short time. The pressure therefore falls, causing a still further reduction in the power, and hence in the speed. The governor therefore continues to open the gate, until the speed is back to normal again. The gates are now too widely opened, and there will be an excess of power, causing the speed to increase above the normal. The governor now begins to close the gates, with the result that the pressure rapidly increases, and causes the speed to continue to increase. In this way great changes in the speed may be caused, unless the governor is made so sluggish that it will not over-run.

We will find an expression for the time required for the water in the pipe to have its velocity changed due to a change in the gate opening. We will take a penstock d feet in diameter, and l feet long, connected to a tangential impulse wheel of the Doble type. In this type of wheel the water flows through a nozzle whose area can be varied by the movement of a needle valve. Let the loss of head in the penstock due to friction, etc. be represented by $b\frac{v^2}{2g}$, let V be the velocity of the water through the nozzle, let v be the velocity of the water in the pipe. Let V = xv, x being a coefficient depending upon the amount the nozzle is open. Then,

$$Aw\left(H - \frac{V^2 - v^2}{2g} - b\frac{v^2}{2g}\right) = A\frac{wl}{g}\frac{dv}{dt}$$

$$\int_{t_1}^{t_2} dt = 2 \times l \int_{v_1}^{v_2} \frac{dv}{2gH - (x^2 - 1 + b)v^2}$$

This expression can be integrated if it is assumed that the change in the valve position is made instantly. That is, the value of x changes instantaneously from x_1 to x_2. Integrating,

$$t_2 - t_1 = \frac{l}{mn_2} \log_e \frac{(m + n_2 v_2)(m - n_2 v_1)}{(m - n_2 v_2)(m + n_2 v_1)} \quad \ldots\ldots\ldots\ldots (63)$$

The values of m and n being

$$m = \sqrt{2gH},$$

$$n = \left(x^2 - 1 + b\right)^{\frac{1}{2}}$$

and H is the static pressure at the valve expressed in feet of head, v_1 and v_2 being the initial and final velocities.

When there is no nozzle at the end of the pipe, the equation reduces to,

$$t_2 - t_1 = \frac{l}{mb} \log_e \frac{m - \sqrt{b}\ v}{m - \sqrt{b}\ v}$$

when the initial velocity is zero, and the velocity after $t_2 - t_1$ seconds is <u>v</u> feet per second.

From eq. (63) can be calculated the time required to make any change in the penstock velocities. Fig. 14 shows how the velocity increases with the time after the valve movement is made. The curve shows clearly the long time required for the last increments of the velocity. A governor must over-run in order to bring the velocity of the water to the normal in as short a time as possible.

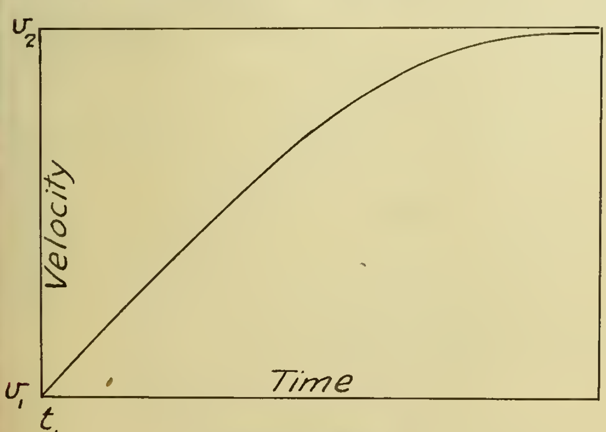


Fig. 14.

THE SURGE TANK OR STANDPIPE.

The surge tank, as defined by Raymond D. Johnson, "is a term applied to a standpipe or storage reservoir placed at the downstream end of a closed aqueduct to prevent undue rise of pressure in case of a sudden diminution of draft, and to furnish water quickly when the gates are opened, without having to wait for the velocity in the long feeder to pick up. When such a device terminates a pipe used to feed water wheels, the changes in load, producing corresponding variations in gate opening, cause the stored water to rise and fall in wave-like swells or surges; hence the name."

The paper by Mr. Johnson in the Transactions of the American Society of Mechanical Engineers in 1908, from which the above quotation was taken, has often been referred to as a correct solution of the surge tank problem. It will be well therefore to point out the errors in his fundamental equations. The paper was discussed by a number of prominent hydraulicians, but no one seems to have looked at the fundamental equations. It is a very good illustration of the fact that an engineer will not take the time to go through the mathematics in a paper of this kind, but will spend hours writing a discussion of the results.

The first portion of the mathematical analysis given by Mr. Johnson will be quoted:

"The best way to approach these problems is through equations of work and energy, and it may be as well to start out by laying down a perfectly general principle which will later become evident.

"The work done within either the stand pipe or the differential regulator, in raisng or lowering the water, is precisely

equal to the work of lifting the water in the conduit through a distance equal to the head due to the velocity lost or gained in the conduit, which change in velocity is in turn due to a change in load.

"The work done within th stand pipe, neglecting all friction would be $\frac{1}{2}y_1^2 \cdot \frac{A}{R}\alpha$ where y_1 = the maximum height of surge, R = the ratio of conduit area "A" to stand pipe area, α = the weight of water. The equivalent energy destroyed or gained in the conduit would be $\frac{(V_2 - V_1)^2}{2g}\alpha LA$ where L is the length of the conduit, $V_2 - V_1$ the ultimate change in velocity due to the load change.

"Equating these we have

$$y^2 = \frac{RL}{g}(V_2 - V_1)^2 \text{ or } y = \sqrt{\frac{RL}{g}}\,(V_2 - V_1).$$

This formula neglects both friction and the bellows action of the wheel gates in keeping step with the wave. This later consideration may usually be neglected if the regulator is otherwise designed correctly, but not unless. The equation given is not the equation of the wave curve, but simply an expression of the y maximum. We will now develop the differential equation of this curve, still for the present neglecting friction and also work out the time or period of vibration of the water pendulum.

"The work done upon the water in the tank when part load is rejected is $y\,dy\frac{A}{R}\alpha$: this may also be written in terms of the velocity change or $\frac{\alpha LA}{g}(V - V_1)dv$, whence we have

$$y\,dy = \frac{LR}{g}(V - V_1)dv$$

$$\int^{y} y\,dy = \frac{LR}{g}\int_{V}^{V_2}(V - V_1)dv$$

Solving for y we have

$$y_r = \sqrt{\frac{RL}{g}} \left\{ (V_2 - V_1)^2 - (V - V_1)^2 \right\}^{\frac{1}{2}}$$

Similarly for accelerating, when part load is demanded we have

$$y_a = \sqrt{\frac{RL}{g}} \left\{ (V_2 - V_1)^2 - (V - V_1)^2 \right\}^{\frac{1}{2}}$$

The maximum value of y can be seen to be the same in both cases or as previously demonstrated:

$$y_1 = \sqrt{\frac{RL}{g}} (V_2 - V_1)$$

The first error made by Mr. Johnson in the above quotation is in the statement that the "energy destroyed or gained in the conduit would be $\frac{(V_2 - V_1)^2}{2g} \alpha LA$, where $V_2 - V_1$ is the ultimate change in the velocity due to the load change." The change in the kinetic energy is actually $\frac{(V_2^2 - V_1^2)}{2g} \alpha LA$, a very different quantity. An idea of the magnitude of the error involved in the incorrect assumption may be had by substituting $V_1 = 10$ ft. per sec., and $V_2 = 11$ft. per sec. in the two equations. Johnson's equation gives as the energy gained αLA instead of the correct value $21 \alpha LA$.

Making the correction, and solving the first problem, we get as the maximum value of y_1 (height of surge),

$$y_1 = \sqrt{\frac{RL}{g}(V_2^2 - V_1^2)}$$

which is quite different from the result given above.

Another error is made by stating that the change in kinetic energy may be represented by $\frac{\alpha AL}{g}(V - V_1)dv$. The true value is $\frac{\alpha AL}{g}\left\{ V^2 - (V - dV)^2 \right\} \doteq \frac{\alpha AL}{g} V dV$.

Since all of Mr. Johnson's work rests on these incorrect assumptions, his work can not be called a solution of the problem of the surge tank. The corrections here indicated can be easily applied to Mr. Johnson's work, and his method can be followed out.

THE OSCILLATION OF THE WATER IN THE PENSTOCK.

Valve at End Fully Closed. Fig. 15 shows a penstock having a reservoir at one end and a stand pipe near the valve. When the valve is suddenly closed, the water will continue to flow into the stand pipe until the kinetic energy of the water in the pipe is all expended. The level of the water in the stand pipe will continue to rise until it is considerably above the level in the reservoir. A flow in the opposite direction will then begin. The period of this oscillation will now be found.

Let A = the area of the cross-section of the reservoir, a= the cross-sectional area of the stand pipe, A_p = the cross-sectional area of the penstock and L = the length of the penstock. Neglect friction. Then if y is the amount the water level in the stand pipe is above (or below) normal at any instant, then from Fig. 14 the difference of head at A and B, is

$$y + \frac{a}{A}y$$

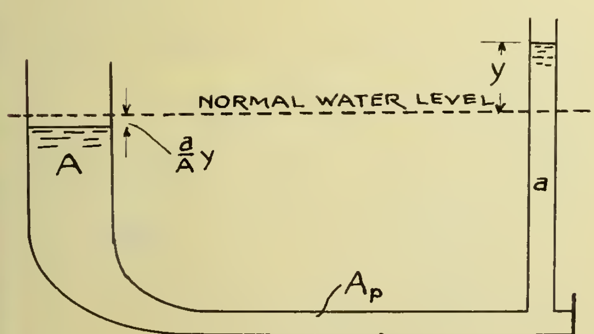


Fig. 15

From the principles of impulse and momentum

$$\frac{w}{144}y\left(1 + \frac{a}{A}\right)A_p\,dt = \frac{wA_pL}{144g}dv$$

Simplifying

$$\frac{dv}{dt} = -\frac{g}{L}\left(1 + \frac{a}{A}\right)y \text{ ....(64)}$$

According to the law of harmonic motion,

$$\text{Acceleration} = -\omega^2 y$$

The value of the angular velocity in a circle producing like motion, is

$$\omega = \sqrt{\frac{g}{L}(1 + \frac{a}{A})}$$

The period of a complete oscillation will be from the laws of harmonic motion,

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{g}{L}(1 + \frac{a}{A})}} \quad \text{.....................(65)}$$

When the area of the stand pipe and the area of the reservoir are equal, a = A, the formula reduces to:

$$T = \pi\sqrt{\frac{2L}{g}}$$

When the reservoir is large as compared to the area of the stand pipe the value of $\frac{a}{A}$ may be taken as zero. The period of vibration is then,

$$T = 2\pi\sqrt{\frac{L}{g}}$$

Which is the time of a complete vibration of a simple pendulum whose length is equal to the length of the pipe line. This will be the most common case, since in almost all installations the reservoir is very large as compared with the penstock or the stand pipe.

