WATER HAMMER

WITH SPECIAL REFERENCE TO THE

RESEARCHES OF

PROFESSOR N. JOUKOVSKY

BY

O. SIMIN

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ERRATA AND REMARKS.

PAPER ON WATER HAMMER.

By O. Simin.

Pages 356 and 365. Figs. 6 and 8. The letter A should be near the "origin," or point where the small pipe leaves the large main.

Page 360. Fig. 7. The selection of a pipe of such length that the pressure wave occupies just four seconds in traversing it, and the selection of points 0, 1, 2, 3 and 4 for illustration, were matters of mere convenience.

Page 362. For = the weight of a unit of volume of water, read γ = the weight of a unit of volume of water.

Page 368. Fig. 9. The letters M and N should appear at the ends of the dotted line pertaining to the inner semi-cylinder.

Page 368. Fourth equation from foot, for R read R_o.

Page 369. Lines 8 and 9. For increase, P, of pressure, read increase, P, of unit pressure.

Page 369. Table II. Title. For Theoretical read Calculated.

Pages 375 and 376. Tables V. and VI. For Velocity, in Feet per Second, read Velocity, v, of water, in Feet per Second.

Page 391. Lines 10 and 11. For dead end of the small pipe, read dead end formed by the small pipe.

Page 393. At end of foot note, add, as in Figs. 20.

Page 374. Line 4 from foot. For the nearest to the beginning of the pipe, read the nearest to the gate.

Page 417. Fourth paragraph from foot. Read The distance on the diagram from the first increase of pressure to each of the three depressions, a, b and c, Fig. 26, expressed in time units, was found to be in all cases equal to the time required for a round trip of the pressure wave from the point where the diagram was taken to the air pocket and back. This enables us readily to locate an air pocket by means of the diagram.

Insert between pages 886 and 887.

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WATER HAMMER.

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INTRODUCTION.

The question of the so-called "water hammer" or "hydraulic shock," caused by stopping the flow in a water pipe, is of great practical importance, as the shock frequently bursts the pipe.†

Although the phenomenon has been long known, yet its nature has not been studied and explained until lately. Previous explanations were little better than guesses, and the effects upon the pipe could be neither foreseen nor avoided.

Efforts to understand the phenomenon were indeed

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* The writer here undertakes merely a compilation of existing information respecting this subject, including especially a digest of the notable work of Prof. N. Joukovsky, at Moscow, in 1897, which, in fact, embodies what is now known of the subject.

The present paper involves no original research. It was written at the suggestion of Mr. John C. Trautwine, Jr., while the writer was engaged in his office. The writer is indebted to Mr. Trautwine and to Mr. Boris Simin for suggestions and for corrections of proofs.

The writer has frequently quoted (in translation) the language used by Professor Joukovsky.

† Geo. M. Peck describes water hammer as follows:

"When a liquid is flowing through a pipe there is a certain amount of energy in the liquid, and if we stop the flow this energy must be used up in some way.

"If the liquid is incompressible, and we stop the flow suddenly, the energy of the liquid is used up in doing work on the pipe by stretching it or increasing its diameter.

"If the liquid is compressible, the energy of the liquid is used up in compressing the liquid and stretching the pipe. Nearly all bursting of pipes is due to a sudden checking of the velocity of the liquid or to the freezing of the liquid." (See *Trans. Am. Soc. of M. E.*, Vol. XXI, December, 1899.)

made, but experiments upon a sufficient scale were lacking, and even as late as in 1897 and 1898, when the subject had been taken up by the American Society of Civil Engineers,* the result of the discussions was that the existing data were insufficient and that more experiments were needed.†

Since then more satisfactory experiments have been made in connection with a general mathematical study of the problem, and formulæ governing the phenomenon have been established and experimentally proved to be correct.

Before describing these experiments and their results, let us glance at the history of the subject from the beginning. We shall thus see how the study gradually developed, the work of one investigator starting from that of another, the study thus gradually growing, from small beginnings, into a broad general mathematical theory, giving the phenomenon its proper place in the science of hydraulics, and furnishing a solid footing for practical men in their engineering work.

A. EARLIEST THEORETICAL STUDIES.

The earliest studies of the propagation of hydrodynamic pressure in pipes with elastic walls were purely scientific, and did not refer to water pipes. They related chiefly to acoustic and physiological phenomena (such as the pulse in animals), which, as we know now, are governed by the same laws as are hydraulic pressure waves‡ in water pipes. Such were the studies of Marey, of Resal (1876), of Gromeka (1883), of Korteveg (1878), and lately of Lamb (1898), who established formulæ for the pressure waves; but these formulæ were hardly thought of

* See *Trans. Am. Soc. of C. E.*, Vol. XXXIX, June, 1898.

† See quotations on page 346.

‡ German, "Stosswellen."

in connection with the engineering question of water hammer.

It will perhaps be surprising to many to learn that water hammer is merely a special case of the phenomena of wave action covered by these formulæ.

While the wave-like transmission of water hammer has indeed been recognized, notably by Frizell, the true nature of its relation to the subject has not hitherto been realized. This may be due in part to the fact that most of the earlier experiments were made with relatively short pipes, in which (the wave traveling with from 3 to 4 times the velocity of sound in air) its transmission could not well be observed and studied in detail, as it was in the experiments of Joukovsky.

B. AMERICAN STUDIES OF THE HYDRAULIC SHOCK IN ITS APPLICATION TO WATER PIPES.

A practical study of water hammer in pipes began in America in 1884. At that time appeared an article, by the late John W. Nystrom, in *Mechanics* (August, 1884),* and the experiments of Ed. B. Weston in Providence, R. I., were described by him in a paper presented to the American Society of Civil Engineers in 1885.†

Mr. Weston's experiments were made upon a rather small scale: the first series with about 111 feet of 6-inch pipe, 58 feet of 2-inch pipe, 99 feet of 1½-inch pipe and 4 feet of 1-inch pipe; the second series with 111 feet of 6-inch pipe, 3 feet of 3-inch pipe, 58 feet of 2-inch pipe,

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* See notice in the *Journal of the Franklin Institute,* April, 1890, page 328, and in *Engineering Record,* August 11, 1894, page 173.

† See *Trans. Am. Soc. of C. E.*, June, 1885, pages 238 and 246½ *Engineering Record,* August 11, 1894, page 173; *Trans. Am. Soc. of C. E.,* June, 1898, pages 1 and 5; *Journal of the Association of Engineering Societies,* February, 1889, page 103.

96 feet of 1½-inch pipe and 4 feet of 1-inch pipe; and the third series with 181 feet of 6-inch pipe, 65 feet of 4-inch pipe, 3 feet of 2½-inch pipe, 1 foot of 2-inch pipe, 6 feet of 1½-inch pipe and 6 feet of 1-inch pipe.

The aim of these experiments was to ascertain the maximum pressure caused by a sudden stoppage of flow. A rapidly closing gate and a steam indicator were used.

"The pressures were recorded by a style, which drew a straight mark upon a stationary paper, and the diagram took no account of the oscillations which succeeded the main shock incident to the stoppage of the current."*

The quick-closing gate was thought to occupy 0.15 second in closing, a time which, as we now know, was sufficient for the pressure wave to run to the end of the pipe and to return before the gate was completely closed. This affected the resultant pressure, and the full pressure, due to closure, could never be obtained. Mr. Weston made no attempt to deduce mathematical formulæ from his experiments.

In 1889, Mr. S. Bent Russell, of St. Louis, endeavored, by means of experiments, to frame an empirical formula for the intensity of water hammer.† He correctly established the fact that the intensity of the shock is proportional to the velocity of the arrested flow of water; but he assumed that it depended also upon the mass of the moving water and consequently upon the length of the pipe line. As seen from later studies, this assumption was incorrect, as was also Mr. Russell's empirical formula.

In 1890 Prof. Irving P. Church made a theoretical study of the subject.‡ Professor Church assumed that the

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* *Trans. Am. Soc. of C. E.*, June, 1898, page 1.

† See *Journal of the Association of Engineering Societies*, February, 1889, page 100.

‡ See *Journal of the Franklin Institute*, April, 1890, page 328, and May, 1890, page 374; *Engineering Record*, August 11, 1894, page 173; *Trans. Am. Soc. of C. E.*, June, 1898, page 2; Joukovsky, page 3.

maximum increase of pressure depends upon the manner of closing the gate valve, which, as has since been shown, is not the case.*

In 1892-93 the subject was studied by Messrs. S. H. Barraclough, B.E., and A.W. Wigglesworth, M.E., graduates of Sibley College, Cornell University, under the auspices of Prof. R. C. Carpenter.†

In the first series of these experiments there were used 375 feet of 6-inch pipe, 150 feet of 3-inch pipe, 33 feet of 2½-inch pipe and 30 feet of 2-inch pipe; in the second series, 53 feet of 1½-inch pipe.‡ The velocity of flow reached 8.6 feet per second.

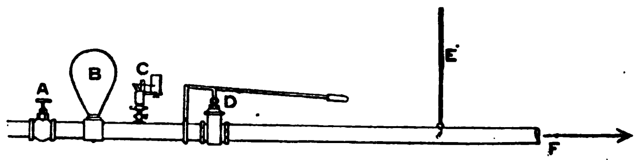

FIG. 1. CARPENTER'S EXPERIMENTAL APPARATUS.

"Water passed through the pipe in the direction from A to F (Fig. 1) under a pressure of 2 atmospheres. The velocity of flow was regulated by gate A and recorded by means of the Pitot tube E. The stoppage of the flow was effected by means of a throttle-gate, D, operated by hand. A pressure indicator, C, showed the variations of pressure, tracing, by means of a pencil, a diagram on a paper placed on a rotating cylinder. Between the gates A and D was placed an air chamber, B, which could also be used as a water chamber or could be removed altogether. Experiments were performed (a) with air chamber, (b) with water chamber and (c) without chamber."§

* Joukovsky, page 3.

† See *Transactions* of the New York meeting of the Am. Soc. of M. E.; *Engineering Record,* August 11, 1894, page 173.

‡ See *Trans. Am. Soc. of C. E.,* June, 1898, page 5.

§ Joukovsky, German edition, page 4.

These experiments were scientifically performed, and Professor Carpenter discusses the subject theoretically, confining himself, however, to the determination of the maximum increase of pressure.

On October 6, 1897, Mr. J. P. Frizell presented to the American Society of Civil Engineers a paper on water hammer, under the title, "Pressures Resulting from Changes of Velocity of Water in Pipes."*

"He was led to the study of this subject from its connection with the regulation of water wheels, drawing through long penstocks, and from the fact that requirements of electric transmission call for greatly increased exactness in the regulation of wheels."

Mr. Frizell made no experiments himself, but he studied the subject as it appeared from the experiments of Messrs. Weston, Carpenter and Church, and the aim of his paper was to give a general explanation of the phenomenon and mathematical formulæ respecting it. Professor Joukovsky's results, which satisfy the requirements of theory, have shown the formulæ derived by Mr. Frizell to be incorrect, but he gives a clear and correct view of the subject in general.

The paper of Mr. Frizell was followed by a discussion, in which many prominent engineers took part. Almost all who discussed the paper expressed the opinion that the subject is an important one, but that data in regard to it are scarce and that more experiments are wanted. Following are quotations from the discussion:

MR. RUDOLPH HERING.—"In view of the fact that opportunities for properly conducting such experiments are rare, the making of even imperfect sets of experiments should rather be encouraged."

MR. GARDNER S. WILLIAMS.—"The literature on the

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* See *Trans. Am. Soc. of C. E.*, June, 1898, page 1; see also short remark in the *Minutes of Proceedings of the Institution of Civil Engineers*, Vol. CXXX, 1897, page 351.

subject is extremely limited, and anything which may throw ever so little light upon it seems worthy of consideration."

MR. HENRY GOLDMARK.—"The theoretical values obtained by the author's formulæ are higher* than those which have generally been used in designing. For this reason it is all the more desirable that their correctness should be practically tested by an extended series of experiments on hydraulic plants in actual operation."

MR. THERON A. NOBLE.—"It is to be regretted that the author could not have supplemented his analysis with an account of some tests to show that the formulæ could be used with safety. Such an investigation, following his analysis, would be very valuable, and it is to be hoped that some engineer who has the opportunity will find out what actually occurs in a long pipe line of uniform diameter when the water is suddenly shut off."

MR. J. P. FRIZELL.—"The paper attempts to deal with an important engineering problem from a mathematical

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* They were much higher also than the actual values obtained in Joukovsky's experiments.

Example given by Frizell: A velocity, $v = 4$ feet per second, is checked in a steel pipe of 60 inches diameter and with wall ¼ inch thick. Modulus of elasticity of steel, $E = 30,000,000$ pounds per square inch.

The speed of pressure wave λ and the additional shock pressure P, as derived from Frizell's and Joukovsky's formulæ, are as follows:

(1) Frizell:

$\lambda = 4272$ feet per second. $P = 230$ pounds per square inch $= 15.65$ atmospheres.

$h = \frac{P}{v} = 3.9$ atmospheres per foot of extinguished velocity.

(2) Joukovsky:

$\lambda = 2556$ feet per second. $P = 138$ pounds per square inch $= 9.4$ atmospheres.

$h = \frac{P}{v} = 2.35$ atmospheres per foot of extinguished velocity.

point of view, and therein differs from the general character of the papers presented to the Society, which more commonly deal with experimental and practical results. By way of excuse for this departure, the author would observe that, while no formula which rests wholly upon theoretical considerations can be used with confidence, it is nevertheless true that a clear comprehension of the theory of a subject is a necessary and indispensable preliminary to any intelligent experiment."

While this discussion was in progress, important experiments upon an unprecedentedly large scale were being conducted in Moscow, Russia.

C. STUDY AND EXPERIMENTS OF N. JOUKOVSKY.*

I. Organization and Character of the Experiments.

When the new Moscow waterworks (now in use) were projected, the question came up as to the velocities which could be safely allowed in water pipes of small diameters.

Mr. Nicholas P. Simin, then chief engineer of the Moscow waterworks, advised following the recommendations of Mr. J. T. Fanning and other prominent engineers,† to allow, in small pipes, velocities not exceeding 1 meter per second, in order not to subject the water pipes to danger of water hammer. This recommendation was not adopted, the Imperial Supervising Commission of the Moscow Waterworks being in favor of higher velocities. Mr. Simin then proposed to organize a series of large-scale experiments upon water hammer in pipes.

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* See "Ueber den Hydraulischen Stoss in Wasserleitungröhren," von N. Joukovsky, 1898. *Mémoires de l'Académie Impériale des Sciences de St. Pétersbourg,* Classe physico-mathématique, Vol. IX, No. 5. (Edited in Russian and in German.)

† See, for instance, "A Practical Treatise on Hydraulic and Water-Supply Engineering," by J. T. Fanning," 1884, page 508, paragraph 495, table 105.

This proposition was adopted, and in the summer of 1897 experiments were started under the charge of N. Joukovsky, professor of the University of Moscow and of the Moscow Imperial Technical Institute.

These experiments were finished in the winter 1897-98, and resulted in the paper of Professor Joukovsky, which contains a thorough discussion of the subject.

Professor Joukovsky's report commences with a mathematical analysis of the phenomenon leading up to the derivation of a formula. He then gives a detailed description of the experiments by which the correctness of the formula was demonstrated, and sums up, in a number of conclusions, the principles of the phenomenon. In the spring of 1898 this paper was presented to the Polytechnic Society in Moscow and to the Imperial Academy of Science in St. Petersburg, and later to the fourth Russian Waterworks Convention, held in Odessa.

Professor Joukovsky gives a complete analytical explanation of the phenomenon in all its phases. His method of reasoning is purely mathematical and deductive. Starting from a general differential equation and using a mathematical method given by Riemann, Joukovsky explained all cases of water hammer by the transmission of a pressure wave in the water pipe and by its reflections from the ends of the pipe.

In the analytical determination of the velocity of transmission of this pressure wave, Joukovsky deduces a formula which fully agrees with the formula given by Korteveg* for the transmission of *sound* in an elastic pipe filled with a compressible liquid. Joukovsky showed by his analysis, and demonstrated by his experiments, that the formula of Korteveg applies not only to acoustic phenomena (for which it was established), but also to water hammer in pipes.

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* D. Korteweg, "Over Voortplating-snelheid van golven in elastische buizen," Leiden, 1878.

The full analytical solution of the problem by Joukovsky is too complicated for introduction here, but the principles involved and the leading formulæ will be explained.

As special cases of the phenomenon of water hammer, Joukovsky has investigated:

1. The influence of dead ends, which increase the shock.

2. The effect of the rapidity or slowness with which the gate is closed.

3. The effect of air chambers.

4. The effect of safety valves.

5. The influence of air pockets and of leaks from pipes upon the form of the pressure curve, enabling us to locate such difficulties by means of the curve.*

The experiments were made with 6-inch, 4-inch and 2-inch pipes.

The 6-inch line was 1066 feet long.

The 4-inch line was 1050 feet long.

The 2-inch line was 2494 feet long.

All these pipes were laid on the surface of the ground in the yard of the pumping plant (Fig. 2); each of the lines formed a loop, starting from the 24-inch waterworks main (also shown on the sketch) and ending at the well, g, where a quick-closing gate was placed.

The 24-inch main, 7007 feet long, was also subjected to experimental water hammer.

All the pipes, including the 2-inch, were of cast iron.

In all the experiments the closing was effected by means of a quick-closing gate, governed by a weight which hung from a pulley, and which was dropped from a certain constant height when the flow was to be stopped.

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* Professor Joukovsky has also theoretically investigated the case of water hammer in a pipe on which a standpipe is placed; but this subject is not included in his paper, here quoted, or in the present paper.

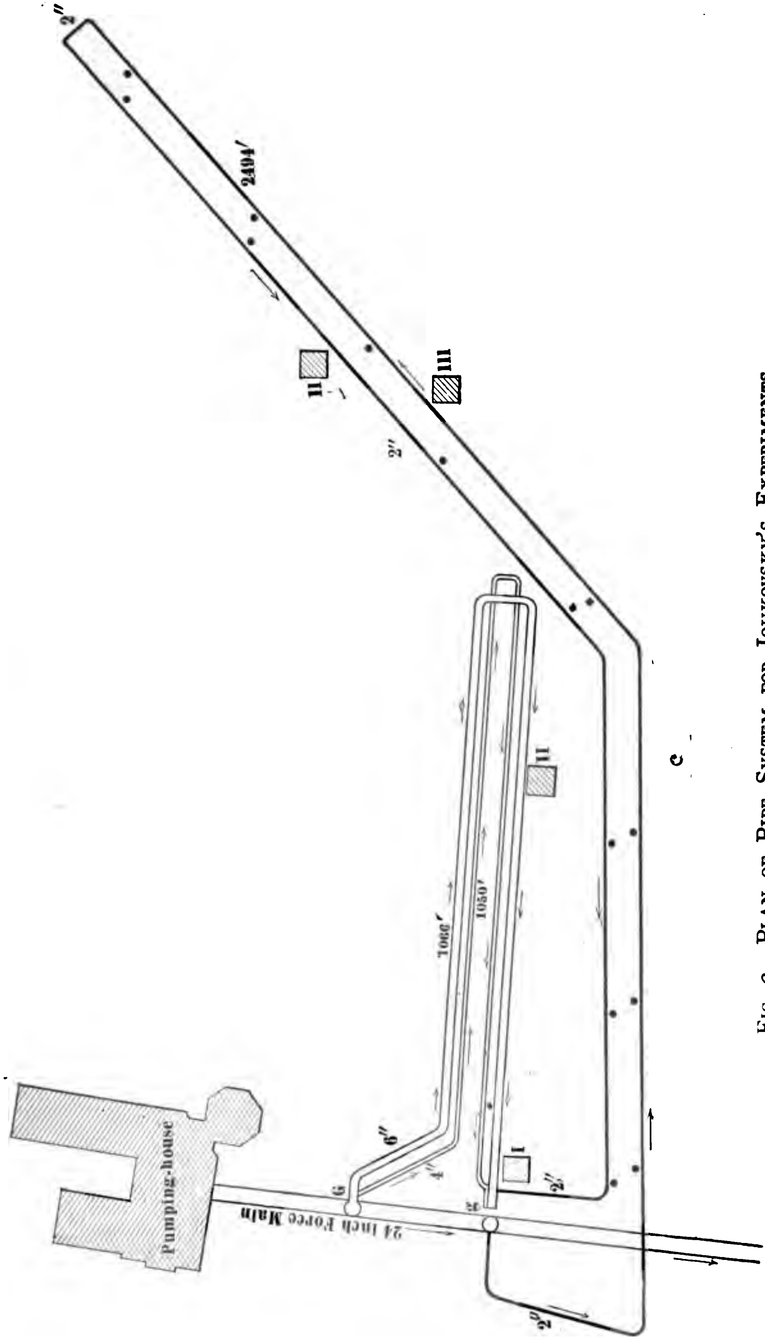

FIG. 2. PLAN OF PIPE SYSTEM FOR JOUKOVSKY'S EXPERIMENTS.

The resulting pressures were recorded by instruments of three kinds, viz: (1) 11 Bourdon manometers, which were placed along the pipe lines at the points shown in Fig. 2; (2) 3 specially constructed Crosby indicators, which were placed in small, portable frame houses, I, II and III, set in different points of the pipe lines; and (3) a Marey chronograph, which stood in the pump house, and which was connected, by means of electric currents, with 2 special manometers, shown by square dots.

All these devices were electrically connected, each half-second being recorded by a special pendulum, which stood in the pump house and which sent electric currents to the Crosby indicators on the pipe lines.

The Marey chronograph marked every 0.01 second by means of a tuning fork, which was connected by electric current with the chronograph.

The Bourdon manometers were used for determining the *intensity* of pressure obtained in water hammer. The Crosby indicators showed the variation of the *intensity* of the pressure and the *speed* of propulsion of the pressure waves. The Marey chronograph served only for the direct determination of the *speed* of the pressure waves.

A great number of pressure diagrams were obtained by means of the indicators and of the chronograph, both kinds of diagrams being drawn on paper placed upon a revolving cylinder.

The indicator curves were especially interesting, as they showed all the oscillations of pressure in different points of the pipe line, records being obtained at the same instant in different points. These curves formed autographic records of the pressure-wave movements.

Many tables were computed from these curves.

II. Explanation of the Phenomenon of Water Hammer.

In cases of water hammer, as already shown by Carpenter, a very rapid oscillation of pressure results, the pressure rising above the normal, then falling below the normal, then again rising, and so on, the amplitude of the vibrations gradually diminishing until the normal pressure is again established. But, owing to the shortness of the pipes used in Professor Carpenter's experiments and to the insufficient exactness of his measuring apparatus, the laws governing this variation of pressure could not then be established.

Joukovsky used Crosby indicators, each with a very rapidly moving ribbon of paper, upon which pressure dia-

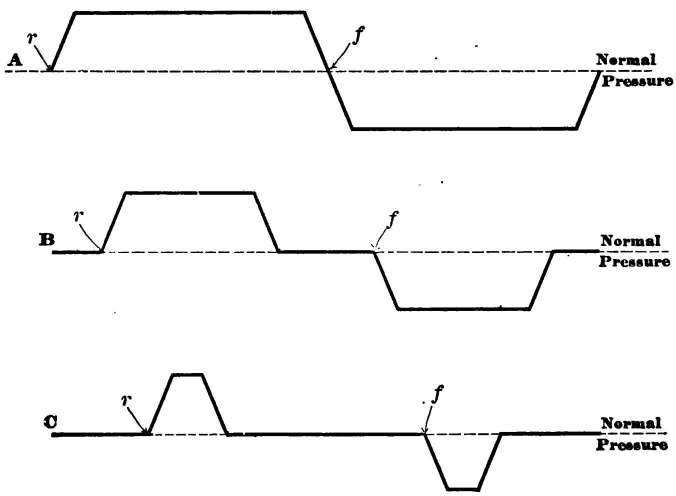

FIG. 3. THREE TYPES OF PRESSURE CURVES.

grams on a large scale were traced, giving a detailed record of the oscillations of pressure.

Curves of three types were obtained, and these types are schematically represented in Fig. 3, in which the horizontal

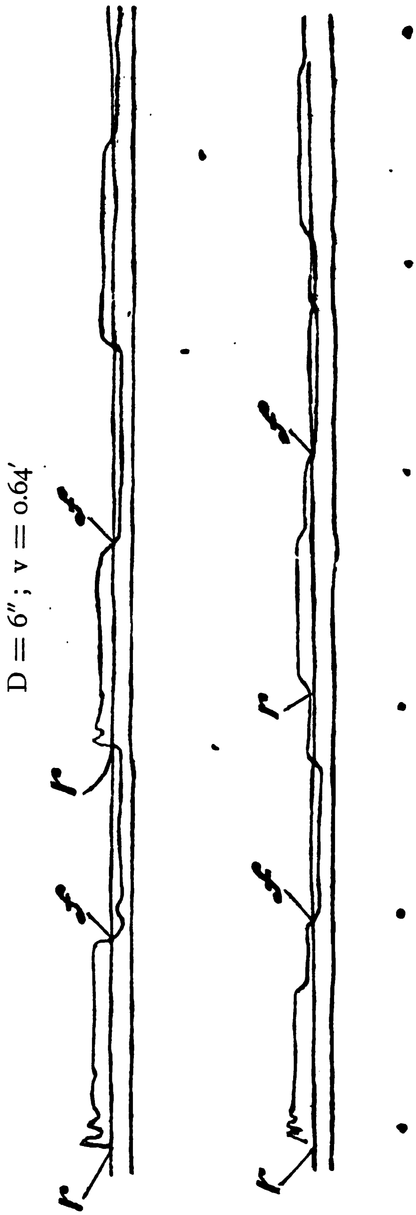

Fig. 4. Indicator Diagrams in Two Points of the 6-inch Pipe.

axes represent time. In Figs. 4 and 5 are reproduced, on a small scale, curves actually obtained by means of the apparatus.

D = 2″; v = 1.8′

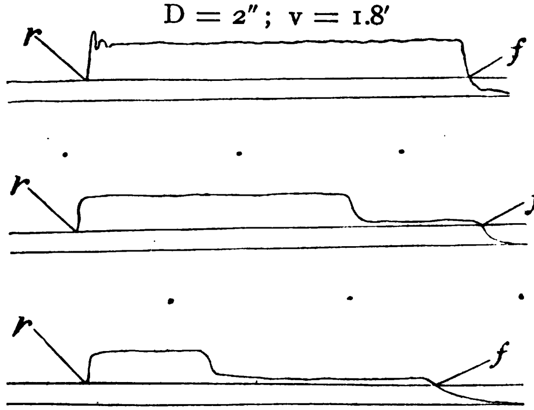

FIG. 5. INDICATOR DIAGRAMS IN THREE POINTS OF THE 2-INCH PIPE.

In each of these diagrams, the time, from the beginning, r, of rise of pressure above the normal, to the beginning, f, of fall below the normal, is the same for all points of the same pipe. Joukovsky found that this time is equal to that required for the wave, with velocity, λ (calculated theoretically and verified experimentally), to travel from the gate to the origin* and back.

At the gate (Fig. 3, A) the transition from supernormal to subnormal pressure, or *vice versa,* is not interrupted by any period of normal pressure; but at all other points in the pipe such periods of normal pressure intervene, giving the "stepped" diagrams of Figs. 3, B and C.

The diagrams obtained have shown also that for any

* By "origin" is meant the beginning of "the pipe," *i. e.,* of that portion of the system which is notably affected by the closing of the gate. Back of the origin the velocity is negligible and the pressure practically unaffected by the closing of the gate. Thus, the origin may be the point where the pipe under experiment leaves a reservoir or a main of large diameter.

given point the period of supernormal pressure is equal to the period of subnormal pressure, and is proportional to the distance between said point and the origin.

The explanation of these facts follows from the theoretical consideration of the phenomenon.

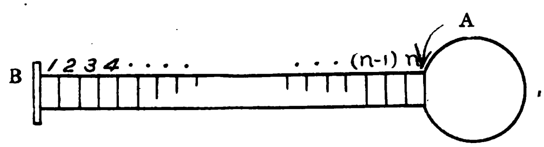

FIG. 6.

In Fig. 6 let A B be a pipe, in which water flows with velocity, v, from the origin, A, past the gate, B. If, now, the flow is suddenly stopped by a rapid shutting of the gate, B, the kinetic energy of the water column, A B, will cause an increase of pressure in the pipe.

Let us consider the column of water, A B, as divided into n very small equal sections, 1, 2, 3, ... (n — 1) and n.

The phenomena of water hammer take place in a series of cycles, each consisting of four processes, as follows:

(1) Section 1, meeting, in the gate, an obstacle to its movement, will be compressed and will stretch the pipe wall surrounding it. All the kinetic energy of this section of water will be used up (a) in its own compression, resulting in the increase of pressure by an increment, P, and (b) in the corresponding stretching of the walls in section 1 of the pipe. As a result of this action, section 1 of the water column has left vacant behind itself a small space, to be occupied by a part of the next arriving section 2. Consequently it is only after section 1 has been stopped and compressed, and after the small space thus left has been filled, that section 2 can be arrested and compressed.