The maximum height to which the water will rise can also be determined from the laws of harmonic motion. If v_s represents the velocity of the water in the stand pipe at the time when the water level is y feet from the normal, and y_{max} represents the maximum height of surge, the from the principles of harmonic motion,

$$v_s = \omega\,(y_{max}^2 - y^2)^{\frac{1}{2}}$$

Substituting the value of ω

$$y_{max} = \frac{(\frac{A}{a}p)v}{\sqrt{\frac{g}{L}(1 + \frac{a}{A})}} \quad \text{.....................(66)}$$

When a = A

$$y_{max} = \sqrt{\frac{L}{2g}}\cdot(\frac{a}{A_p}v)$$

When the value of A is very great as compared with the area a, the expression for y_{max} becomes,

$$y_{max} = \sqrt{\frac{L}{g}} \cdot \frac{a}{A_p} \cdot v$$

No account has been taken of friction in this derivation. The equations resulting when friction is considered are very difficult to solve and graphical approximate methods must be used.

PART TWO - EXPERIMENTAL WORK

DESCRIPTION OF APPARATUS. The experiments were made in the Hydraulic Laboratory of the University of Illinois. The pipe line consisted of 740 feet of 2 inch steel pipe, laid in rectangles, one above another, of about 75 ft. by 2 1/2 ft. A Lunkenheimer quick closing valve was used for shutting off the flow quickly. Air chambers made by capping 6 inch pipe with blind flanges were used. Three sizes were used, 10 ft., 3 ft., and 2 ft. lengths of 6 inch pipe.

The pressures were measured by means of Crosby indicators, steam indicators being used for the lower pressures, and hydraulic indicators for pressures exceeding 100 lb. per sq. in. The indicator connection was made with a 3/4 inch pipe from tees which were put in the line at about every 75 feet for this purpose. In this way it was possible to determine the pressure at any point in the pipe line.

The pressures were recorded on a drum rotated at a uniform rate by an electric motor. Different speeds could be given the drum by the arrangement of the belts. The drum was of wood, and was about 12 inches in diameter. The paper on which the records were made, was about 6 inches wide and long enough to go around the drum, to which it was attached by means of thumb tacks. The photograph of the recording apparatus Fig. 16 shows the construction of the apparatus clearly.

The velocity of flow was obtained by weighing the water discharged in a given time. In order to save time in this part of the work, a vertical jet method of measuring water was used. This was done by making two sizes of orifices in two 2 in. caps. One of these was then screwed on the pipe, and the water allowed to discharge vertically. The height of the jet was a very accurate indicator of

the quantity of water being discharged, and hence of the velocity of the water in the pipe. A rod was clamped beside the orifice, and was graduated to read the velocity of water in the pipe directly. By means of the two sizes of orifices, accurate measurement could be made of all of the velocities obtained in the experiments. The photograph of this measuring device, Fig. 17, shows how it looks when in operation.

In some of the experiments an electric attachment which marked half seconds on the diagram, was used. It consisted of an electro-magnet with a pencil attached to the armature. The pulsations were caused by a heavy pendulum dipping into a globule of mercury at the bottom of the swing, making connection in the battery circuit. The pendulum was quite heavy, so that it would swing for half an hour when started going.

The water used in the experiments came from a standpipe, 4 feet in diameter, and 60 feet high. By means of an altitude governor on the pump used, any height of water in the standpipe could be maintained.

Some of the experiments to be described, were made by Mr. M. S. Mc Collister and Mr. W. A. North, and reported in their graduating thesis, in 1910. Most of the experiments have been made by the author, beginning with experimental work in the spring of 1906.

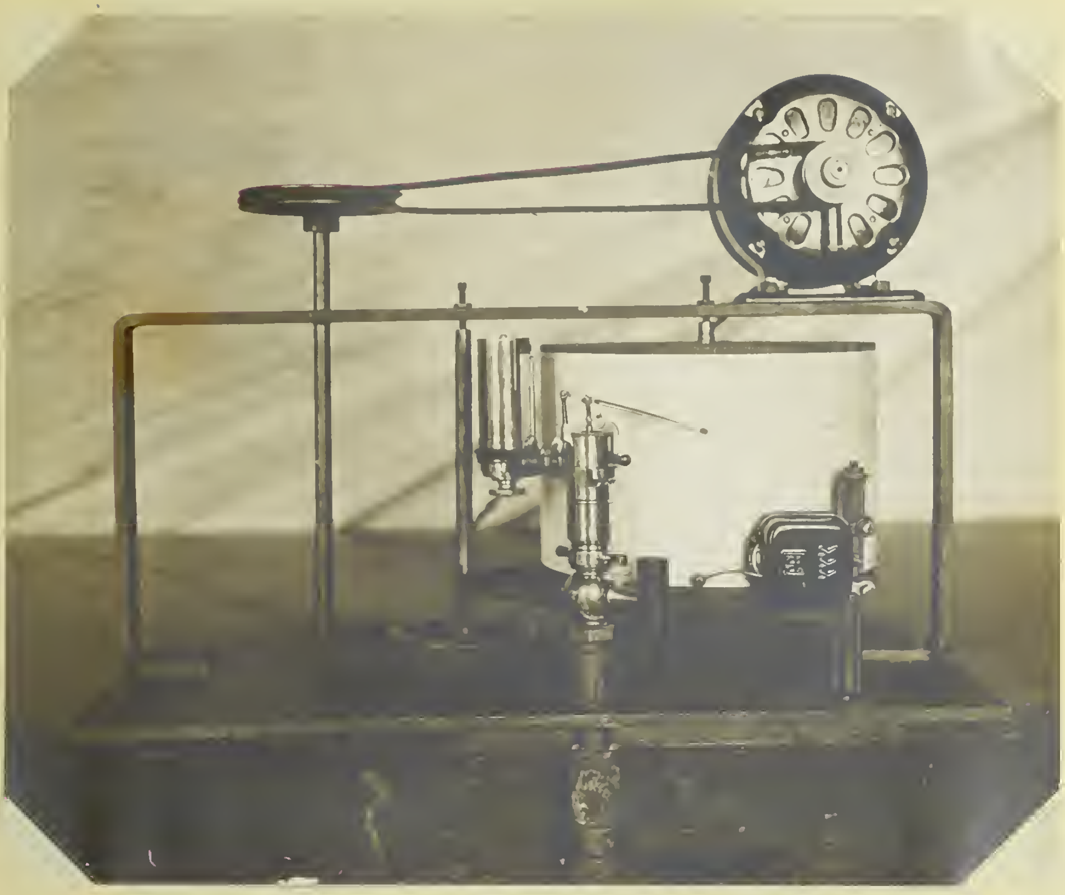


FIG. 16.

THE RECORDING APPARATUS

FIG. 17.

THE VERTICAL JET MEASURING APPARATUS.

FIG. 18.

AIR CHAMBERS

MAXIMUM PRESSURE DUE TO THE SUDDEN CLOSING OF A VALVE

The formula for the maximum pressure due to the sudden closing of a valve at the end of a long line of pipe has been verified by several experimenters working on pipes of several diameters. There is no doubt as to the substantial accuracy of the formula which was derived in another part of this thesis. The results of the experiments of Joukovsky, Gibson, and North and Mc Collister are tabulated below.

JOUKOVSKY - 24 INCH PIPE.

Velocity of water in feet per second.	Pressure in Atmospheres.	
	From diagrams	Calculated
0.18	0.45	0.54
0.56	1.81	1.68
0.55	1.66	1.65
0.54	1.77	1.62
0.55	1.80	1.65
0.41	1.23	1.23
0.40	1.27	1.20
0.16	0.42	0.48
0.16	0.42	0.48
0.09	0.29	0.27

JOUKOVSKY - 6 INCH PIPE.

Velocity of water in feet per sec.	Pressures in Atmospheres From Diagrams	Calculated
3.3	15.7	13.2
1.9	7.3	7.6
0.6	3.0	2.4
1.4	6.0	5.6
3.0	12.1	12.0
4.0	15.6	16.0
5.6	25.2	22.4
7.5	29.0	30.0
7.5 Pipe	11.7 burst	30.0

JOUKOVSKY - 4 INCH PIPE.

Velocity of water in feet per sec.	Pressures in atmospheres. From Diagrams	Calculated
3.3	13.3	13.2
1.9	7.8	7.6
4.1	15.8	16.4
9.2	35.0	36.8
2.9	11.3	11.6
0.5	2.0	2.0
1.1	4.4	4.4

JOUKOVSKY - 2 INCH PIPE. (CAST IRON)

Velocity of water in feet per sec.	Pressures in Atmospheres. From Diagrams	Calculated
4.52	18.5	18.1
4.30	17.8	17.2
4.16	17.0	16.6
3.67	15.1	14.7
3.67	14.5	14.7
3.66	14.6	14.6
1.79	6.3	7.2
1.76	7.3	7.0
0.64	2.8	2.6
1.52	6.3	6.1
1.52	6.3	6.1
4.23	17.3	16.9

GIBSON - 3.75 INCH PIPE. (CAST IRON)

Velocity of water in feet per sec.	Pressures in lb. per sq. in. From Diagrams (Average of four exper'ts)	Calculated
.363	19.5	19.5
.551	29.3	29.7
.720	37.7	38.9
1.094	57.5	59.0
1.444	73.8	77.9

NORTH AND MC COLLISTER - 2 INCH PIPE. (STEEL)

Velocity of water in feet per sec.	Pressures in lb. per sq. in.	
	From Diagrams	Calculated $P = 59.7v$
0.5	30	29.9
0.6	37	35.8
0.7	42	41.8
0.8	48	47.8
0.9	55	53.7
1.0	61	59.7
1.1	67	65.6
2.6	153	155.2
3.0	175	179.1
3.4	196	203.0
3.5	200	209.0
3.9	239	232.8
4.0	240	238.8
4.4	271	262.7
4.5	270	268.7
4.9	290	292.5