Now the kinetic energy of section 2 must be expended in some way. Will it increase the pressure upon the gate, which has already been caused by the arrest of section 1? No, and for the following reason:

The pressure upon the gate depends entirely upon the pressure, P, sustained by section 1, which is now in static condition.

The pressure upon the gate could therefore be increased only if section 1 could be farther compressed, and this could take place only if the pressure upon the surface between it and section 2 (which we may imagine to be a thin piston) could be greater from the side of section 2 than it is from the side of section 1; and this is impossible, because section 2 has only the same kinetic energy as section 1, and this energy will (as in the case of section 1) be used up entirely in compressing the water of the section (section 2) only to the same additional pressure, P, and in stretching that part of the walls surrounding section 2.

The same is true of each following section, 3, 4, ... (n — 1) and n; each of these sections, as it is arrested, being compressed to the pressure, P.

During process (1) a small quantity of water flows from the reservoir into the pipe, to occupy the space formed by the compression of the water and the extension of the pipe walls.

Finally, when all the sections have been arrested, the entire column will be under the pressure, P. The entire energy of the water column is now stored (as potential energy) in elastic deformation, viz, in the compression of the water column and in the extension of the pipe walls.

But this condition cannot be maintained; for

(2) As soon as the additional pressure, P, has been produced in the last section, n, the water in that section will again expand, and the walls of that section of the pipe will again contract, restoring the original conditions in that section, and pushing the water of that section back into

the reservoir* from which the pipe issues, and restoring the original normal pressure in section n.

This operation will now be repeated by each section (n — 1), ... 4, 3, etc., in turn, until all the potential energy, stored in the water column when it was under the pressure, P (neglecting the portion lost in friction), has been reconverted into kinetic energy.

During process (2) the water which entered the pipe during process (1) is forced back into the reservoir.

The condition of the water column is now what it was just before the gate was closed, except that its velocity, v, has now the opposite direction, *i. e.*, toward the origin.

(3) The kinetic energy of the water column, moving toward the origin or away from the gate, is now reconverted into potential energy, which manifests itself in an extension of volume of the water to a subnormal pressure,† beginning with section 1, and concluding only when the entire water column has been reduced to the subnormal pressure.

During process (3) water continues flowing from the pipe into the reservoir.

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* We here use the word "reservoir" to denote the supply of water, back of the origin, whatever may be its character. See page 355, footnote.*

† As in other cases of vibration, the subnormal pressure, thus produced, should always fall as much below the normal as the supernormal pressure rose above the normal, except that the amplitude of the vibrations should be gradually diminished by friction. But the experiments have shown (see pressure diagrams, Fig. 13) that the fall of the subnormal below the normal pressure is much less than the rise of the preceding and following supernormal pressure above the normal; and this discrepancy is very considerable when the subnormal pressure approaches or even falls below the atmospheric pressure. In such cases water vapor and air are disengaged from the water, and possibly some air is inhaled through the pores in the walls of the pipe. These factors, of course, raise the subnormal pressure above what it should theoretically be.

(4) When the subnormal pressure has been established throughout the length of the pipe, and all the water has come to rest, the water from the reservoir will again direct itself into the pipe, restoring the normal pressure, first in section n, next to the reservoir, and then, in rapid succession, in the other sections (n — 1), . . . 4, 3, etc., until, when the normal pressure reaches the gate, we have once more the conditions which existed just before the gate was closed, viz, the normal pressure is restored and the water is moving toward the gate with the original velocity, v.

We have now followed these pulsations of pressure (with the accompanying transformations of energy and flow of water into and back from the pipe) through a complete cycle of four movements, each extending through the length of the pipe. For convenience, we may consider two successive movements of this kind as a "round trip" through the pipe.

The gate remaining closed, the whole process is now repeated in a second cycle, which, in turn, is followed by a third, and so on, the amplitude of the pressure vibrations gradually diminishing (because of friction) until the pipe and the water come to a state of rest.

But although the *intensity* of the pressure becomes gradually less, the *time* required for each cycle remains constant for all repetitions.

This propagation of pressure, consisting of its transmission through all points of the length of the pipe, each point successively repeating the same periodical movements, is, in its nature, simply a case of wave motion, like that of a sound wave.

The velocity of wave propagation is independent of the intensity of the pressure, and depends only upon the properties of the medium through which the propagation takes place—in the case of water hammer, upon the elasticity of the water and of the pipe.

We now find that the time required for each cycle of pressure transmission (including 2 round trips through the pipe) is equal to $\frac{4l}{\lambda}$, where l is the length of the pipe and λ is the velocity of wave propagation. This is the period of an entire pressure vibration in every point of the pipe.

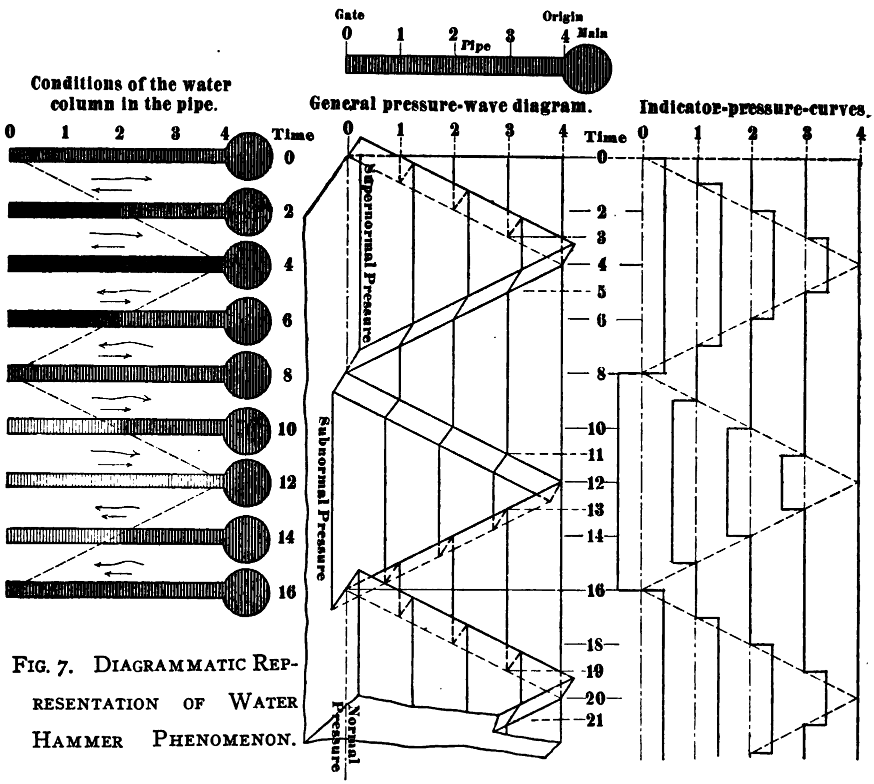

FIG. 7. DIAGRAMMATIC REPRESENTATION OF WATER HAMMER PHENOMENON.

A general diagrammatic representation of the principles of water hammer, as just described, is given in Fig. 7. Here we see, on the right, pressure curves of the same character as those in Fig. 3, corresponding to the different points of the pipe indicated at the top. On the left of

Fig. 7 the conditions of the water column in the pipe are shown at different instants during the propagation of the pressure waves back and forth along the pipe, sections under supernormal, normal and subnormal pressure being indicated by heavy, medium and light shading, respectively. To the right of the diagonal lines, crossing these sections, the pressure is always normal. To the left it is supernormal in the second, third and fourth sections from the top, and subnormal in the sixth, seventh and eighth.

Arrows drawn with straight lines show the direction of water flow from reservoir to pipe or *vice versa,* and arrows drawn with waving lines show the direction of propagation of the pressure wave.

Thus, in the uppermost space we have a *waving* arrow pointing to the *right,* and a *straight* arrow pointing to the *left,* indicating that, during the period between the two moments represented by the first and second figures, respectively, the *pressure wave* is traversing the pipe *from the gate toward the origin,* while *water* is flowing in the opposite direction, viz, *from the large main into the pipe.*

It will be noticed that we always find *normal* pressure between the head of the wave and the *origin,* and *abnormal* pressure between the head of the wave and the *gate;* in other words, that disturbances of pressure come always from the gate and the restoration of normal pressure comes always from the origin. Hence, the pressure at any point remains normal so long as the head of the pressure wave (moving in either direction) is between such point and the gate; and the nearer the point is to the origin, the longer will be its periods of normal and the shorter its periods of supernormal and subnormal pressure, and *vice versa.*

Formula for the Velocity, λ, of the Pressure Wave.

The formula for the velocity, λ, of propagation of the pressure wave was deduced by Joukovsky, by means of a purely deductive mathematical analysis, from the differential equation representing the phenomenon of water hammer.

This formula is:

$$\lambda = \frac{1}{\sqrt{\frac{\gamma}{Kg} + \frac{2R_0\gamma}{eEg}}} = \frac{1}{\sqrt{\frac{\rho_0}{K} + \frac{2R_0\rho_0}{eE}}},$$

where

$2R_0$ = the diameter of the pipe.

e = the thickness of the walls of the pipe.

E = Young's linear modulus of elasticity of the material of the pipe,

$= \frac{\text{unit stress}}{\text{unit stretch}}$,

= 1,000,000 kilograms per square centimeter for cast iron.

= 10^{10} kilograms per square meter for cast iron.

K = the volumnar modulus of elasticity of water.

= 21,000 atmospheres.

= 300,000 pounds per square inch.

γ = the weight of a unit of volume of water.

g = the acceleration of gravity.

$\rho_0 = \frac{\gamma}{g}$ = the density (mass per unit of volume) of water.

As already mentioned, this formula for the velocity of the pressure wave fully coincides with Korteveg's formula for the velocity of sound waves in an elastic pipe filled with a compressible liquid.

Korteveg's formula is:

$$\lambda = \frac{\lambda_1 \lambda_2}{\sqrt{\lambda_1^2 + \lambda_2^2}},$$

where

λ_1 = the velocity of the sound wave in an inelastic pipe filled with a compressible liquid.

This velocity is

$$\lambda_1 = \sqrt{\frac{Kg}{\gamma}} = \sqrt{\frac{K}{\rho_0}} .$$

For water, λ_1 = 1435 meters per second = 4711 feet per second.

λ_2 is the velocity of the pressure wave in an elastic pipe filled with an incompressible liquid. This velocity, as given by Resal in discussing the pulsation of the blood in the arteries of animals, is

$$\lambda_2 = \sqrt{\frac{eE}{2R_0\rho_0}} = \sqrt{\frac{eEg}{2R_0\gamma}} .$$

Substituting these values of λ_1 and λ_2 in the formula of Korteveg, we obtain exactly the formula deduced by Joukovsky for the velocity of the pressure wave in an elastic pipe filled with an elastic liquid.

Applying this formula, Joukovsky calculated the theoretical value of λ for the pipes used in his experiments,* obtaining the values given in the following table, in which all the steps of the calculation are shown.

The formulæ of Joukovsky being entirely rational, and not empirical, any system of units may be used at pleasure, care being taken, of course, to adhere to one and the same system of units throughout any one equation.

* In all the experiments of Joukovsky these were cast-iron waterworks pipes, of the type standardized by the Russian Waterworks Conventions. For pipes where E or $\frac{e}{R_0}$ differs from the corresponding value for these pipes, other results will be obtained.

TABLE I.

TABLE OF THEORETICAL CALCULATION OF λ FOR THE PIPES USED IN THE EXPERIMENT OF JOUKOVSKY.

$2R_0$, Diameter of the Pipe, Inches.	e, Thickness of Walls of Pipe, Inch.	$\sqrt{\frac{e}{2R_0}}$	For Cast Iron, $\sqrt{\frac{Eg}{\gamma}}$ Feet per Second.	$\lambda_1 = \sqrt{\frac{Kg}{\gamma}}$ Feet per Second.	$\lambda_2 = \sqrt{\frac{e}{2R_0}} \times \sqrt{\frac{Eg}{\gamma}}$ Feet per Second.	λ, feet per Second. $\lambda = \frac{\lambda_1 \lambda_2}{\sqrt{\lambda_1^2 + \lambda_2^2}} = \frac{1}{\sqrt{\frac{\gamma}{Kg} + \frac{2R_0\gamma}{eEg}}}$	λ Obtained from the Experiments. (See pp. 383, 384, 387.)
1	2	3	4	5	6	7	8
2	$\frac{10}{32}$	$\frac{1}{8}\sqrt{10}$	$\sqrt{\frac{10^{10} \times 9.8}{1000 \times 0.3048^2}} = 32479$	$\sqrt{\frac{21012 \times 10000 \times 9.8}{1000 \times 0.3048^2}} = 4710$	12838	4424	4375
4	$\frac{11}{32}$	$\frac{1}{8}\sqrt{\frac{11}{2}}$			9520	4228	4200
6	$\frac{13}{32}$	$\frac{1}{8}\sqrt{\frac{13}{3}}$			8449	4116	4100
24	$\frac{22}{32}$	$\frac{1}{8}\sqrt{\frac{11}{12}}$			3885	2996	3313

Formula for the Maximum Additional Pressure, P, Produced by the Closure of the Gate.

Joukovsky found that

$$P = \frac{v\lambda\gamma}{g} = v\lambda\rho_0 ,$$

where

v = the extinguished velocity of water in the pipe.
λ = the speed of propagation of the pressure wave in the pipe.
γ = the weight of a unit volume of water.
g = the acceleration of gravity.
$\rho_0 = \frac{\gamma}{g}$ = mass of unit of volume of water = its density.

Joukovsky gives two different methods of deduction of this formula. One of these methods is based upon the equation of the conservation of energy; the other upon a mathematical expression of the fact that the space, vacated by the compression of the water and the stretching of the pipe, always equals the volume of water which arrives during the same time with the given velocity, v, to fill said space.

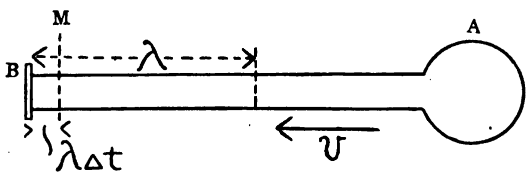

FIG. 8.

We now give this second method of deducing the formula:

Let us suppose that water is flowing, with velocity v, through the pipe, A B, Fig. 8. The gate B is now suddenly closed and the flow is stopped, the stoppage proceeding from the gate through the whole length of the pipe with velocity λ per second. In other words, during the first second, following the moment of closure of the gate, a column of water, of length λ, is arrested. Those parts of the water column which are farther from the gate than the length λ will not be stopped during this first second.

During an interval of $\triangle$ t seconds, the stoppage of flow proceeds through a length, B M $= \lambda \triangle t$, to the point M; compressing the water, stretching the pipe walls, and thus bringing an increase, P, of pressure throughout the length, M B, to the left of M.

In A M, to the right of M, the water is still flowing with the original velocity, v; consequently, during the interval of $\triangle$ t seconds, the cross-section, M, whose area is πR_0^2, has passed a volume of water

$$Q = \pi R_o^2 . v . \Delta t \qquad (1)$$

By virtue of the stretching of the pipe walls, the volume of the pipe, B M, is increased by the quantity

$$q = (\pi R^2 - \pi R_o^2) . \lambda . \Delta t \qquad (2)$$

where

$R_0 =$ the normal inside radius of the pipe.

$R =$ the inside radius of the pipe after it has been stretched by the pressure, which has been increased by P.

But the compression of the water represents a further

volume $$q' = \pi R_o^2 (\lambda \Delta t) \left(\frac{\rho - \rho_o}{\rho_o} \right) \qquad (3)$$

where

$\rho_0 =$ the density of the water before compression.

$\rho =$ the density of the water under the pressure increased by P.

$\frac{\rho - \rho_0}{\rho_0} =$ the relative increase of density of the water.

Now Q and q + q′ are necessarily equal. Hence,

$$\pi R_0^2 v \Delta t = (\pi R^2 - \pi R_0^2)\lambda \Delta t + \pi R_0^2 \left(\frac{\rho - \rho_0}{\rho_0}\right)\lambda \Delta t$$

$$\left.\begin{array}{c}\text{Volume}\\ \text{of water}\\ \text{added}\end{array}\right\} = \left\{\begin{array}{c}\text{Increase of}\\ \text{volume of pipe}\end{array}\right\} + \left\{\begin{array}{c}\text{Space vacated}\\ \text{by compression}\\ \text{of water.}\end{array}\right.$$

$$Q \quad = \quad q \quad + \quad q'$$

Consequently, dividing by $\pi R_0^2 \triangle t$, we find

$$v = \left(\frac{R^2 - R_0^2}{R_0^2} + \frac{\rho - \rho_0}{\rho_0}\right)\lambda = \left(\frac{R - R_0}{R_0} \cdot \frac{R + R_0}{R_0} + \frac{\rho - \rho_0}{\rho_0}\right)\lambda$$

But, R being very nearly $= R_0$, we may substitute 2 for $\frac{R + R_0}{R_0}$;

Hence $$v = \left(2\,\frac{R - R_0}{R_0} + \frac{\rho - \rho_0}{\rho_0}\right)\lambda \qquad (4)$$

Here $\frac{R - R_0}{R_0}$ is the stretch of the pipe, $R - R_0$, per unit of the original radius, R_0, due to the stress in its walls produced by the additional pressure, P, of the water per unit of surface.

On a semicylinder of unit length, this additional pressure gives a resultant force, $P.\,\overline{KLMN} = P.\,2\,R_0$ (see Fig. 9), which gives, in the walls, $\overline{KL}$ and $\overline{MN}$, of thickness, e, an additional stress, per unit of cross-section,

$$\frac{2\,R_0\,P}{\text{Sum of Areas } \overline{KL},\, \overline{MN}} = \frac{2\,R_0\,P}{2\,e} = \frac{R_0\,P}{e}$$

But $\frac{R_0\,P}{e} : \frac{R - R_0}{R_0}$ = modulus of elasticity, E, of the pipe walls;

Hence $$\frac{R-R_0}{R_0} = \frac{R_0 P}{Ee}.$$

Now $\frac{\rho - \rho_0}{\rho_0}$, being the increase of density per unit of normal density, is numerically equal to the reduction of

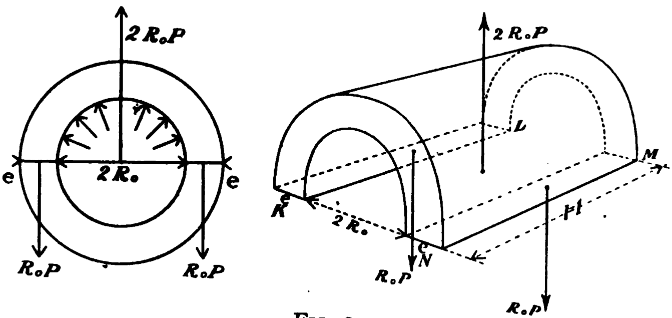

FIG. 9.

volume in unit mass caused by the additional pressure, P; and K is the volumnar modulus of elasticity of water.

Hence $$K = \frac{P}{\frac{\rho-\rho_0}{\rho_0}}; \quad \text{and} \quad \frac{\rho-\rho_0}{\rho_0} = \frac{P}{K}.$$

Hence, substituting, in equation (4), the values just found for $\frac{R - R_0}{R_0}$ and $\frac{\rho - \rho_0}{\rho_0}$,

$$v = \left(\frac{2 R_0 P}{Ee} + \frac{P}{K}\right)\lambda = \frac{P\lambda}{\rho_0}\left(\frac{2 R \rho_0}{Ee} + \frac{\rho_0}{K}\right).$$

But (see p. 362) $$\lambda = \frac{1}{\sqrt{\frac{\rho_0}{K} + \frac{2 R_0 \rho_0}{Ee}}}.$$

Consequently $$v = \frac{P\lambda}{\rho_0 \lambda^2} = \frac{P}{\rho_0 \lambda};$$

and $$P = v\lambda\rho_0.$$

Now ρ_0, the original density of the water, is equal to the weight of a unit of volume of water divided by the acceleration of gravity; or

$$\rho_0 = \frac{\gamma}{g}$$

Therefore, the maximum additional pressure, caused by water hammer, is

$$P = \frac{v \lambda \gamma}{g} \qquad (5)$$

From this formula (5) we see that *the increase, P, of pressure, stands in direct proportion to the extinguished velocity, v, of flow; that it does not depend on the length of the pipe, and that it is directly proportional to the velocity, λ, of propagation of the pressure wave;* the last-named quantity, being expressed by the formula

$$\lambda = \frac{1}{\sqrt{\frac{\rho_0}{K} + \frac{2 R_0 \rho_0}{Ee}}},$$

depends on the size, thickness and kind of pipe and on the density and modulus of elasticity of the liquid.

Substituting numerical values in the formula

$$P = hv = \frac{v \lambda \gamma}{g}$$

Joukovsky finds that, for the pipes with which he experimented, each foot per second of extinguished velocity of flow causes an increase of pressure as given in atmospheres in the following table:

TABLE II.*

THEORETICAL VALUES OF ADDITIONAL SHOCK PRESSURE.

$2 R_0$ Diameter of Pipes, in Inches.	$h = \frac{P}{v}$ Additional Pressure, per Foot per Second of Extinguished Velocity of Flow, in Atmospheres.
2	4.066
4	3.886
6	3.783
24	2.754

* Here the calculated values of λ (Table I, page 364, column 7) are taken.

From this table we see that, *for pipes of small diameters (2 inches to 6 inches), the increase, h, of pressure, for each foot of extinguished velocity of flow v, equals about 4 atmospheres.*

$$P = 4\ v \text{ atmospheres.}$$

For pipes of large diameters h is somewhat less.

These results, obtained from the formula, correspond well with those obtained during the experiments on the diagrams of the indicators. (See, for instance, the tables on pages 383, 384, 387.)

III. Experimental Proof of the Correctness of Joukovsky's Theory.

1. Maximum additional shock pressure, P, in different points of the pipes, as measured by means of Bourdon manometers.

Eleven Bourdon manometers, with frictional pointers for the marking of the maximum readings, were placed on the 4-inch pipe at different points.

Before the gate was opened, all the frictional pointers were set near the pointers of the manometers, which showed the static pressure in the main, which was equal to 4.5 atmospheres, effective. Then the gate was opened to the desired extent, and water allowed to flow from the pipe, the rate being determined by measurement of the discharge in the tank. Next the weight on the gallows frame was dropped, quickly closing the gate, and the water hammer thus effected. Finally the readings of all the frictional pointers were noted. The results of such experiments, performed upon the 4-inch pipe on June 23 and 24, 1897, are given in Table III, page 371.

TABLE III.

PRESSURE EXPERIMENTS WITH MANOMETERS ON 4-INCH PIPES, JUNE 23, 24, 1897.

No. of Experiment.	Velocity of Water in Feet per Second. v.	Readings of Manometers in Atmospheres. Manometer No. 1	2	3	4	5	6	7	8	9	10	11	Average of Readings of Manometers Nos. 1-10.	Average P.	P According to the Approximate Formula $P = 4v$.
1	2	3											4	5	6
1	7.0	40	37	48	37	36	48	38	38	45	38	8	40.5	36.0	28.0
2	7.0	40	40	53	38	42	48	38	38	47	38	8	42.2	37.7	28.0
3	4.7	28	28	29	26	26	38	27	30	27	27	7	28.6	24.1	18.8
4	6.4	28	26	25	25	25	34	27	27	28	26	7	27.1	22.6	25.6
5	2.8	18	15	15	15	15	18	16	17	17	16	5.5	16.2	11.7	11.2
6	2.6	18	14	14	12	13	18	15	15	15	14	5.5	14.8	10.3	10.4
7	9.9	50	50	68	50	52	50	37	44	53	34	7	48.8	44.3	39.6
8	3.5	29	27	29	25	25	37	25	27	27	26	7	27.7	23.2	14
9	4.0	22	23	23	20	22	29	21	24	23	22	6	22.9	18.4	16
10	4.0	25	23	22	20	23	27	21	23	23	22	6	22.9	18.4	16

Looking over the pressures observed at different points of the pipe, we see that *the shock was propagated along the pipe without being weakened.*

Manometer No. 11, standing at the end of the pipe nearest to the main, shows, as might be expected, pressures approaching that of the main; therefore its readings were not included in calculating the averages. The fluctuations, observed in the readings of the manometers, are explained by the inertia of the bent tube of the Bourdon manometer and by the fact that, with high velocities, v, the maximum pointers, arrested by friction, sometimes marked, not the first but the second pressure wave.* The same reasons must be assigned for the fact that the pressure, P, determined from the average of the readings of the manometers, is higher than when calculated according to the formula $P = 4 v$.

During the experiments with the 2-inch pipe, 11 manometers were used, No. 11 being placed near the main. The results obtained from these experiments on September 1 and 23, 1897, are given in Table IV, page 373.

This table also shows that *the pressure is transmitted undiminished throughout the length of the pipe* (the readings of the tenth manometer, which stood near the end of the pipe, were sometimes even higher than those of No. 1).

For the reasons already given, variations, due to imperfection of the measuring apparatus, are here found, as well as in Table III, page 371. Better results were obtained by means of indicators (see pp. 383, 384, 387.)

2. *Speed of Pressure Wave Propagation, λ, Measured by Means of a Marey Chronograph.*

In order to measure the speed of propulsion of the pressure waves, use was made of a Marey chronograph, which gave readings corresponding to 0.01 second.

* The second pressure wave may be greater than the first when two water columns are directed against one another. See pp. 381, 382.

TABLE IV.

PRESSURE EXPERIMENTS WITH MANOMETERS ON 2-INCH PIPES, SEPTEMBER 1 AND 23, 1897.

No. of Experiment.	Velocity of Water in Feet per Second.	Readings of Manometers in Atmospheres. Manometer No.											Average of Readings of Manometers Nos. 1-10.	Average P.	P According to the Approximate Formula $P = 4v$.
	v.	1	2	3	4	5	6	7	8	9	10	11			
1	2	3											4	5	6
1	4.4	27	23	25	24	30	30	33	32	30	28	5	28.2	23.7	17.6
2	4.4	30	24	25	22	34	30	32	30	32	30	5	28.9	24.4	17.6
3	3.3	20	18	18	20	25	23	28	30	22	24	5	22.8	18.3	13.2
4	3.2	20	18	18	20	20	26	30	32	23	24	5	23.1	18.6	12.8
5	4.5	30	30	20	23	23	20	27	25	35	29	5	26.2	21.7	18.0
6	4.4	25	30	20	25	35	25	27	26	27	30	5	27.0	22.5	17.6
7	4.4	29	30	20	25	35	20	27	26	27	30	5	26.9	22.4	17.6

The observations were made thus: In two points of the 4-inch pipe, at a distance of 700 feet apart along the pipe, two U-shaped manometric tubes were placed, which, at the moment of increase of pressure, unbent and pushed forward metallic rods, controlling electrical contacts. Each rod, after being once pushed out, was held in the new

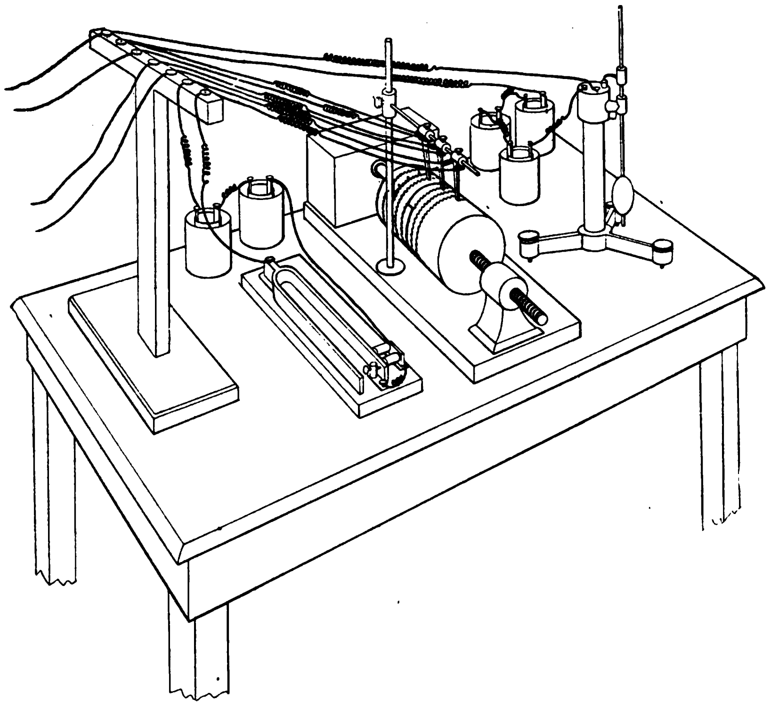

FIG. 10. THE MAREY CHRONOGRAPH.

position by means of a spring. The contact made by the first rod (the nearest to the beginning of the pipe) sent an electric current to the Alexeievskaia pump house, where the Marey chronograph was placed. (See Fig. 10.) This current lifted the armature of the chronograph, which carried a pen, tracing a diagram on a revolving cylinder covered with blackened paper. At the moment when the pressure wave arrived at the second manometer, similar to the first, it cut off the current (by pushing the rod), and the armature, with the pen, returned to its original position. In this way the pen of the chronograph drew a hump, the length of which, expressed in time units, showed the duration of the 700 feet run of the pressure wave. The time was obtained in hundredths of a second, which were marked on the same cylinder by a wave-like line made by a special pen. This pen was moved by an electric current, interrupted by a tuning fork making 100 vibrations per second.