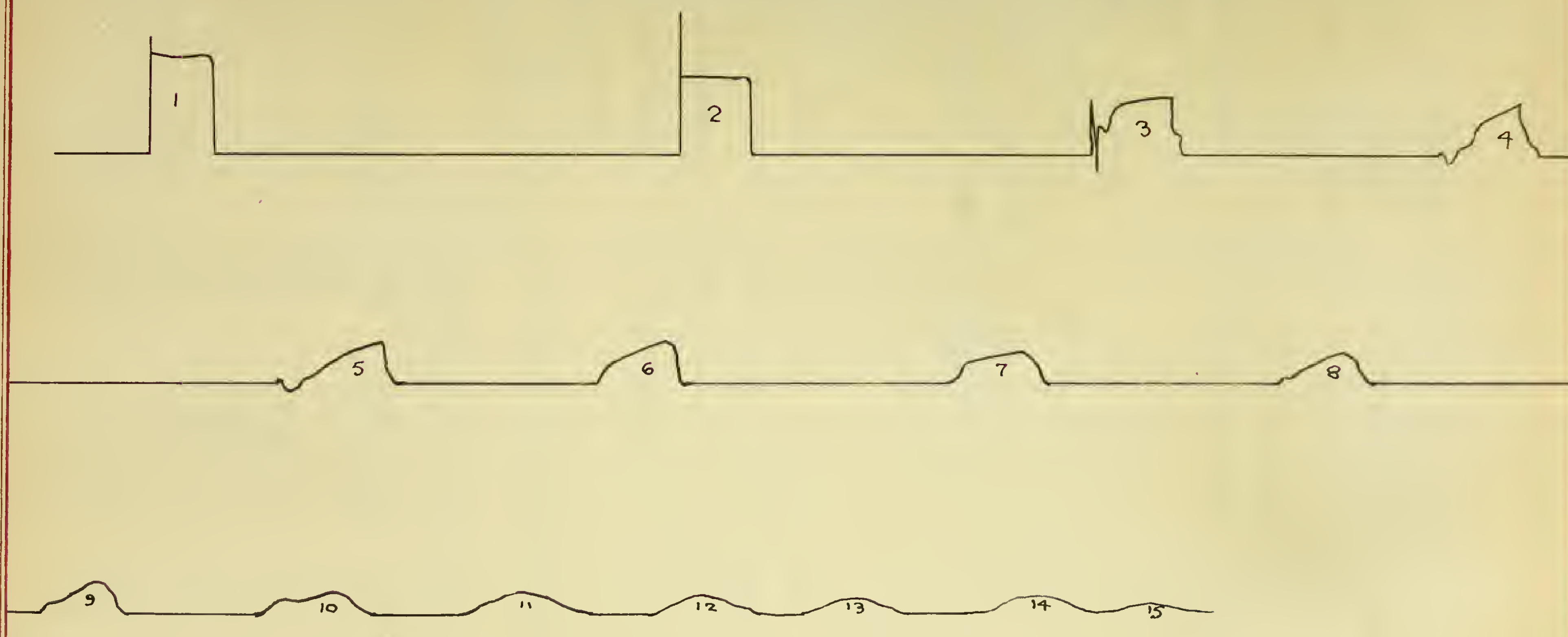


Fig. 19.

Diagram Showing Recurrence of Pressure Pulsations.

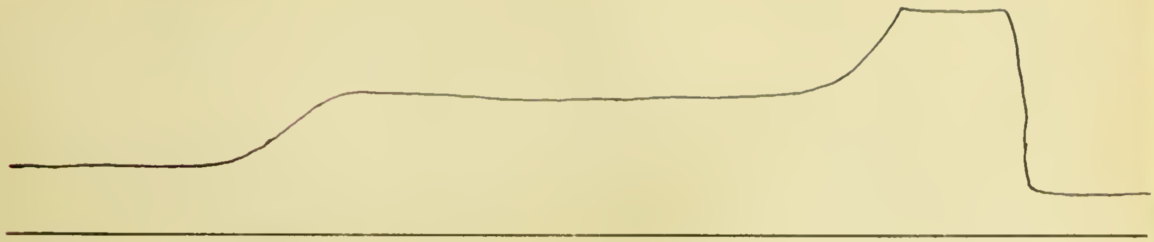
LEAKS

Experiments were made to locate a leak by means of the water hammer diagram, and to determine its magnitude. Sample diagrams are given to illustrate the drop caused on the pressure diagram. The distance from the beginning of the rise of the pressure to the beginning of the drop in pressure, represents to some scale the distance to the leak. The most satisfactory way to find the distance to a leak from the diagram, is to multiply the length of the pipe by the ratio of the distance (scaled from diagram) from the first rise in pressure to the drop in pressure caused by the leak, to the distance from the first rise in pressure to the final drop in pressure. This is the second method described under the theoretical discussion of this subject. By this method it is not necessary to determine the velocity of the pressure wave in the water in the pipe, and more accurate work can therefore be done. The value of the velocity of transmission of the pressure wave can be estimated by means of Table 2, but considerable variation must be expected from this value.

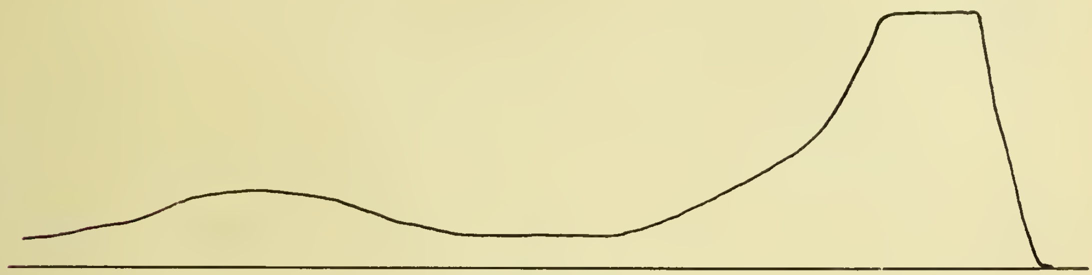
The following tabulations are from the thesis of North and Mc Collister, 1910.

LEAK 149 FEET FROM THE VALVE.

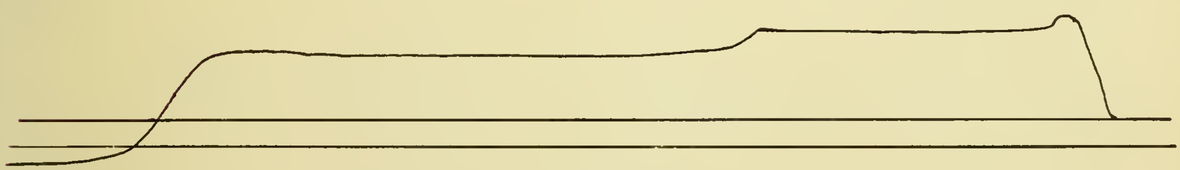
Trial	Computed dist. ft.	Trial	Computed dist. ft.	Trial	Computed dist. ft.
1	152	6	151	11	158
2	154	7	147	12	155
3	143	8	154	13	156
4	148	9	152	14	158
5	145	10	152	15	143



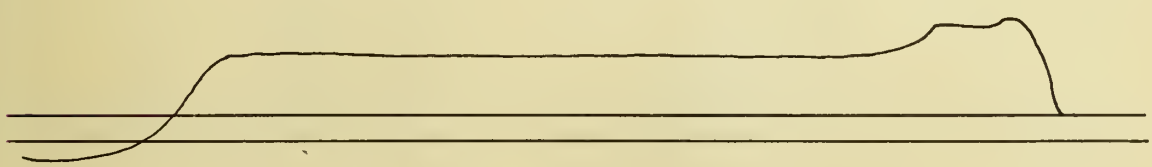
LEAK 149 FEET FROM THE VALVE.



LEAK 149 FEET FROM THE VALVE.



LEAK 380 FEET FROM THE VALVE.



LEAK 90 FEET FROM THE VALVE.

FIG. 20.

The expression for the quantity of a leak was checked by measuring the quantity flowing from the valve in the pipe, 149 feet from the valve at the end. In this way the quantity of the leak could be varied. A pressure gauge was placed near the leak for determining the dynamic pressure at the leak. The results are not as close as could be wished, but are quite close when the character of the apparatus is considered.

The experiments of North and Mc Collister made under the direction of the author, are tabulated on the following pages. The values of p given by these tables are actually 0.6 lb. per sq. in. too high. With this correction the results would be a little nearer the true value of the quantity flowing from the leak.

THE QUANTITY OF A LEAK DETERMINED FROM THE WATER HAMMER DIAGRAM.

Pressure in lb per sq in			Quantity in cu ft per sec	
P	P_1	p	Observed	Calculated
45	30	20.0	0.0087	0.0078
46	32	20.0	"	0.0071
76	54	19.5	"	0.0074
140	97	16.3	"	0.0093
150	100	15.5	"	0.0090
155	110	15.0	"	0.0077
170	115	14.0	"	0.0086
200	160	12.5	"	0.0050
260	170	8.0	"	0.0077
280	160	5.0	"	0.0077
280	170	5.0	"	0.0069
270	160	6.5	"	0.0085
30	19	20.0	0.0109	0.0082
80	48	18.5	"	0.0105
86	51	18.5	"	0.0109
140	90	16.5	"	0.0108
150	115	16.5	"	0.0063
170	105	15.0	"	0.0108
170	110	14.0	"	0.0094
270	140	5.0	"	0.0087
270	120	4.8	"	0.0085
260	170	6.0	"	0.0068
62	36	19.3	0.0131	0.0108
220	130	7.3	"	0.0082
460	290	13.0	"	0.0138

(Continued)

Pressure in lb per sq in			Quantity in cu ft per sec	
P	P_1	p	Observed	Calculated
56	36	20.0	0.0131	0.0089
74	46	19.0	"	0.0098
170	125	14.0	"	0.0068
125	90	16.0	"	0.0072
105	62	17.3	"	0.0102
250	120	7.5	"	0.0116
78	47	18.8	"	0.0105
170	110	14.0	"	0.0094
285	120	5.5	"	0.0117
97	54	18.0	0.0153	0.0123
180	100	12.5	"	0.0117
60	32	19.3	"	0.0121
240	120	8.5	"	0.0118
130	80	16.5	"	0.0107
150	80	15.0	"	0.0132
230	100	10.0	"	0.0151
48	26	20.0	"	0.0116
280	110	5.0	"	0.0115
175	110	13.8	"	0.0101
215	120	11.0	"	0.0115
56	28	19.5	0.0174	0.0131
58	29	19.0	"	0.0130
68	33	18.5	"	0.0137
160	75	15.0	"	0.0156
185	85	14.3	"	0.0163
200	95	12.3	"	0.0140

(Continued)

Pressure in lb per sq in			Quantity in cu ft per sec	
P	P_1	p	Observed	Calculated
275	90	5.0	0.0174	0.0132
270	95	6.5	"	0.0146
130	65	16.5	"	0.0148
185	110	13.2	"	0.0110
290	95	4.5	"	0.0127
54	24	19.0	0.0196	0.0150
67	31	18.5	"	0.0145
85	47	17.5	"	0.0117
50	24	19.0	"	0.0131
102	57	16.7	"	0.0122
280	80	4.8	"	0.0139
127	55	15.7	"	0.0164
260	90	6.5	"	0.0143
280	90	4.8	"	0.0130

EXPERIMENTS ON RELIEF VALVES

In order to check the theoretical formula for the area of relief valve required for any given service, it was necessary to make the experiments with known areas of opening. Fig. 29 shows the method of experimenting. A 2 in. nipple was capped as shown. The cap was machined smooth on the inside, and a small hole was drilled. The hole was lightly plugged, and the valve was opened, and when the flow had become steady the velocity of the water in the pipe was measured. The valve was then suddenly closed and a water hammer diagram taken. The plug being placed in loosely was blown out as soon as the pressure exceeded static pressure, and the whole opening was then clear to reduce the maximum pressure. The pressures were obtained in this manner for a range of velocities. The coefficient of discharge was then determined. A slightly larger opening was then bored in the cap and another set of experiments made.

The accompanying plates give the result of the experiments. The values of the pressures are seen to be too low for the smaller sizes tested. This is because too great pressure was put upon the indicator pencil. It was a somewhat difficult matter to close a valve suddenly with one's right hand and keep a light pressure on the indicator pencil with the left. The later experiments are much better in this respect.

In Fig. 27 are plotted all of the theoretical curves for the sizes of orifice tested, and on the same sheet are plotted the experimental results with the 1/2 inch Lunkenheimer relief valve on which experiments were also run. It will be seen that the 1/2 inch valve is equivalent to an orifice from 5/16 inch to 3/8 inch in diameter, depending somewhat upon the pressure for which the valve is set.

A curve is also given showing the coefficient of discharge of the 1/2 inch relief valve. On this plate are also shown the equivalent circular orifices whose coefficient of discharge are taken as 0.60.

Below is given a table showing the coefficient of discharge of the 1/2 inch relief valve, for various openings, based on the nominal area of the relief valve and also based on the actual opening.

Distance valve is raised from its seat (Inches)	Coef. of disch. based on nominal area.	Coef. of disch. based on actual opening.
0.0139	0.075	0.675
0.0278	0.145	0.652
0.0416	0.173	0.567
0.0555	0.189	0.425
0.111	0.247	0.278
0.167	0.326	0.244
0.222	0.356	0.201
0.333	0.458	0.172
0.444	0.505	0.142

The principal difficulty in the use of the formula given, is in the determination of the correct value of the coefficient of discharge. Since the size of the relief valve required to keep the pressure below any given value is a direct function of this coefficient, this factor is important. There will need to be further experiments on all of the principal types of relief valves before accurate design can be made.

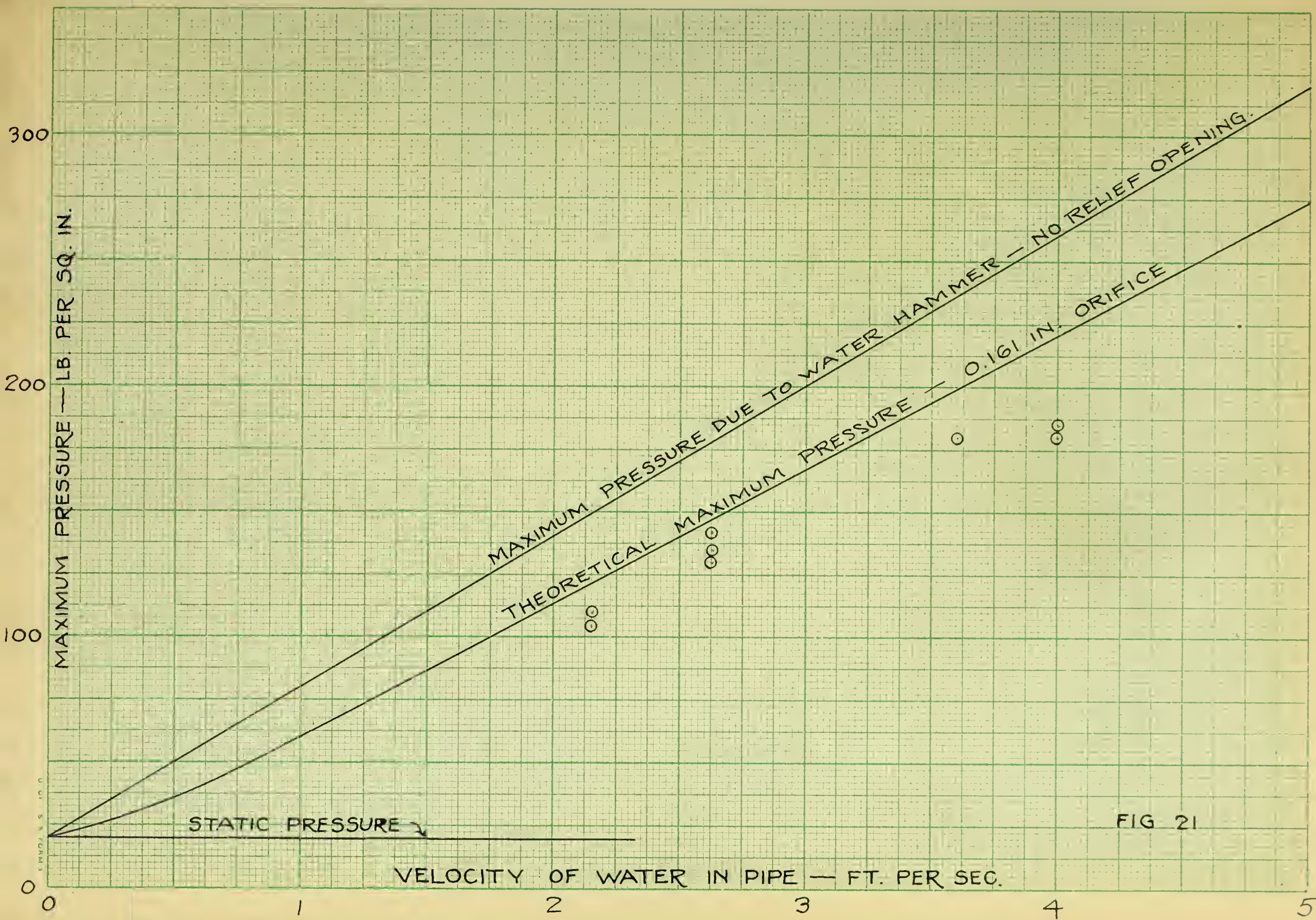
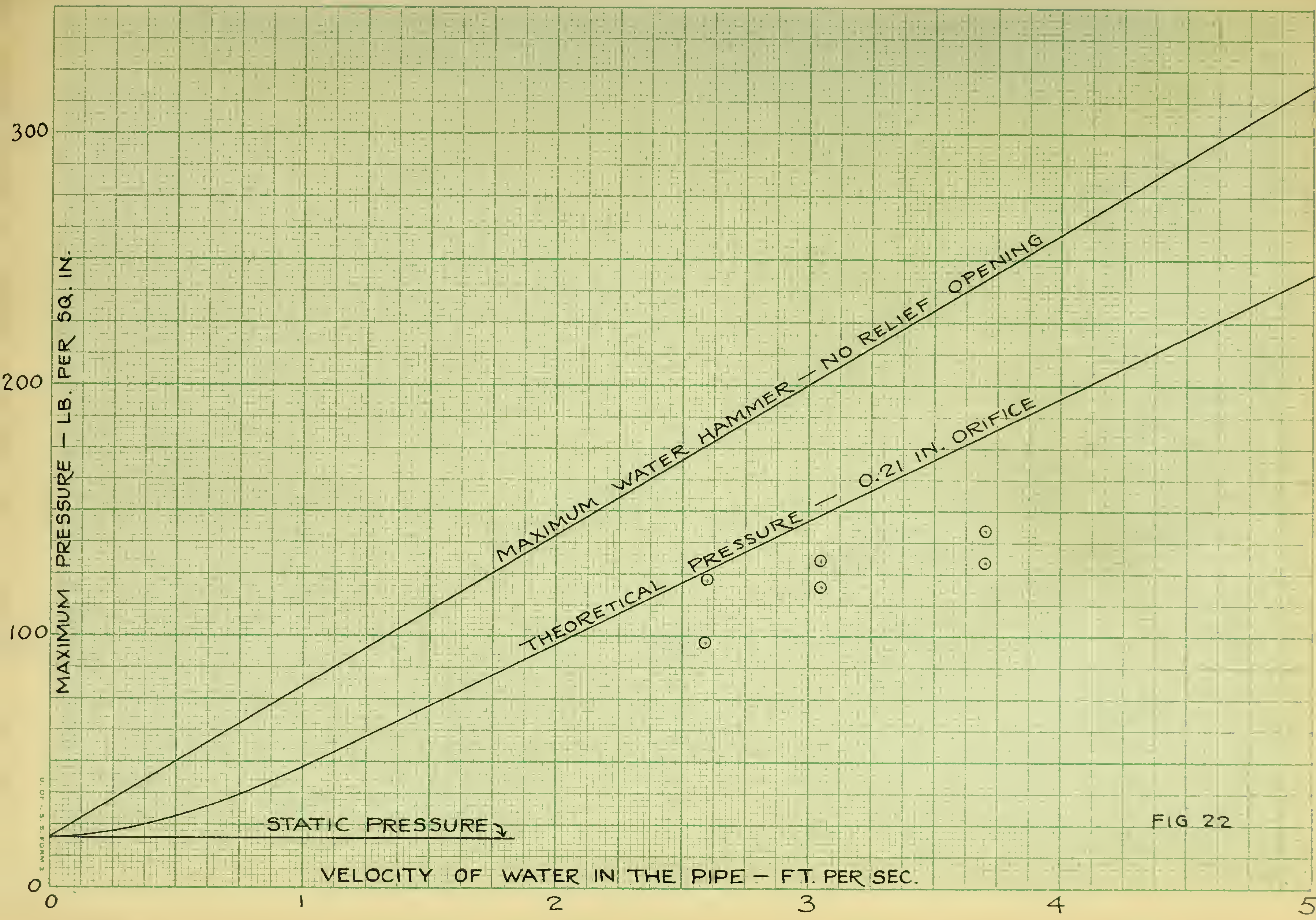

MAXIMUM PRESSURE — LB. PER SQ. IN.
VELOCITY OF WATER IN PIPE — FT. PER SEC.
MAXIMUM PRESSURE DUE TO WATER HAMMER — NO RELIEF OPENING.
THEORETICAL MAXIMUM PRESSURE — 0.161 IN. ORIFICE
STATIC PRESSURE
0
100
200
300
0
1
2
3
4
5

FIG 21



MAXIMUM PRESSURE – LB. PER SQ. IN.
VELOCITY OF WATER IN THE PIPE – FT. PER SEC.
MAXIMUM WATER HAMMER — NO RELIEF OPENING
THEORETICAL PRESSURE — 0.21 IN. ORIFICE
STATIC PRESSURE
0
100
200
300
0
1
2
3
4
5