Fig. 10 shows the chronograph and the tuning fork. The pendulum (also seen in this figure) made an electric contact every half-second, sending a current to the Marey chronograph and to the observation houses containing the Crosby indicators, marking, upon the diagrams, the dots which served to co-ordinate them in time.

Data obtained from these experiments on the 4-inch pipe with the chronograph on June 22 and 24, 1897, are given in the table below.

TABLE V.

OBSERVATIONS FOR VELOCITY, λ, OF PROPAGATION OF PRESSURE WAVE IN 4-INCH PIPE, BY MEANS OF THE CHRONOGRAPH, JUNE 22 AND 24, 1897.

No. of Experiment.	Velocity, in Feet per Second.	Duration of a 700 Feet Run, in Seconds.
1	10.8	0.170
2	4.6	0.160
3	3.1	0.140
4	3.5	0.180
5	4.0	0.140
6	3.9	0.160
7	4.1	0.165
8	7.1	0.190
9	9.1	0.180

According to this table, the average 700 feet run of the wave lasted 0.165 second; this corresponds to the velocity $\lambda = \frac{700}{0.165} = 4242$ feet per second, which is quite near to the theoretical velocity calculated by Joukovsky, equal to 4228. (See Table I, page 364.)

Similar observations were performed also with the 2-inch pipe, the manometric apparatus being placed at a distance of 1246 feet apart. In Fig. 2 the locations of the manometers are indicated by square dots. The data obtained from these experiments on September 23, 1897, are given in the following table:

TABLE VI.

OBSERVATIONS FOR VELOCITY, λ, OF PROPAGATION OF PRESSURE WAVE IN 2-INCH PIPE, BY MEANS OF THE CHRONOGRAPH, SEPTEMBER 23, 1897.

No. of Experiment.	Velocity, in Feet per Second.	Duration of a 1246 Feet Run, in Seconds.
1	3.07	0.306
2	1.80	0.302
3	1.80	0.297
4	0.80	0.297
5	1.54	0.300

The average duration of the run of the wave, from one point to the other, 1246 feet, is here = 0.300 second, which gives a velocity $\lambda = \frac{1246}{0.300} = 4153$ feet per second for the 2-inch pipe. This velocity is less than the theoretical 4424 feet per second (see Table I, page 364), and even less than that in the 4-inch pipe, whereas, according to the calculations, the velocity in the 2-inch pipe should be the greater.

Joukovsky suggests that this discrepancy may be due to residual magnetism in the armature of the Marey chronograph.

Other observations (pages 383, 384), performed by means of a more accurate method (p. 382), have shown that, in the 2-inch pipe, λ is a little greater than in the 4-inch pipe, as it should be also according to the calculation.

3. Determination of λ and P by Means of Crosby Indicator Diagrams.

(a) The diagrams and the method of obtaining them.

The Crosby indicators were placed in portable houses I, II, III. House I stood always near the gate, g (Fig. 2), while II and III were placed on the pipe line to be tested, usually at one-third and two-thirds of its length.

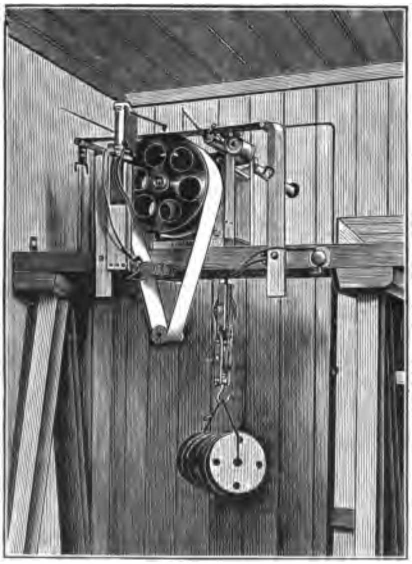

FIG. 11. THE CROSBY INDICATOR.

Fig. 11 shows the interior of one of these houses, with the Crosby indicator used in these experiments. The pressure cylinder of the indicator, with its piston and springs, was identical with the ordinary steam engine indicator; but the arrangement for moving the paper was a special one, constructed by Mr. Oldenborger. It consisted of a cylinder of about 8 inches diameter, with horizontal axis, revolved by means of a weight-driven mechanism; a band of paper was placed over the cylinder, and held in tension by the weight of a roll, placed in the loop, as shown. On this band a dot was made every half-second, through a typewriter ribbon, by the pointer shown, which was actuated by a current governed by the pendulum in the pump house.

The pressure cylinder of the indicator was connected with the water pipe by means of a small metal tube.

Strong springs were used, in order to diminish, as far as possible, the agitation of the indicator pencil by inertia.

The observations were performed as follows: Upon electric signals from the observer in house I, the observers in houses II and III took the atmospheric, the hydrostatic and the hydrodynamic pressure, each being represented by a horizontal line traced by the pencil of the Crosby indicator.

While the line of hydrodynamic pressure was being traced, the velocity of flow was determined by means of the measuring tank.

The observer at house I then closed the circuit operating the apparatus for making simultaneous half-second marks on the revolving paper in the three houses, and gave the order to close the gate by means of the falling weight. Starting from this moment, the pencils of the indicators in the three houses traced the diagrams of pressure of water hammer.

Samples of diagrams obtained are given in Figs. 4 and 5.

The first curve on Fig. 4 was obtained in house I, at the gate; the second in house II. The diameter of the pipe was 6 inches, the velocity of flow, v = 0.64 feet per second. The points under the curves indicate the time in half-seconds.

We see that the shape of the curves is different in the two cases, and in their general form these contours fully agree with the theoretical curves shown in Fig. 3.

Fig. 5 shows curves simultaneously obtained in houses I, II and III with a 2-inch pipe, velocity, v = 1.8 feet per second, and water-hammer pressure P = 7 atmospheres. We see that the first curve consists only of an upper step, the second of a long upper step and a short middle step, and the third of a short upper step and a long middle step. Expressed in time units, the distance between the com-

D = 6″; v = 5.6′

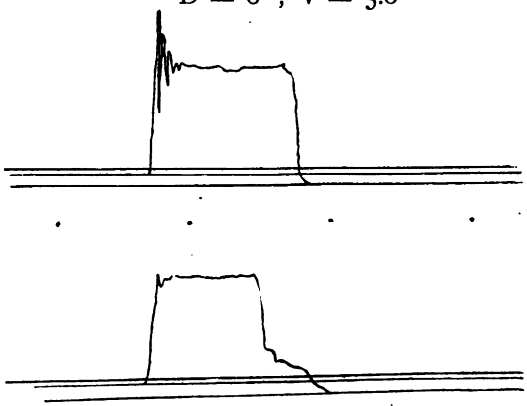

FIG. 12. INDICATOR DIAGRAMS IN TWO POINTS OF THE 6-INCH PIPE.

mencement of increase of pressure to the higher step and the commencement of its falling below this step must be equal, on each of the curves, to a round trip of the pressure wave from the point of observation to the main. During the observation the distances between the main and the points of observation were in the ratio of 3:2:1. The lengths of the higher steps of the curves of the diagrams are also in the ratio of 3:2:1.

Fig. 12 gives diagrams, obtained in houses I and II,

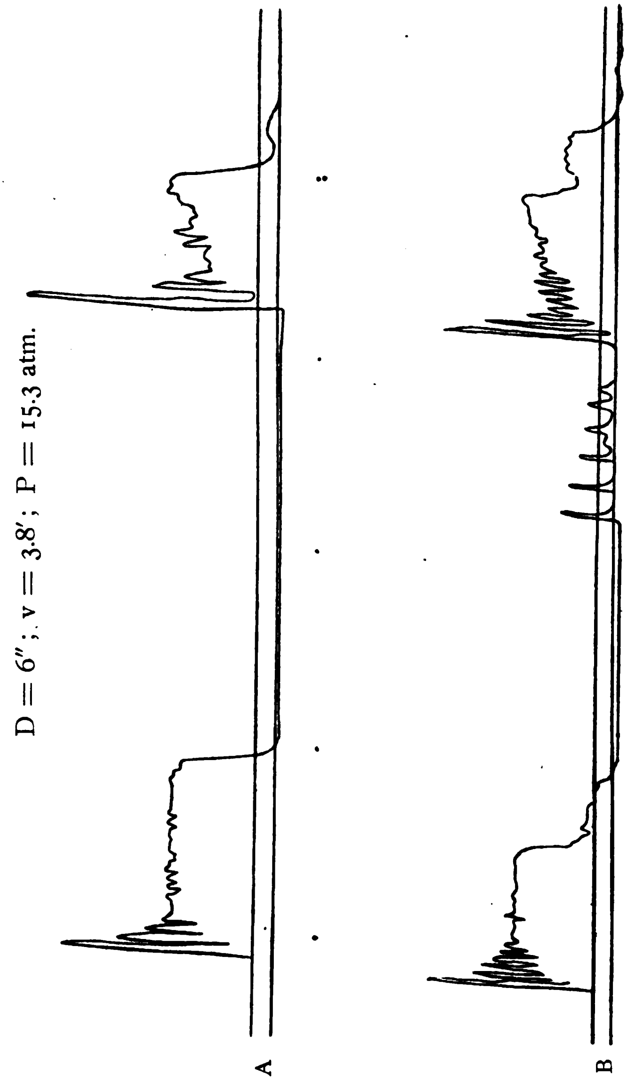

FIG. 13. INDICATOR DIAGRAMS, SHOWING THE EFFECT OF SEPARATION OF WATER COLUMN.

with a 6-inch pipe, velocity, $v = 5.6$ feet per second, and water-hammer pressure = 25 atmospheres.

The curves of the first increase of pressure are generally in complete accordance with the theory in cases of both small and great velocities of flow, and they enable us to determine the values of the velocity, λ, of propagation of the pressure wave, and of the additional pressure, P, due to water hammer.

The violent fluctuations, shown at each sudden increase of pressure, are attributed by Joukovsky to the effect of the small pipe leading to the Crosby indicator, which acted as a dead end (see page 387), momentarily increasing the pressure in the pipe, special care having been taken to minimize the effect of inertia in the moving parts of the apparatus.

The irregular shape of the middle step of the second curve is caused by the increase of pressure in the main itself, due to the arrival of the wave at the origin. This increase made the middle step higher than normal.

The curve of Fig. 13, A, was obtained in house I, which stood near the gate on the 6-inch pipe. The velocity of flow was 3.8 feet per second, and the shock pressure 15.3 atmospheres. It will be noticed that the lowest step of this curve is much longer than the upper step; also that the second rise of the curve begins with a violent oscillation, which runs higher than that of the first rise.

The same peculiarities characterize Fig. 13, B, which was obtained in the same experiment in house II. Moreover, in this case, in that part of the diagram where the middle step of static pressure should appear, we see a number of small separate peaks never observed on the curves obtained at house I.

Joukovsky believes that these eccentricities are due to the fact that, in this experiment, the subnormal pressure fell below the atmospheric pressure, as shown in the diagrams. This caused a parting of the water column and some disengagement and possible inhalation of air and liberation of water vapor, and this combined column of water, air and vapor occupied a longer time for its movements than would normally be the case. This accounts for the length of the portion of the diagram occupied by the curve of subnormal pressure; and the separate shocks, due to collisions between the several portions of the broken column, and their increased velocities when rushing into the partial vacuum formed, explain the irregular character of that part of the curve representing the second rise of pressure.

Joukovsky notes that the portion of the curve which represents the first period of supernormal pressure, and which was obtained while the whole column was under constant pressure, and before the sub-atmospheric pressure had appeared, was very regular, and served perfectly for the determination of the velocity, λ, of propagation of the pressure wave.

The determination of the velocity, λ, from the pressure diagrams was based on the fact that, during every cycle, the pressure wave makes two round trips through the pipe. (See page 359.) The time of a round trip was measured along the horizontal axis of the diagrams, taking, as a scale, the distances between the half-second points marked by the pendulum (the large scale of the diagrams could be read to hundredths of a second). Knowing the length of the pipe through which the wave traveled, it was easy to find the velocity.

The additional pressure, P, was determined from the diagrams by measuring the height of the first rise of the pressure curve above the line of hydrodynamic pressure; the scale was determined by the elasticity of the spring used in the indicator.

The following tables give the record of the experiments on the 2, 4, 6 and 24-inch pipes, and the values obtained for velocity, λ, and pressure, P.

TABLE VII.

(*b*) OBSERVATIONS OF WAVE VELOCITY, λ, AND PRESSURE, P, IN A 2-INCH PIPE, BY MEANS OF INDICATORS, SEPTEMBER 23, 1897.

No. of Experiment.	Velocity of Water in Feet per Second. v.	Duration of Round Trip of Wave in Seconds. From the Gate to the Origin. House I.	House II.	House III.	From House II to the Origin.	From House III to the Origin.	Duration of Gate Closure in Seconds. τ	P in Atmospheres. From Diagrams. House I.	House II.	House III.	Calculated P = 4 V.
1	2	3	4	5	6	7	8	9	10	11	12
1	4.52	1.16	1.15	1.15	0.77	0.38	0.08	18.5	18.0	18.0	18.1
2	4.30	1.13	1.15	1.15	0.78	0.39	0.06	17.8	17.5	16.7	17.2
3	4.16	1.14	1.13	1.13	0 78	0.40	0.06	17,0	16.6	16.0	16.6
4	3.67	1.15	1.13	1.13	0.76	0.37	0.06	15.1	15.0	14.5	14.7
5	3.67	1.14	1.13	1.14	0.75	0.40	0.05	14.5	14.4	14.6	14.7
6	3.66	1.14	1.13	1.13	0.76	0.39	0.06	14.6	14.6	15.0	14.6
7	1.79	1.14	1.14	1.13	0.76	0.39	0.05	6.3	5.9	6.3	7.2
8	1.76	1.14	1.14	1.13	0.76	0.39	0.06	7.3	7.3	7.2	7.0
9	0.64	1.14	1.15	1.14	0.75	0.39	0.06	2.8	2.8	2.5	2.6
10	1.52	1.14	. .	1.15	. .	0.39	0.05	6.3	6.3	6.3	6 1
11	1.52	1 13	1.13	1.13	0.75	0.38	0.06	6.3	6.3	6.1	6.1
12	4.23	1.14	1.13	1.13	0.76	0.39	0.07	17.3	16.7	16.1	16.9

The length of the 2-inch pipe was 2494 feet.
The pressure wave made a round trip through this pipe in 1.14 seconds.
Hence, the velocity of the pressure wave in a 2-inch pipe is

$$\lambda = \frac{2494 \times 2}{1.14} = 4375 \text{ feet per second.}$$

The calculated value of λ for a 2-inch pipe is 4424 feet per second. (See Table I, page 364.)

In this experiment the distances of houses I, II, III from the origin being equal to 2494, 1640, 822 feet were proportional to 3 : 2 : 1; and we see that the durations of round-trip of wave from these houses to the origin, being equal to 1.14, 0.78, 0.39 seconds, are in the same proportion.

TABLE VIII.

(*c*) Observations of Wave Velocity, λ, and Pressure, P, in a 4-inch Pipe, by Means of Indicators, November 4, 1897.

No. of Experiment.	Velocity of Water in Feet per Second, v.	Duration of a Round Trip of Wave in Seconds. House I.	House II.	Average from a Number of Cycles.	Duration of Gate Closure in Seconds τ	P in Atmospheres. From Diagrams. House I.	House II.	Calculated P = 4 v.
1	2	3	4	5	6	7	8	9
1	3.3	0.49	0.51	. .	0.04	13.3	13.3	13.2
2	1.9	0.50	0.50	. .	0.04	7.8	7.8	7.6
3	4.1	0.49	0.50	. .	0.03	15.8	15.9	16.4
4	9.2	0.49	0.50	. .	0.04	35.0	35.9	36.8
5	2.9	0.49	0.50	. .	0.05	11.3	11.3	11.6
6	0.5	0.50	0.50	0.50	0.04	2.0	2.5	2.0
7	1.1	0.50	0.49	0.51	0.04	4.4	4.3	4.4

The length of the 4-inch pipe was 1050 feet.
The pressure wave made a roundtrip through this pipe in 0.50 second.
Hence, the velocity of pressure wave in a 4-inch pipe is

$$\lambda = \frac{1050 \times 2}{0.50} = 4200 \text{ feet per second.}$$

The calculated value of λ for a 4-inch pipe is 4226 feet per second. (See Table I, page 364.)

TABLE IX.

(*d*) Observations of Wave Velocity, λ, and Pressure, P, in a 6-inch Pipe, by Means of Indicators, November 20, 1897.

No. of Experiment.	Velocity of Water in Feet per Second, v.	Duration of a Round Trip of Wave in Seconds. House I.	House II.	Average from a Number of Cycles	Duration of Gate Closure in Seconds, τ.	P in Atmospheres. From Diagrams. House I.	House II.	Calculated P = 4 v.
1	2	3	4	5	6	7	8	9
1	3.3	0.52	. .	0.52	0.03	15.7	15.7	13.2
2	1.9	0.52	0.52	0.52	0.03	7.3	7.1	7.6
3	0.6	0.52	0.52	0.52	0.04	3.0	3.0	2.4
4	1.4	0.51	0.52	0.52	0.04	6.0	6.1	5.6
5	3.0	0.52	. .	0.52	0.03	12.1	11.44	12.0
6	4.0	0.51	0.51	0.52	0.03	15.6	15.2	16.0
7	5.6	0.52	0.52	0.51	0.04	25.2	25.2	22.4
8	7.5	0.51	. .	0.53	0.04	29.0	29.0	30.0
9	7.5	0.51	Pipe burst.			11.7	11.3	30.0

The length of the 6-inch pipe was 1066 feet. The pressure wave made a round trip through this pipe in 0.52 second.
Hence, the velocity of pressure wave in a 6-inch pipe is

$$\lambda = \frac{1066 \times 2}{0.52} = 4100 \text{ feet per second.}$$

The calculated value of λ for a 6-inch pipe is 4116 feet per second. (See Table I, page 364.)

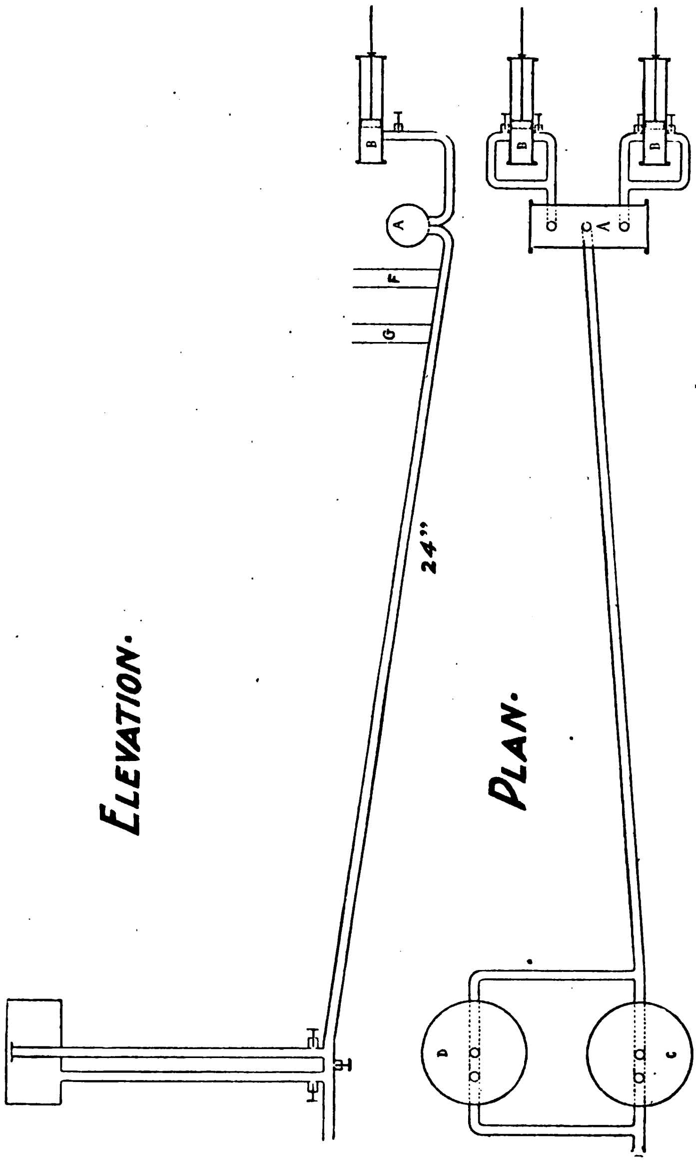

FIG. 14. THE 24-INCH MAIN, USED FOR EXPERIMENTS.

(*e*) *Observations of Wave Velocity, λ, and Pressure, P, in the 24-inch Main, by Means of an Indicator.*

The 24-inch main (see Fig. 14), upon which the observations were performed, was at that time the chief main of the waterworks of Moscow, leading from the Alexeievskaia pumping plant to the Krestovsky water towers. Its length, from the gate valve (where the water hammer was effected) to the Krestovsky water towers, was 7007 feet; the length of the pipe, between the gate valve and the air chamber in the pump house, was 210 feet.

During the observations the pumps were separated from the pipe by a gate, and the air chamber, filled with water, remained in connection with the pipe, precautions being taken to drive out all the air. The water flowed from the Krestovsky reservoir. The water hammer was produced by means of the same gate as before. The indicator, placed in house I, was connected with the 24-inch pipe near the gate. Weaker springs were used in the Crosby indicator than in the other experiments. The pendulum marked on the indicator not each half-second, as before, but each full second.

Some moments after the dropping of the weight which closed the gate, the pressure wave arrived at the water chamber, compressing the water there, and the pressure was gradually propagated along the 24-inch main, the water gradually coming to rest.

Under these conditions no sudden increase of pressure was recorded on the diagrams, and the shape of the curve, Fig. 15 (because of the influence of the water chamber), was the same as if the gate had been closed gradually.*

Omitting the formula and the theoretical explanation of Joukovsky referring to the influence of the water chamber, we give on page 387 a table of some of the results obtained July 25, 1897:

* See pages 392-397.

TABLE X.

OBSERVATIONS OF WAVE VELOCITY, λ, AND PRESSURE, P, IN A 24-INCH PIPE, BY MEANS OF INDICATORS, JULY 25, 1897.

No. of Experiment.	Velocity of Water in Feet per Second, v.	Duration of Round Trip of Wave in Seconds.	P in Atmospheres.	
			From Diagrams.	Calculated $P = 3v$.*
1	2	3	4	5
1	0.18	. .	0.45	0.54
2	0.56	4.24	1.81	1.68
3	0.55	4.39	1.66	1.65
4	0.54	4.20	1.77	1.62
5	0.55	4.18	1.80	1.65
6	0.41	4.20	1.23	1.23
7	0.40	4.18	1.27	1.20
8	0.16	. .	0.42	0.48
9	0.16	. .	0.42	0.48
10	0.09	. .	0.29	0.27

The length of the 24-inch main from the gate to the reservoir was 7007 feet.

The pressure wave made a round trip through the pipe in 4.23 seconds. (average).

Hence, the velocity of pressure wave in the 24-inch main is

$$\lambda = \frac{7007 \times 2}{4.23} = 3313 \text{ feet per second.}$$

The calculated value of λ for a 24-inch pipe is 2996 feet per second. (See Table I, page 364.)

* Using the formula $P = h\,v$, we take here $h = 3$; this value corresponds to $\lambda = 3313$ feet per second, as obtained from these experiments. The value, $h = 2.7$, as given for a 24-inch pipe on page 369, corresponds to the calculated value of λ.

IV. MISCELLANEOUS PHENOMENA CONNECTED WITH WATER HAMMER.

1. Increase of the Intensity of Water Hammer in Case of Passage of the Pressure Waves into Smaller Pipes with Dead Ends.

A doubling of water-hammer pressure occurs when the pressure wave is reflected from a dead end of a branching pipe.

Observations in this connection were made upon the passage of the pressure waves from a 4-inch into a 2-inch

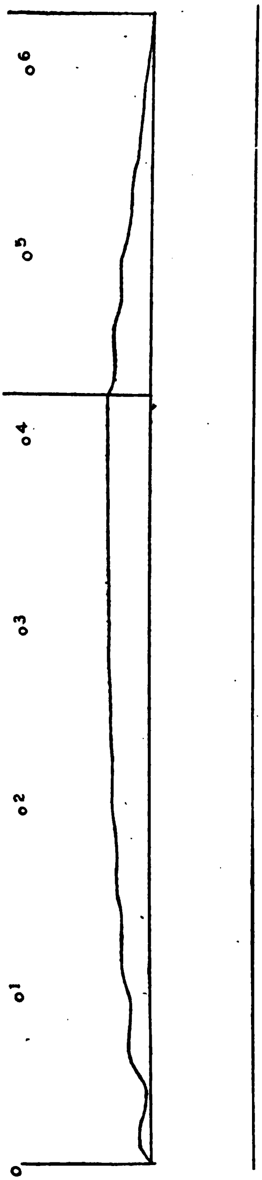

Fig. 15. Indicator Diagram Obtained on the 24-inch Main.

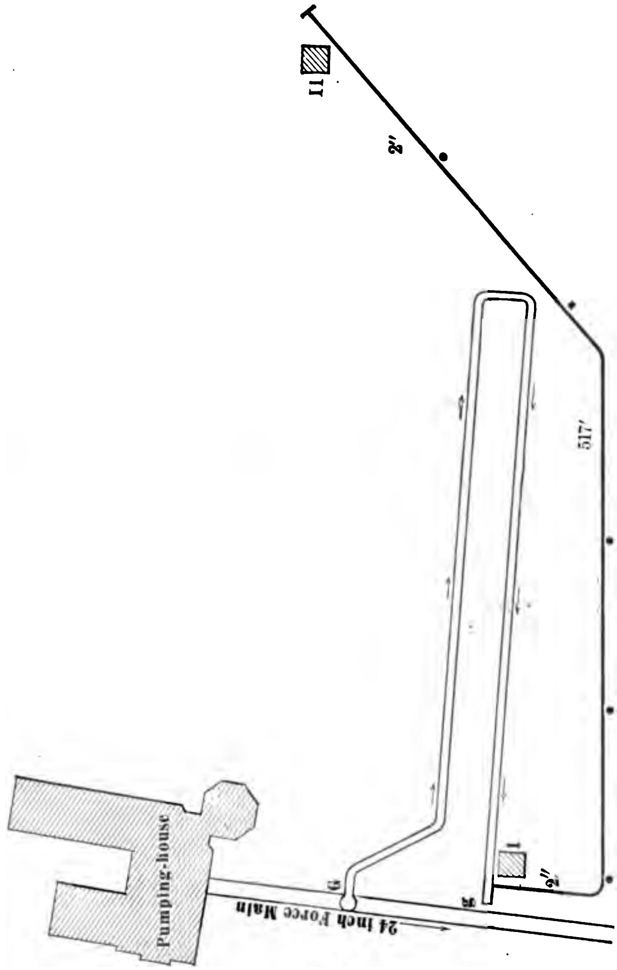

FIG. 16. DEAD ENDS, EXPERIMENTAL PIPINGS.

pipe. For this purpose that portion of the 2-inch pipe line was used which was located between houses I and II. (See Fig. 16.) The length of this section was 517 feet, and it was connected, near house I, with the end of the 4-inch pipe, 1050 feet long, which was left, as before, connected with the outlet gate valve and with the indicator of house I. The tube leading to this indicator started from the 4-inch pipe near the point of its connection with the 2-inch pipe. The end of the 2-inch pipe was connected with the indicator of house II and terminated in a gate.

Before the beginning of the observations, water was allowed to flow from this gate, in order to make sure that there was no air in the 2-inch pipe. Then the gate was closed, so that the branch formed a dead end. The gate valve of the 4-inch pipe was then opened, and water let

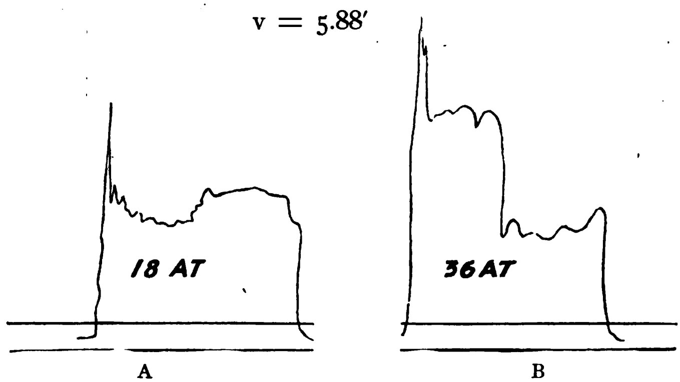

FIG. 17. INDICATOR DIAGRAMS IN THE CASE OF A DEAD END.

out from it in the usual way; the quantities of water were measured, and the hydrodynamic pressures recorded in houses I and II (near the beginning and at the end of the 2-inch pipe). Then the weight (closing the gate, at the outlet of the 4-inch pipe) was dropped, and the pressure diagrams recorded.