FIG 22

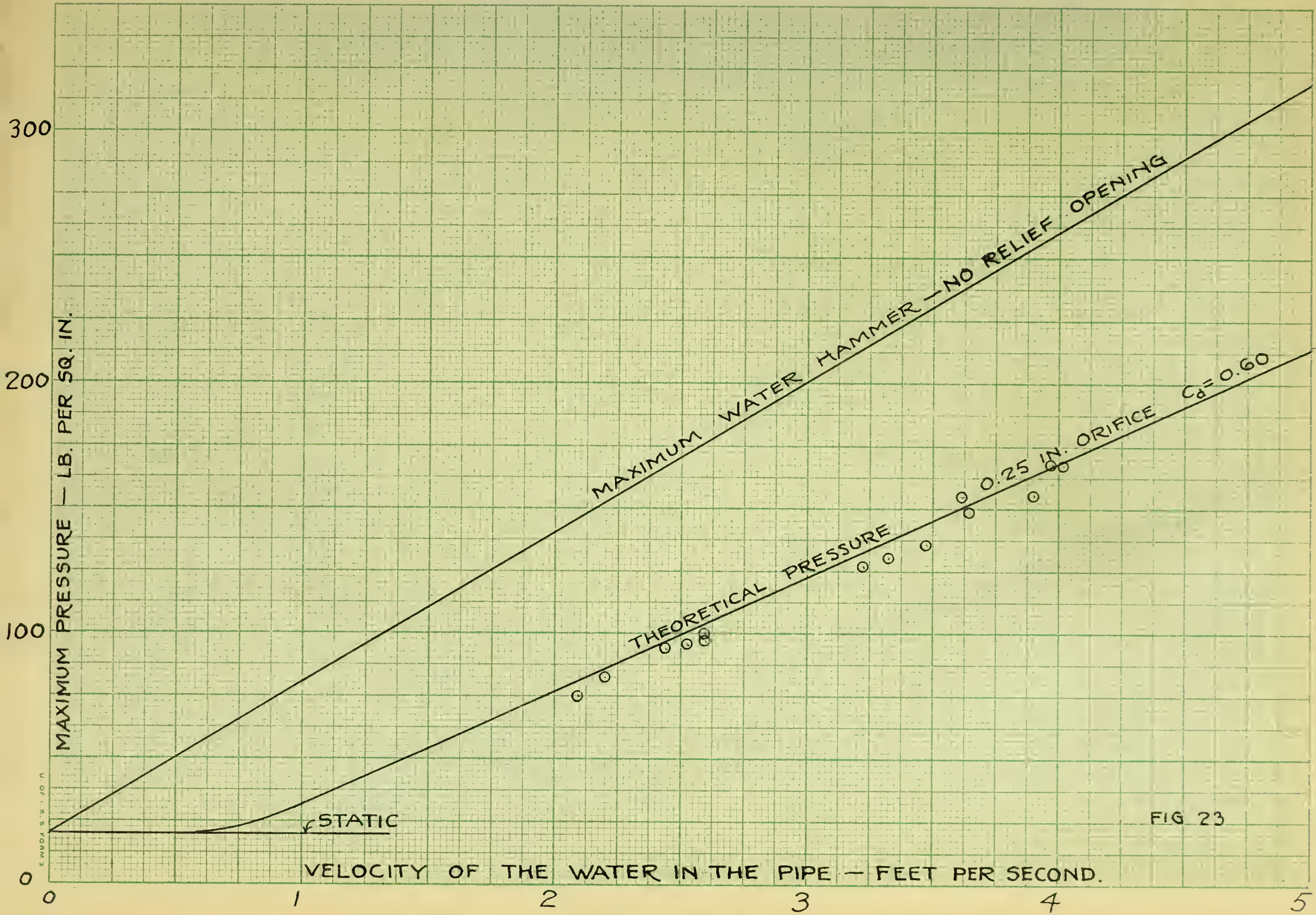

300
200
100
0
MAXIMUM PRESSURE — LB. PER SQ. IN.
MAXIMUM WATER HAMMER — NO RELIEF OPENING
THEORETICAL PRESSURE 0.25 IN. ORIFICE $C_d = 0.60$
STATIC
0
1
2
3
4
5
VELOCITY OF THE WATER IN THE PIPE — FEET PER SECOND.

FIG 23

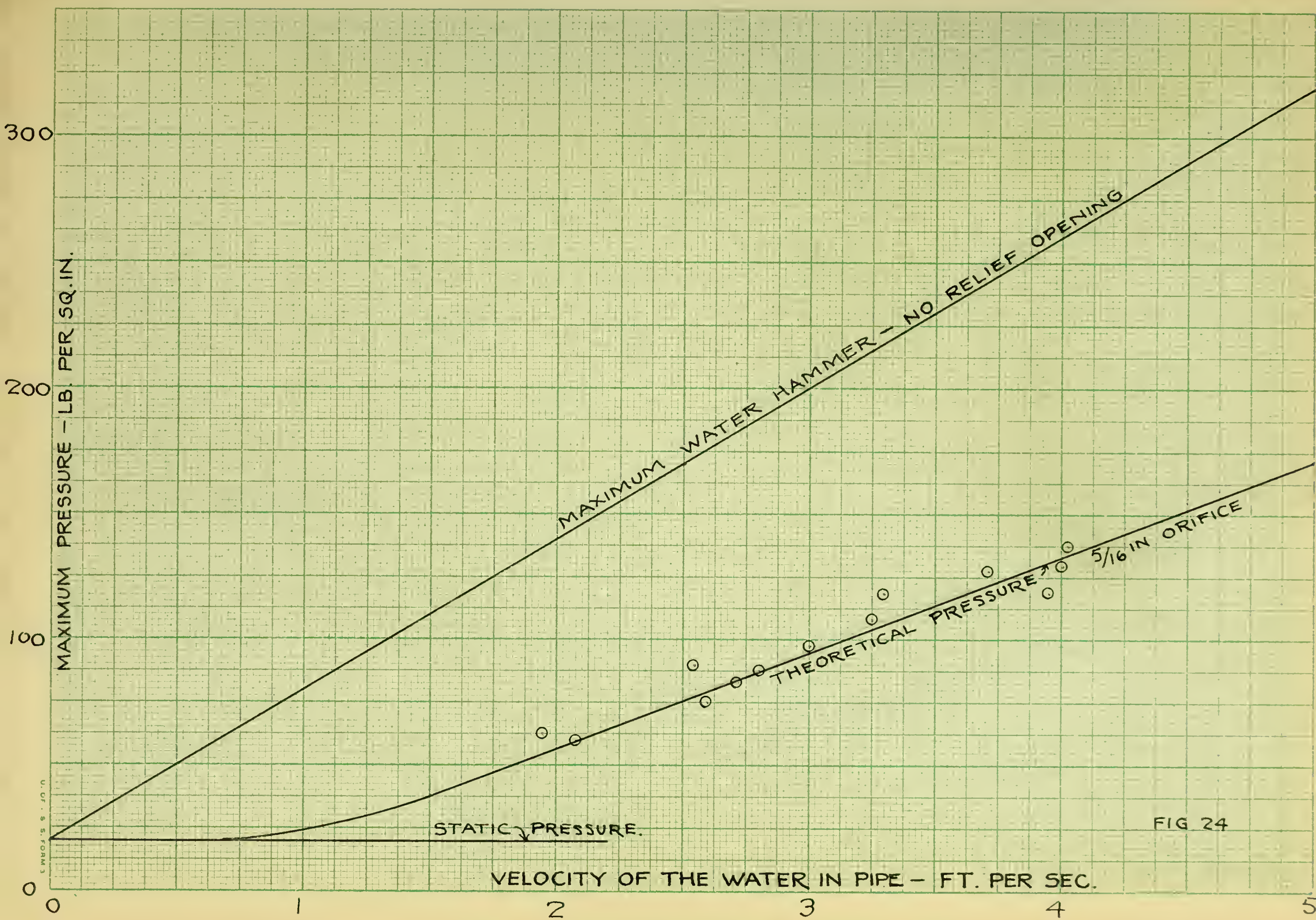
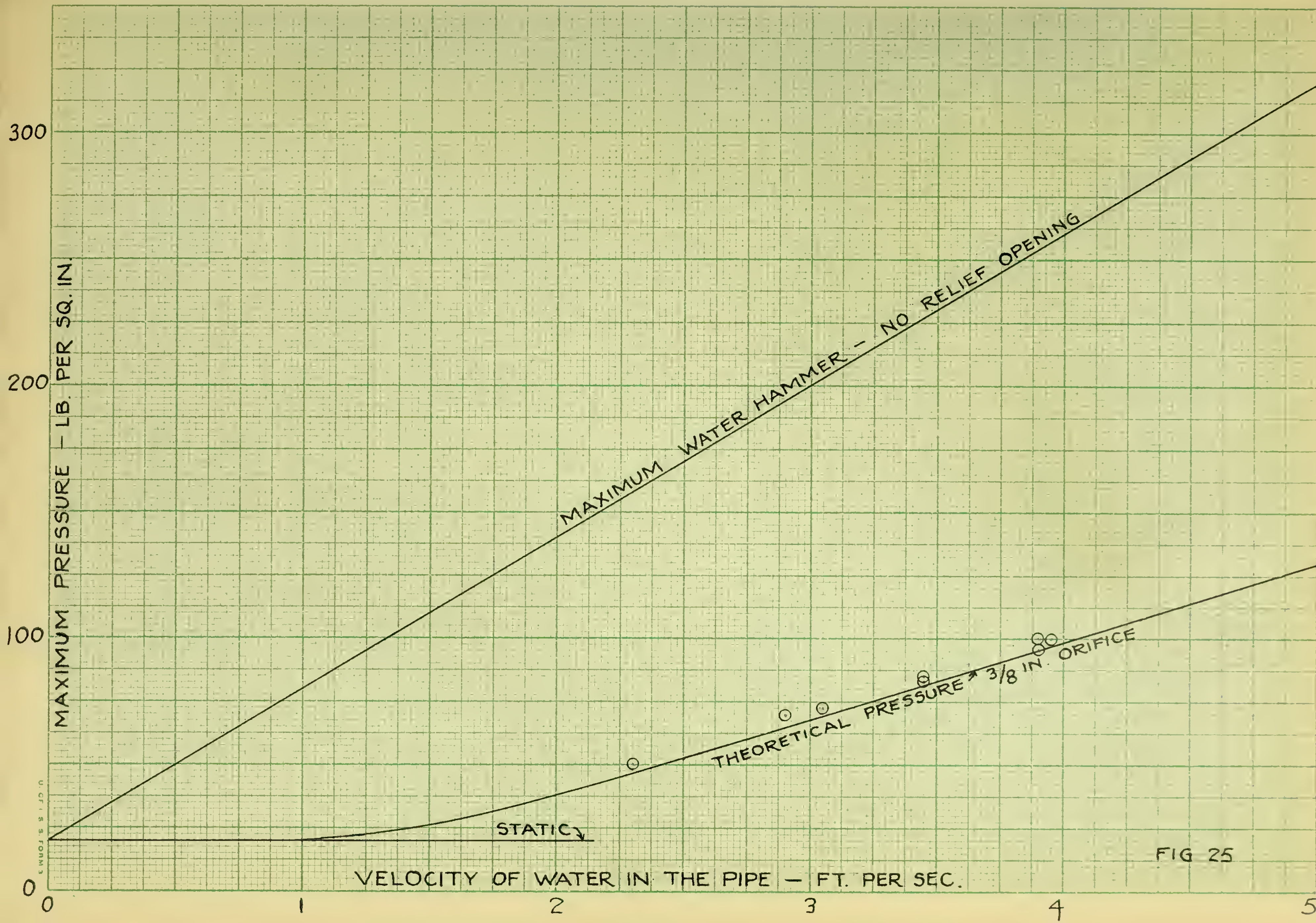