From the principles already established, Joukovsky concluded that the shock pressure in the small pipe, after and owing to the reflection of the pressure wave from the dead end, would be double that in the large pipe; and this was confirmed by the results of the experiments. For instance, with a velocity of 5.88 feet per second in the 4-inch pipe, the observed pressure in the 4-inch main was 18 atmospheres, while that in the 2-inch pipe was 36 atmospheres. See Fig. 17, where diagram A was taken in house I on the 4-inch pipe and diagram B was taken in house II on the 2-inch pipe.

Similar experiments were afterward made upon the 2-inch pipe connected with the 6-inch pipe. Here also the experiments sustained Professor Joukovsky's conclusion that the pressure would be doubled in the dead end of the small pipe.

The pressure in the larger pipe is increased when the doubled pressure arrives from the dead end, and if this occurs at a moment when the pressure in the large pipe is still supernormal, a very high pressure may result.

If there is a system of several dead ends, as for instance in Fig. 18, a combination of their increased pressures may

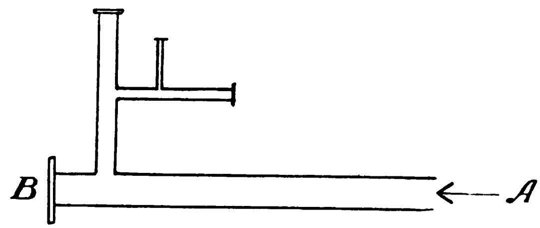

FIG. 18.

raise the pressure in the pipe to a dangerous extent. As will be shown below,* the increase of pressure may be totally prevented by the use of safety valves.

2. *Reflection of the Pressure Wave from the Open End of a Discharging Branch Pipe.*

For these experiments the same combination of 2-inch and 6-inch pipes was used as before (Fig. 16); but the gate at the end of the 2-inch pipe was kept open, the water flowing out from it during the entire experiment.

* See page 413.

At first the gate at the end of the 6-inch pipe was opened, and its discharge was measured. Then this gate was suddenly closed, and the pressure diagrams were recorded in house I. These showed that, so far as the transmission of the pressure wave is concerned, the phenomena, in the flowing water in the branch pipe, follow the laws already established for a pipe in which the flow is checked.

The reflection of the wave from the open end of the 2-inch pipe takes place as it would if this end discharged into a shallow reservoir of water, subjecting the discharge only to atmospheric pressure, which, in this case, is the "normal" (main or reservoir) pressure. Hence the pressure is not doubled (as in the case of a dead end), but diminished.

With the 6-inch and 2-inch pipes described, the pressure in the larger pipe, after the arrival of the reflected wave from the open end of the branch pipe, was about 90 per cent. of the maximum.

3. Intensity of Pressure as Affected by Length of Time Occupied in Closing the Gate.

Water hammer may be almost, if not entirely, obviated by closing the gate slowly.

As this subject is one of considerable practical importance, it will be well to examine, with some care, the underlying principles.

Let

t = duration of closure of the gate.
v = velocity of flow of water.
h = pressure obtained per unit of extinguished velocity, v.
v h = P = the full maximum shock pressure.
P′ = the permissible shock pressure.
l = length of the pipe line.
λ = velocity of the propulsion of the pressure wave.

Let us express in diagrams the successive changes of pressure which we should expect in different points of a pipe line in case of water hammer.

Suppose a pipe, A B (Fig. 19), of length l, which the pressure wave, with velocity, λ, traverses during a time, $t = 8$ seconds $= \frac{l}{\lambda}$ and $l = \lambda t$. For instance, if $\lambda = 1$ kilometer per second, then the length of the pipe will be $l = \lambda t = 1 \times 8 = 8$ kilometers.

In the pipe A B (Fig. 19), let A be the reservoir (or large main from which the pipe is fed) and B the gate, the closing of which produces the water hammer; and let a manometer gauge be placed at the point marked 2, distant 2 kilometers from the gate B.

First, let us suppose that the total closure consists of 8 instantaneous partial closures, 1 second apart, each shutting off abruptly one-eighth of the area of the pipe opening, the first partial closure occurring at the end (1) of the first second, and the last, or eighth, at the end (8) of the eighth second.

Each of these 8 partial closures sends its own weak pressure wave through the pipe, each such wave following exactly the laws already laid down for the case of instantaneous closure, and thus produces a diagram of its own. These diagrams are represented in the upper part of the figure.

Summing up, algebraically, the several partial pressures for the several waves, at each second, as given in the upper diagram, we obtain the influence diagram (covering all the waves) for point 2, as given in the lower figure.

In the same way were obtained the influence diagrams for the other points, given in Fig. 20, B.* The several

* Where our supposed series of abrupt partial closures is replaced by a gradual closure, in which equal areas are shut off in equal times, the broken diagrams are, of course, replaced by straight lines.

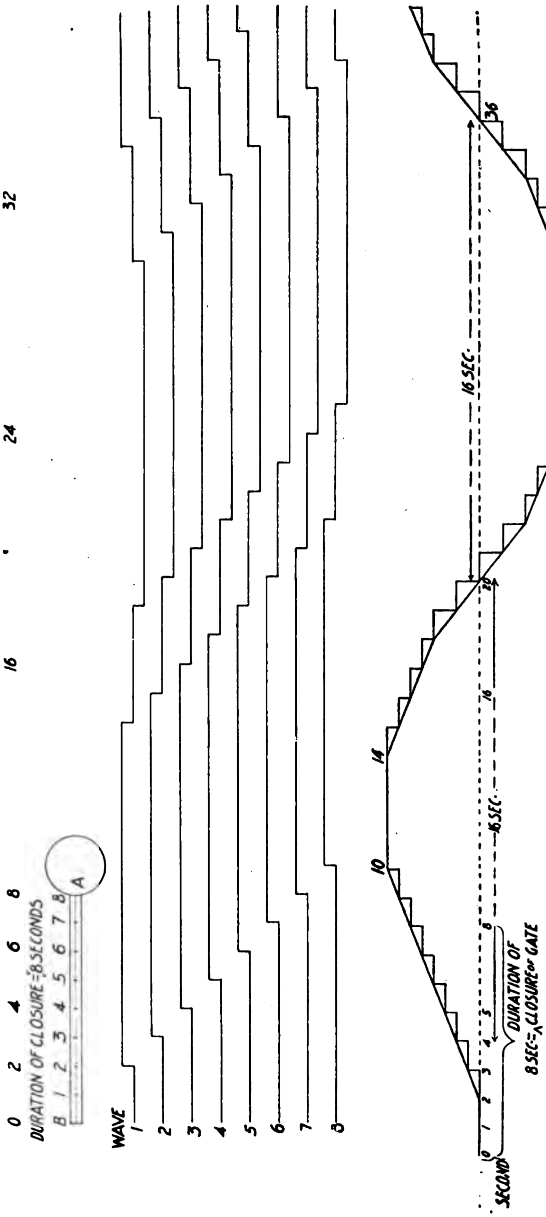

DIAGRAM FOR POINT 2 OF THE PIPE.

FIG. 19. PRESSURE DIAGRAMS IN THE CASE OF SLOW CLOSURE OF THE GATE.

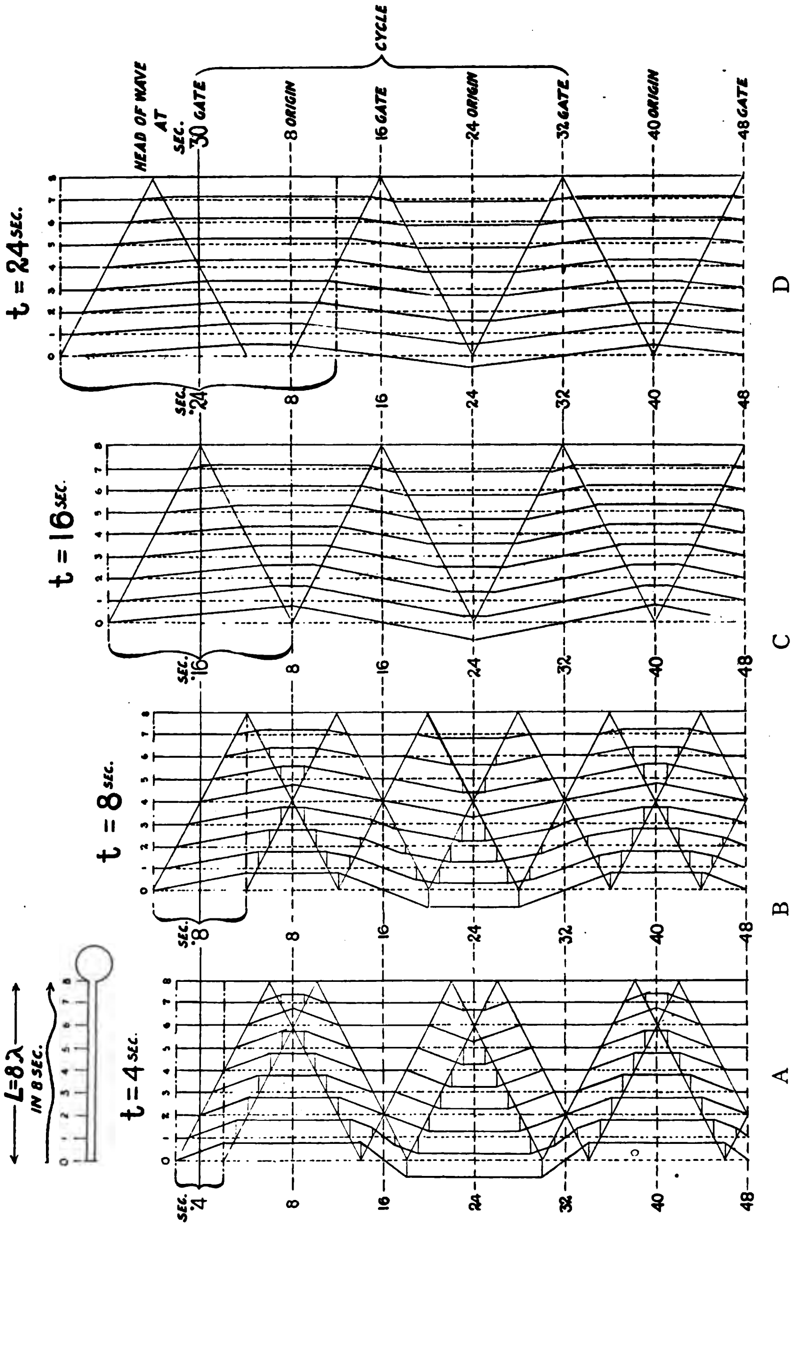

Fig. 20. Pressure Diagrams in the Case of Slow Closure of the Gate.

sets of diagrams (A, B, C, D) of Fig. 20 show the effect of varying the time occupied in closure. In each set pressure diagrams for eight points taken in equal distances along the pipe are traced.

A comparison of the several diagrams in Fig. 20 shows:

1. That the first increase of pressure, due to a uniform closure of the gate in t seconds, proceeds at the rate $\frac{P}{t}$, and that subsequent changes of pressure, at points whose distance from the gate is such that the time required for a round trip of the wave from the point to the gate and back is not less than t, proceeds at the same rate $\frac{P}{t}$; whereas, at the gate, the changes of pressure (after the first increase) proceed at *double* this rate, or rate $= 2\frac{P}{t}$; and at all intermediate points the total change is made partly at the rate $\frac{P}{t}$ and partly at the rate $2\frac{P}{t}$. (See Fig. 20, A, diagram for point 1.)

2. That the maximum shock pressure reaches its full value, P, only at points so far distant from the origin that the time, required for the round trip of the wave from the point to the origin and back, is longer than the time, t, occupied in closing the gate.

Thus (see figures) if t = 8 seconds, the full maximum pressure, P, is felt (for an instant only) at point 4, whence the wave travels to the origin and back in 8 seconds = t.

3. That from the last point at which the full maximum pressure, P, arrives (for an instant only), the maximum pressure, P′, actually exerted, diminishes uniformly from P′ = P, at said point, to P′ = 0, at the origin. In other words, at any point in this portion of the pipe, P′ is less than P in the same proportion as the time, t_r, required for

a round trip of the wave from said point to the origin and back, is less than t; or $P' = P \frac{t_r}{t}$

Hence $$t = t_r \frac{P}{P'}$$

But $t_r = \frac{2l_r}{\lambda}$* and P = h v.

Hence $$t = \frac{2l_r}{\lambda} \cdot \frac{hv}{P'}$$

For a point at the gate, $l_r = l$,

$$t = \frac{2l}{\lambda} \cdot \frac{hv}{P'} .$$

This gives the minimum time, t, which must be occupied in closing the gate, if the shock pressure is not to exceed the permissible pressure, P′, in any point of the pipe.

$$If \quad t \geqq \frac{2l}{\lambda} ; \; then \; \frac{P}{P'} \geqq 1 ; \; or \; P \geqq P'.$$

Hence we see:

4. That if the time, t, occupied in closing the gate, is not greater than the time, $\frac{2l}{\lambda}$, required for a round trip of the wave from the gate to the origin and back, the full maximum pressure, P, will be felt in some part or all of the pipe, and not otherwise.

4. Efficiency of Air Chambers in Reducing Water Hammer.

As already shown by the experiments with the 24-inch pipe,† the effect of a large water chamber, placed at the end of the pipe line (near the gate), is analogous to that due to a prolongation of the time of closing the gate valve.

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* l_r = distance from the point in question to the origin.

† See page 386.

The effect of an air chamber is quite similar, but much more considerable. Joukovsky placed an air chamber on the 2-inch experimental pipe between houses II and III, at a distance of 1070 feet from the gate. (See Fig. 2.) Air chambers of different volumes were used for experiments. The gate was closed in the usual way, and pressure curves were recorded in each of the three houses; the most interesting, however, were those obtained in houses I and III.