MAXIMUM PRESSURE – LB. PER SQ. IN.
300
200
100
0
MAXIMUM WATER HAMMER – NO RELIEF OPENING
THEORETICAL PRESSURE 5/16 IN ORIFICE
STATIC PRESSURE.
VELOCITY OF THE WATER IN PIPE – FT. PER SEC.
0
1
2
3
4
5

FIG 24



MAXIMUM PRESSURE — LB. PER SQ. IN.
VELOCITY OF WATER IN THE PIPE — FT. PER SEC.
MAXIMUM WATER HAMMER — NO RELIEF OPENING
THEORETICAL PRESSURE 3/8 IN. ORIFICE
STATIC
0
100
200
300
0
1
2
3
4
5

FIG 25

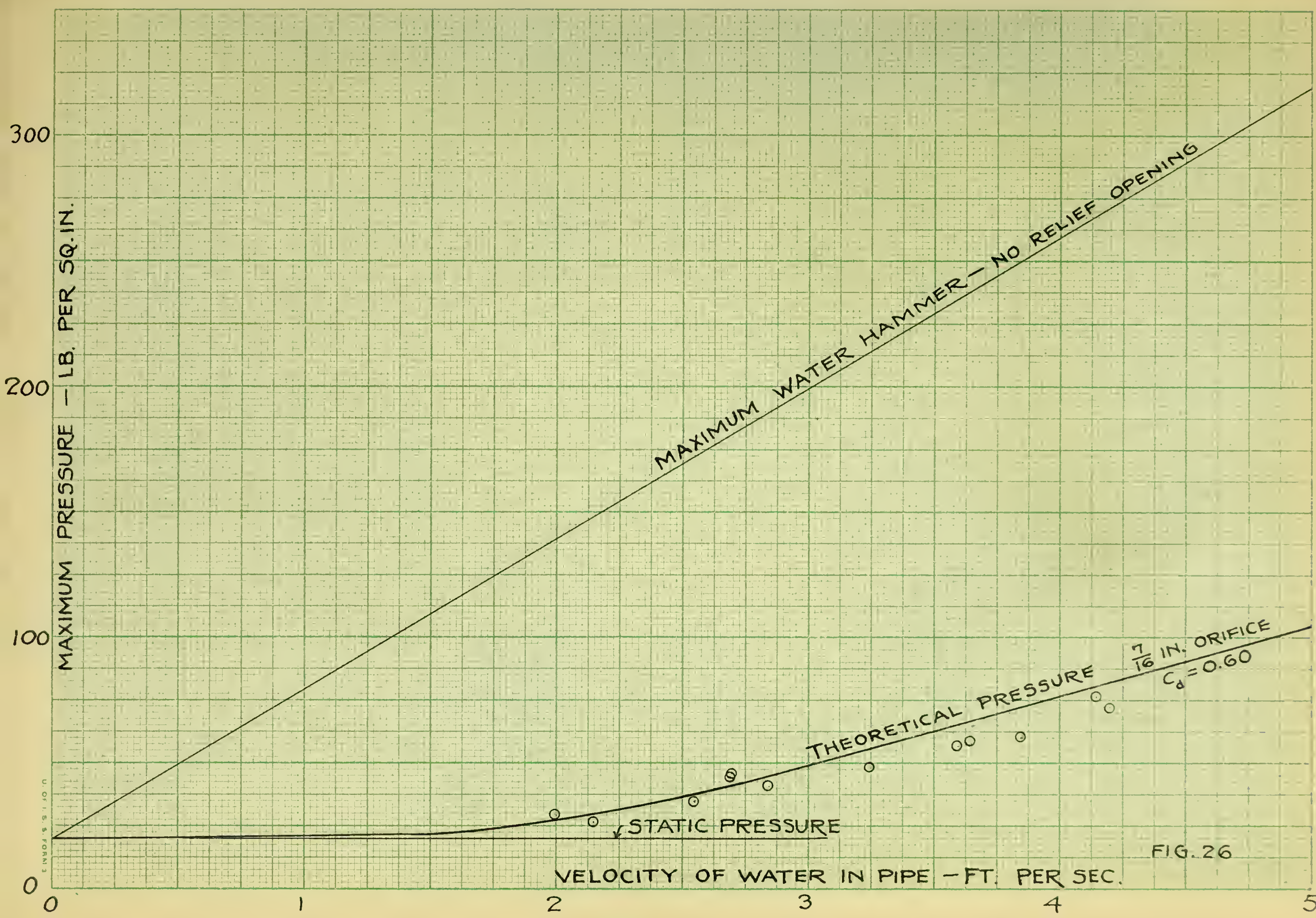

MAXIMUM PRESSURE – LB. PER SQ. IN.
300
200
100
0
0
1
2
3
4
5
VELOCITY OF WATER IN PIPE – FT. PER SEC.
MAXIMUM WATER HAMMER – NO RELIEF OPENING
THEORETICAL PRESSURE
7/16 IN. ORIFICE
Cd = 0.60
STATIC PRESSURE
U. OF I. S. S. FORM 3

FIG. 26

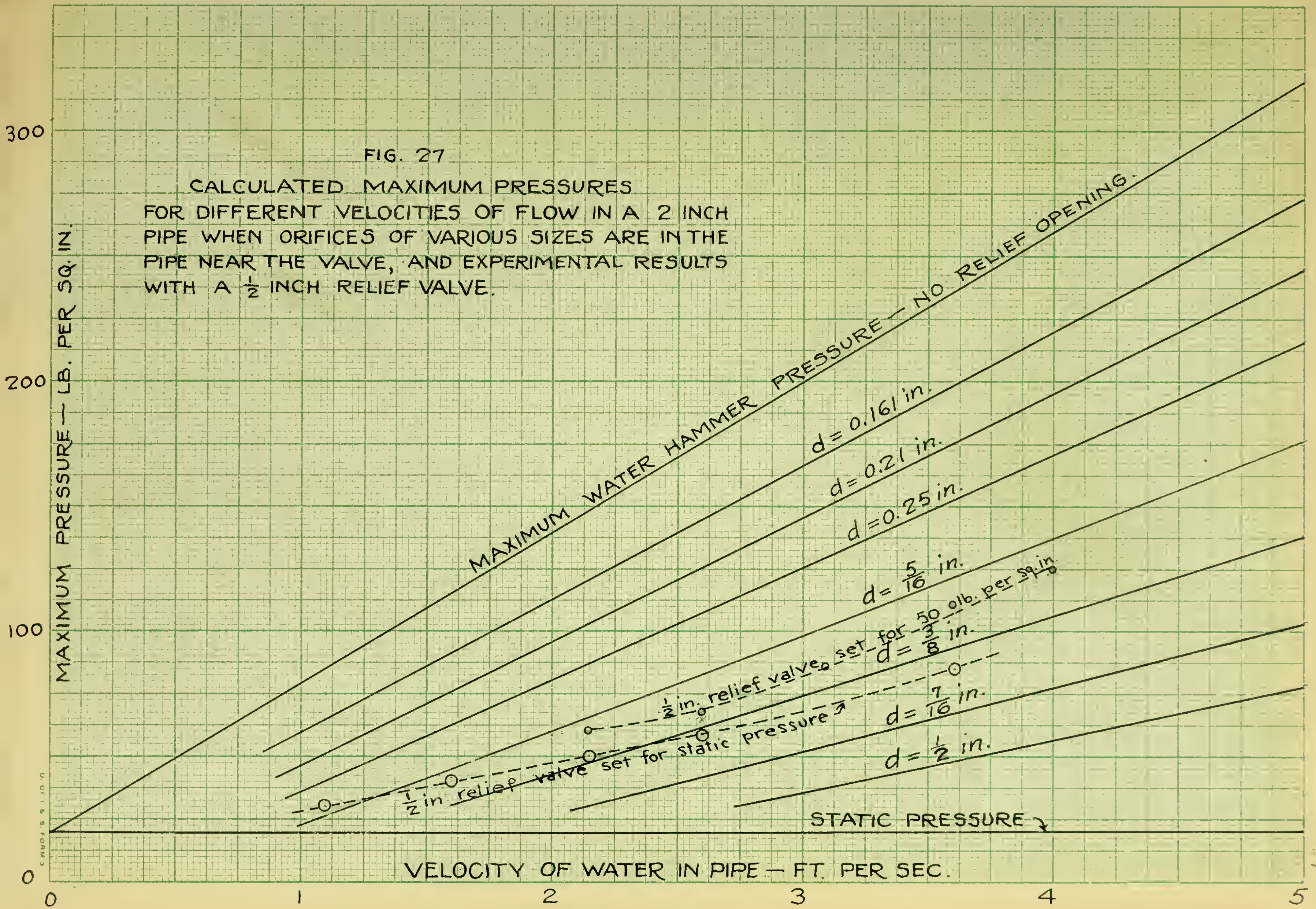


FIG. 27

CALCULATED MAXIMUM PRESSURES FOR DIFFERENT VELOCITIES OF FLOW IN A 2 INCH PIPE WHEN ORIFICES OF VARIOUS SIZES ARE IN THE PIPE NEAR THE VALVE, AND EXPERIMENTAL RESULTS WITH A $\frac{1}{2}$ INCH RELIEF VALVE.

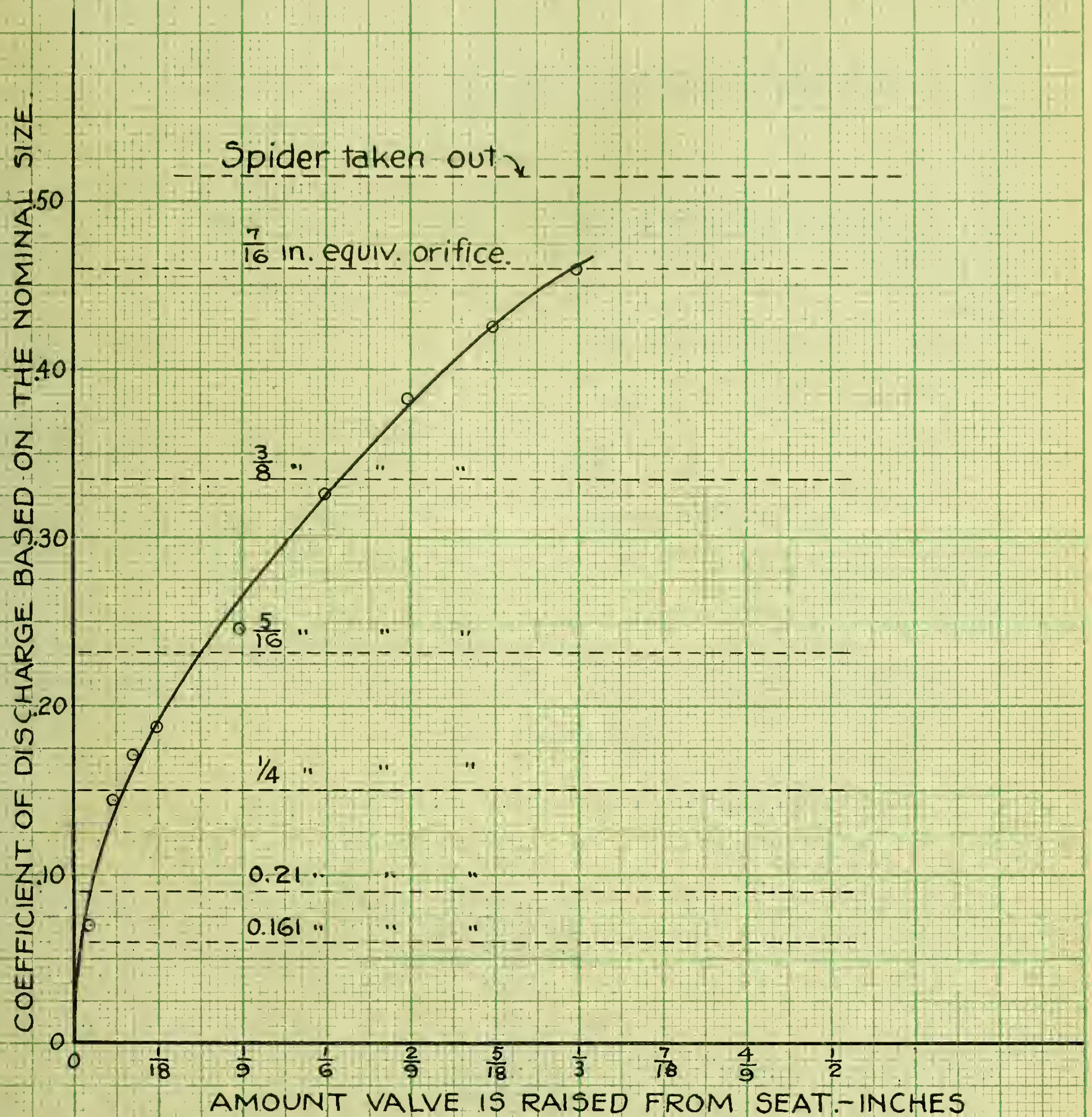


FIG. 28.

COEFFICIENTS OF DISCHARGE OF A 1/2 IN. LUNKENHEIMER RELIEF VALVE.

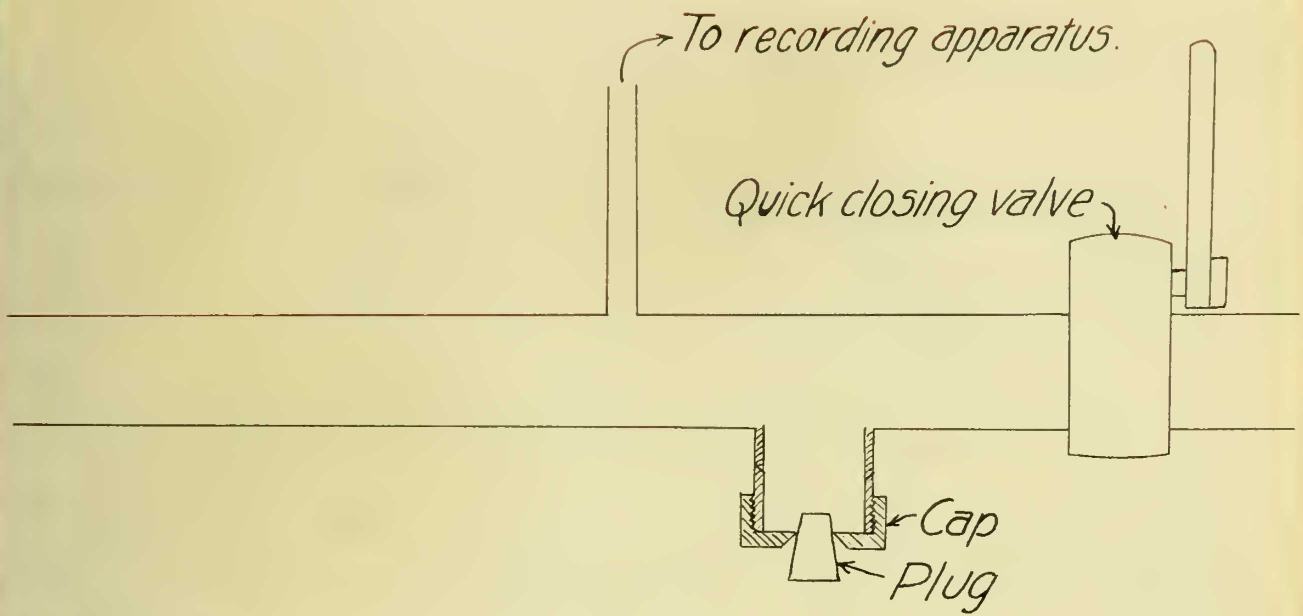


FIG. 29.

Arrangement of Apparatus for Relief Orifice Experiments

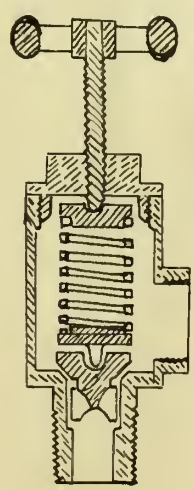


FIG. 30.

Lunkenheimer Relief Valve.

THE TIME OF THE EFFECTIVE CLOSURE

Under the heading "The Velocity of Flow and the Pressure at the Valve During the Time the Valve is Closing", it was shown theoretically that the time of the effective closure is only the time required to make the last 15 to 20 per cent of the closure. By effective closure is meant that portion of the closure during which the valve resistance increases rapidly. Fig. 13 shows the manner in which the pressure will theoretically rise during the time the valve is closing, providing the closure is made in less time than is required for the pressure wave to make the round trip to the open end.

In order to verify the conclusions given above, an attachment was made to the recording apparatus, by which a pencil connected to the valve, so as to move as the valve moved, recorded the valve position on the water hammer diagram. Fig. 31 is traced from one of the diagrams obtained in this manner. The very short time of the effective closure will be noted.

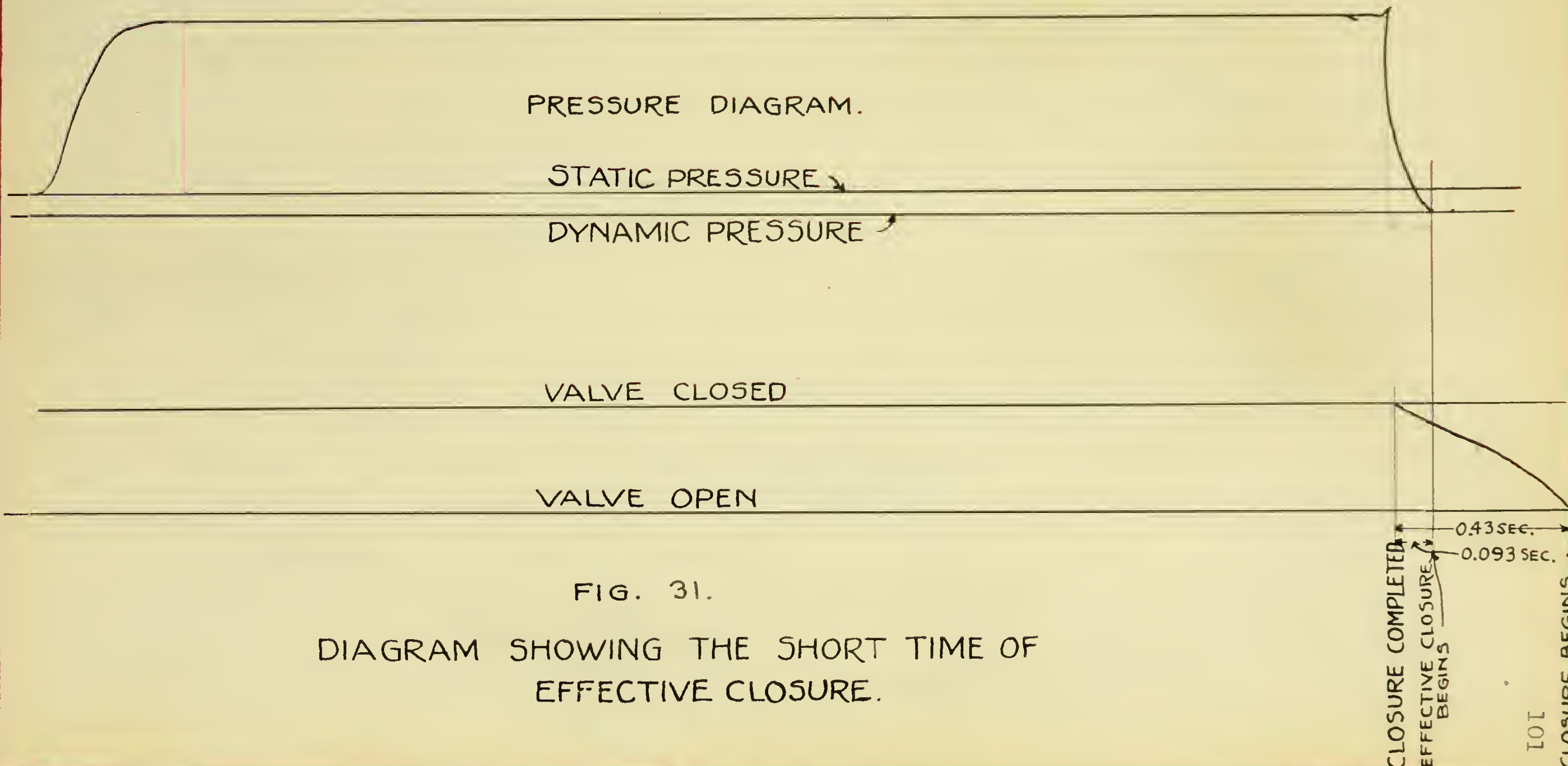


FIG. 31.

DIAGRAM SHOWING THE SHORT TIME OF EFFECTIVE CLOSURE.

THE ACCELERATION OF THE WATER IN THE PIPE

Experiments on the time required for the water in the pipe to gain velocity when the valve is opened, were made by opening the valve suddenly and closing at a later time. The water hammer diagram shows a drop in pressure when the valve was opened, and a sudden rise in pressure when the valve was closed. The distance on the diagram between the drop and the rise in pressure represents to some scale the time the valve was open. The drum rotated at a uniform rate, and a time pencil marked half seconds; it was therefore possible to determine the time accurately. The water hammer pressure recorded on the diagram, is a measure of the velocity of the water in the pipe at the instant the valve is closed. A number of such experiments were made, and the results are platted in Fig. 32. The line in the figure represents the theoretical values obtained from the equation derived on page 66.

In the derivation of this equation it was assumed that the water in the pipe is incompressible. The experimental results are seen to fit the theoretical equation in a general way, but some of the points lie a considerable distance from the curve. This is probably because of the elastic vibration of the column of water in the pipe. If the pressure in the pipe at the valve is P lb. per sq. in. above atmospheric pressure; when the valve is suddenly opened the water in the pipe near the valve immediately expands and takes atmospheric pressure. This causes a velocity in the pipe of $\frac{P}{h}$ ft. per sec., and this flow continues until a wave of low pressure has run to the source and a wave of increased pressure has run back to the valve. The velocity of flow through the valve is then suddenly

increased. This operation is repeated over and over again, until the flow has become steady.

This boosting of the velocity of the water at the valve can be clearly seen when the end of the pipe is turned up and an orifice put on. The height of the jet issuing through the orifice represents the velocity of the water at any instant. The jet rises by jumps when the valve is suddenly opened.

In Fig. 34 are shown some diagrams taken when the valve was suddenly opened and a little later suddenly closed.

WATER HAMMER DUE TO SUDDEN PARTIAL OPENING OF A VALVE. It seems paradoxical that water hammer may be caused by the opening of a valve at the end of a pipe, but in certain cases this may occur. When the valve is only partially opened, the pressure will be reduced to near atmospheric pressure, and a wave of this pressure will run from the valve to the open end. When this wave gets to the source a wave whose pressure is as much above the pressure of the source as the low pressure wave was below it, will run to the valve. If the valve is nearly closed, it will act as a reflecting surface, and double the pressure (diminished by the relief valve action of the partially closed valve) In Fig. 33 are tracings of diagrams taken in this manner. Notice how the pressure rises in steps.

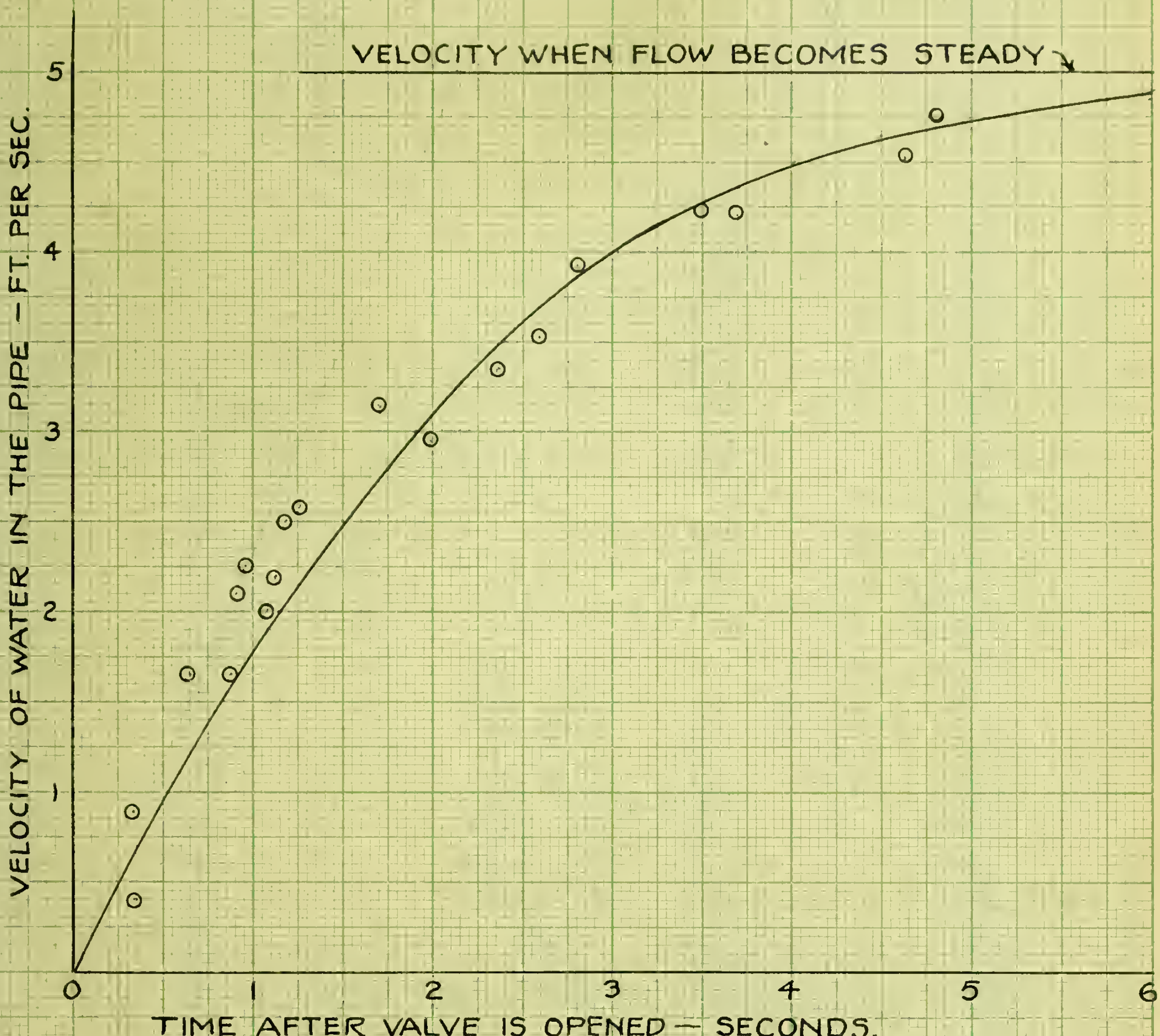


FIG 32
ACCELERATION CURVE
FOR 730 FEET OF 2 INCH
PIPE — 50 FOOT HEAD
COMPUTED FROM

$$T = 3.18 \log_{10} \frac{56.6 + 9.35V}{56.6 - 9.35V}$$

VALVE SUDDENLY OPENED.

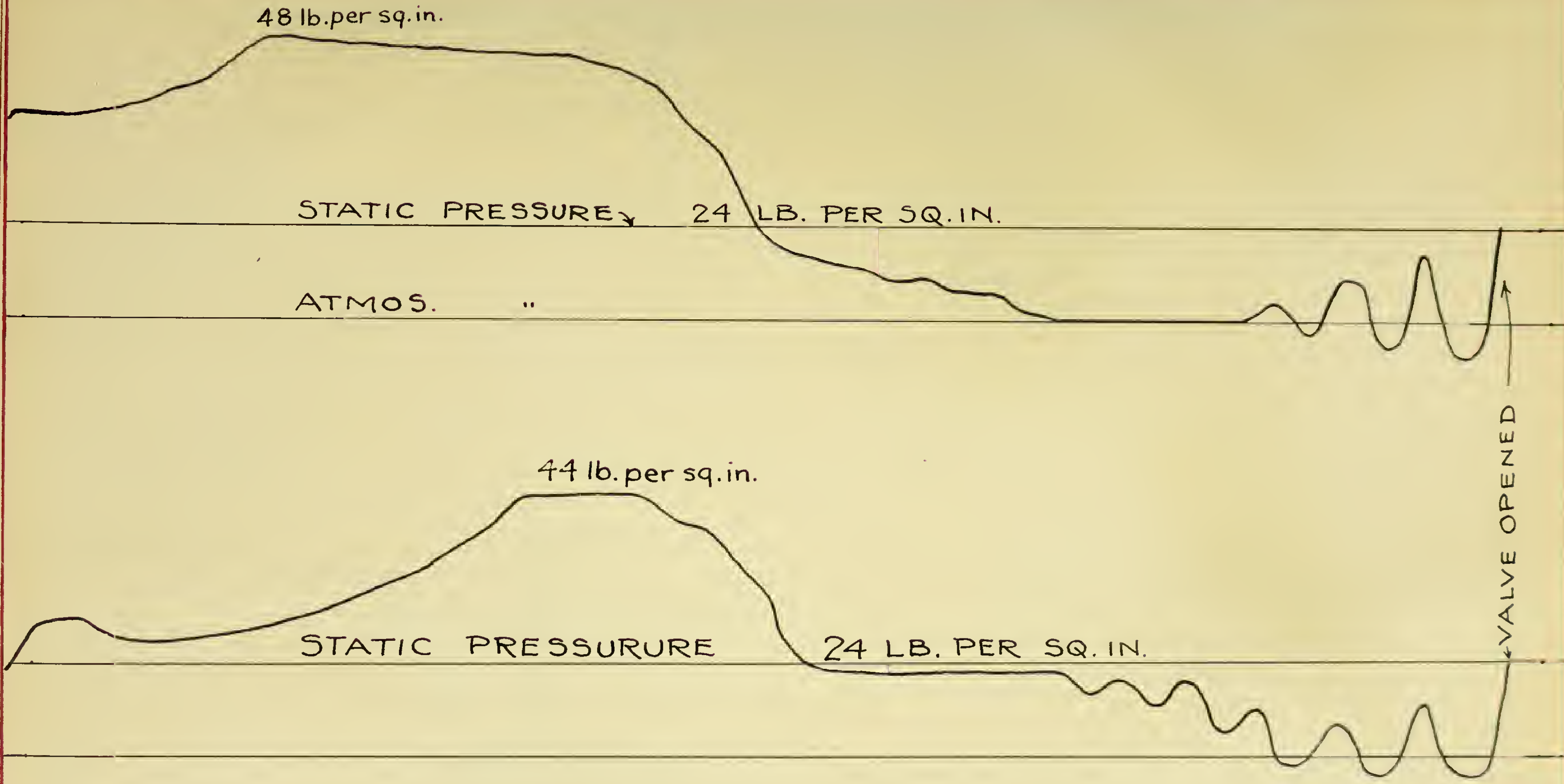


FIG. 33

WATER HAMMER CAUSED BY A SUDDEN PARTIAL OPENING OF A VALVE.

FIG. 34

DIAGRAMS FOR DETERMINING THE ACCELERATION OF WATER IN PIPE.