Figs. 21 and 22 show curves obtained with a small air chamber (about 60 cubic inches), the velocity being 4.4 feet per second. The curve in Fig. 21 was obtained in house I, between the gate and the air chamber, while that in Fig. 22 was obtained in house III, between the air chamber and the origin. We see, from the curves, that an air chamber of that small size caused no lowering of the first stage of the curve obtained between the gate and the air chamber, which showed a pressure of 17.3 atmospheres, which is very nearly equal to the theoretical P = h.v = 4 atmospheres × 4.4 = 17.6 atmospheres. As to the second stage of this curve, the air chamber even *increased* the pressure, in the second stage, to almost 1.3 times the pressure in the first stage. The pressures in the third and the following stages are notably diminished. The curve in Fig. 22, obtained between the air chamber and the origin, shows a slightly reduced maximum pressure (14.6 atmospheres); and the stages of the curve become rounded and rapidly lower.

Quite different results were obtained when the size of the air chamber was materially increased. Fig. 23 shows a curve obtained in house I (between air chamber and gate) with the use of an air chamber of 548 cubic inches, or 9 times that used in the experiments just described. The velocity of flow was 1.8 feet. This curve is very much like those usually obtained without the use of air chambers; the pressure wave is reflected from a large air cham-

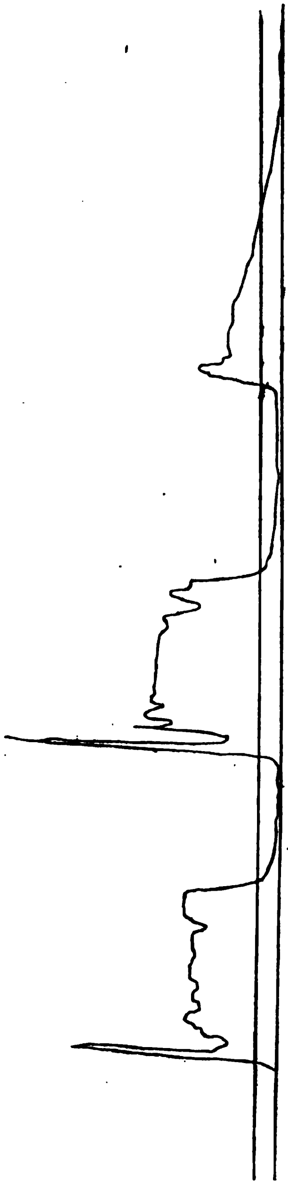

FIG. 21. Diagram taken between the Gate and the Air Chamber.

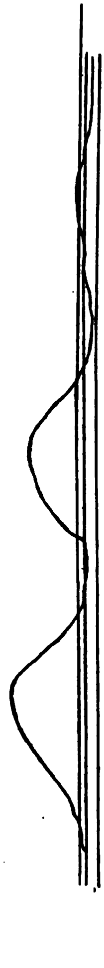

FIG. 22. Diagram taken between the Air Chamber and the Origin.

INDICATOR DIAGRAMS IN THE CASE OF A SMALL AIR CHAMBER.

ber as from the origin. The pressure here is 7.1 atmospheres, which corresponds closely with the theoretical pressure, P = 4 v = 7.2 atmospheres. The diagram recorded in this case between the air chamber and the origin is a

FIG. 23. INDICATOR DIAGRAM TAKEN BETWEEN THE GATE AND A LARGE AIR CHAMBER.

straight line, coinciding with the straight line of the original hydrodynamic pressure, showing that an air chamber of this size does not allow a water hammer of the given intensity to pass through it.

Explanation of Principles.

Suppose a water pipe, A B, on which an air chamber, C, is placed (see Fig. 24).* Upon sudden closure of the gate,

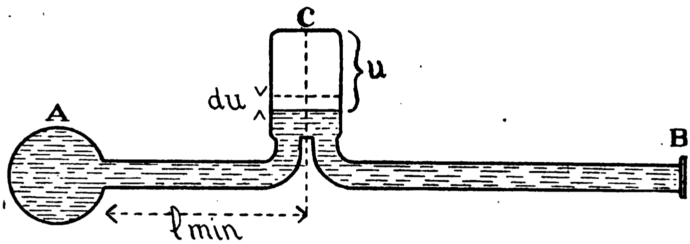

FIG. 24.

B, the water near the gate is compressed† to the pressure P = h.v atmospheres, and, after some moments, the pres-

* It is only for the sake of clearness that the 2 pipes are shown entering the air chamber separately. This does not affect the principle involved.

† See page 356.

sure wave, moving with velocity λ, will arrive at the air chamber, C. At that moment, the pressure at G, as well as in the pipe between the air chamber and the origin, is still the original hydrodynamic pressure, p_1. Therefore, the compressed water finds an outlet into the air chamber, which it enters with the original velocity v (which is now in the inverse direction).

Hence, the presence of the air chamber causes the pressure wave to be reflected not from the main, A, but from the air chamber, C.

We see also that the air chamber causes no reduction of the shock pressure in the section, B C, of the pipe between the gate and the air chamber. The experiments of Joukovsky with air chambers have even shown an increase of pressure in this section at the beginning of the second cycle, this increase (which in one case reached 50 per cent.; see Table XI, page 411, last column) being due to the compression of the air in the chamber and its subsequent reaction, which throws the water toward the gate, where a subnormal pressure already exists.

Let us now pass to the effect of the air chamber upon the section of the pipe, C A, between the air chamber and the origin. The water here continues to flow in the original direction, A C, and with the original velocity, v, until the pressure wave arrives at the air chamber, C. From this moment the pressure in the chamber increases because of the water coming into it from both sides (from B C and from A C). But the increase of pressure in the chamber, and the compression of the air there, gradually stop the entrance of water into it, and *affect the part, A C (and that part only), in the same way as if the flow from the pipe, A C, were stopped by a slow closure of a gate placed on the pipe at C.*

Conclusion: It has already been explained that a slow closure of a gate, lasting longer than a double run of the pressure wave through the length of the pipe,* prevents the exertion of the maximum shock pressure.

It is evident that the flowing of water into the air chamber will last longer with a large than with a small volume of air; consequently, the stoppage of the column of water, A C, will be slower, and the shock pressure less, in the first case. Therefore, the air chamber must be of large dimensions. As already stated, small air chambers do not accomplish their purpose, being even harmful, increasing the additional pressure in the section, B C, between the gate and the air chamber.

We see, then, that the air chamber reflects the pressure wave, allowing to pass through it only that part of the shock pressure which corresponds to the size of the chamber; in other words, *only that pressure is exerted in the pipe, A C (between the air chamber and the origin), to which the air in the chamber has been raised.*

Formula for the Required Volume of Air Chamber.

The formula for the determination of the size of the air chamber, which will allow to pass through it only a shock pressure of given intensity, is deduced as follows:

Let u = variable volume of air in the air chamber.
v = original velocity of flow in the pipe.
d = diameter of the pipe.
$P = h\,v$ = maximum additional shock pressure due to the checking of the velocity, v.
P_a = maximum additional shock pressure which will appear in the air chamber and will be carried along the pipe beyond the air chamber.
P_u = variable additional pressure in the air chamber, corresponding to the variable volume, u.

* See page 396.

k = a constant numerical value, characterizing the thermodynamic properties of the gas contained in the chamber. (In the case of air, k = 1.4).

t = duration of a round trip of the pressure wave (with velocity λ) from the air chamber to the nearest end (distance = l_{min}) of the pipe (whether the gate or the origin) and back;

$$t = \frac{2\, l_{min}}{\lambda}$$

p_0 = the original hydrostatic pressure in the air chamber.

u_0 = volume of air in the air chamber corresponding to the original hydrostatic pressure, p_0.

p_1 = original hydrodynamic pressure in the air chamber.

u_1 = volume of air in the air chamber corresponding to the original hydrodynamic pressure, p_1.

At the moment when the pressure wave reaches the air chamber (which still has the original hydrodynamic pressure, p_1), the water will enter the chamber from *both* sides with the *original velocity*, v. It is evident that the volume of water, flowing into the chamber, must be equal to the reduction of the volume of air due to its compression. This compression and the resulting increase of pressure in the chamber gradually reduce the velocity of the water flowing into it. When the additional pressure in the chamber becomes = P_a, that is to say, when the total pressure there is = $p_1 + P_a$, the flow of water into the chamber will cease.

From the general theory of water hammer, we know that P stands in direct proportion to the reduction of velocity of flow in the pipe ($P = v.h$).* Consequently, if some additional pressure, P_u, has appeared in the air chamber, we know that the velocity of flow in the pipe has been reduced by $\frac{P_u}{h}$ and is equal to $\left(v - \frac{P_u}{h}\right)$. This is the velocity with which water will flow from both sides into the air chamber when the additional pressure there is P_u.

We thus obtain the equation: The reduction of air volume in the chamber ($-du$) during each small interval of time (dt) is equal to the volume of water, which, during this interval of time (dt), has entered the air chamber from both sides.

The velocity of the entering water is, as has been already shown, $\left(v - \frac{P_u}{h}\right)$.

The area of the cross-section of the pipe is $\frac{\pi d^2}{4}$.

Hence, in a unit of time, each of the pipes will bring into the air chamber $\frac{\pi d^2}{4}\left(v - \frac{P_u}{h}\right)$ cubic units of water.

During the interval, dt, each pipe will bring in

$$\frac{\pi d^2}{4}\left(v - \frac{P_u}{h}\right)dt$$

of water. Two pipes will let in twice this amount of water, and this amount will be equal to the reduction of volume of air in the chamber. Consequently,

$$-du = \frac{\pi d^2}{2}\left(v - \frac{P_u}{h}\right)dt.$$

This is the differential equation of air compression in the chamber by the water entering it.

The flowing of water into the air chamber and the compression of air therein will continue until the state of compression of water in the air chamber (in the form of a

* See page 369.

wave, moving with velocity λ) arrives at the end of the pipe nearest to the chamber, and will return thence to the air chamber with a lowered pressure. In other words, it will continue until the water begins to flow from the air chamber, which will result in a lowering of pressure. In order to find the amount of additional pressure, which will originate in the air chamber and will be transmitted along the pipe beyond the air chamber, we must *integrate* the above equation for the time, t.

The process of the compression of air in the chamber being rapid, we take it to be *adiabatic,* which is expressed by the physical law

$$u^k . p = \text{constant.}$$

Beginning with the equation on page 404

$$-du = \frac{\pi d^2}{2}\left(v - \frac{P_u}{h}\right)dt \qquad (1)$$

and, substituting $v = \frac{P}{h}$, (according to the equation $P = v\ h$) we have

$$-du = \frac{\pi d^2}{2}\left(\frac{P - P_u}{h}\right) dt \qquad (1')$$

From the equation of the adiabatic compression we have:

$$u_1^k p_1 = u^k (p_1 + P_u) = \text{Constant} \qquad (2)$$

$$u^k = \frac{u_1^k p_1}{p_1 + P_u} \qquad (2')$$

Differentiating this, we obtain:

$$k u^{k-1} du = \frac{-u_1^k p_1 \, dP_u}{(p_1 + P_u)^2}$$

consequently

$$du = -\frac{u_1^k p_1 \, dP_u}{k u^{k-1}(p_1 + P_u)^2} \qquad (3)$$

From equation (2′) we see that

$$u^{k-1} = \frac{u_1^k p_1}{u(p_1 + P_u)} \; ;$$

and that $$u = \sqrt[k]{\frac{u_1^k p_1}{p_1 + P_u}} \; ;$$

consequently $$u^{k-1} = \frac{u_1^k p_1}{\sqrt[k]{\frac{u_1^k p_1}{p_1+P_u}} \cdot (p_1 + P_u)} .$$

Substituting for u^{k-1}, in equation (3), its value, as just given, we find that

$$du = -\frac{1}{k} \cdot \frac{u_1 p_1^{\frac{1}{k}} \, dP_u}{(p_1 + P_u)^{\frac{k+1}{k}}} \qquad (4)$$

Substituting, for d u, in equation (1), the expression (4), we obtain:

$$\frac{1}{k} \cdot \frac{u_1 p_1^{\frac{1}{k}} \, dP_u}{(p_1 + P_u)^{\frac{k+1}{k}}} = \frac{\pi d^2}{2}\left(\frac{P - P_u}{h}\right) dt \; ;$$

or

$$k \cdot \frac{\pi d^2}{2} \cdot \frac{dt}{p_1^{\frac{1}{k}} h} = u_1 \frac{dP_u}{(p_1 + P_u)^{\frac{k+1}{k}}(P - P_u)}$$

For convenience, let

$$z = \frac{p_1 + P_u}{p_1 + P} = \frac{\textit{pressure in air chamber}}{\textit{total max. pres. in pres. wave.}} \qquad (5)$$

Then

$$P_u = z(p_1+P)-p_1\ ;\quad dP_u = (p_1+P)dz.$$

Substituting h by $\frac{P}{v}$ and using the adopted designations, we write:

$$\frac{k\pi d^2}{2}\cdot\frac{v\,dt}{p_1^{\frac{1}{k}}P} = u_1\frac{(p_1+P)\,dz}{(p_1+P_u)^{\frac{k+1}{k}}\left[P-z\,(p_1+P)+p_1\right]},$$

$$\frac{k\pi d^2}{2}\cdot\frac{p_1(p_1+P)^{\frac{k+1}{k}}}{p_1^{1+\frac{1}{k}}P}\,v\,dt =$$

$$= u_1\frac{dz}{\left(\frac{p_1+P_u}{p_1+P}\right)^{\frac{k+1}{k}}\frac{(p_1+P)(1-z)}{p_1+P}};$$

$$\frac{k\pi d^2}{2}\left(\frac{p_1+P}{p_1}\right)^{\frac{k+1}{k}}\frac{p_1}{P}\,v\,dt = u_1\frac{dz}{z^{\frac{k+1}{k}}(1-z)}. \qquad (6)$$

Let us now integrate this equation for the time, t, during which the air in the air chamber is compressed and the additional pressure, P_u, increased from 0 to P_a. From our designation for z, it follows that the corresponding limits of the integration will be

$$z_0 = \frac{p_1}{p_1+P} \qquad and \qquad z_t = \frac{p_1+P_a}{p_1+P}.$$

The integration gives us:

$$\frac{k\pi d^2}{2}\left(\frac{p_1+P}{p_1}\right)^{\frac{k+1}{k}}\frac{p_1}{P}\,v\,t = u_1\int_{z_0}^{z_t}\frac{dz}{z^{\frac{k+1}{k}}(1-z)}. \qquad (7)$$

For brevity, let

$$\psi(z) = \int_{z_0}^{z_t} \frac{dz}{z^{\frac{k+1}{k}}(1-z)} \qquad (8)$$

For convenience, we will express u_1 in terms of the volume of the air chamber, u_0, corresponding to the static pressure, p_0.

According to the law of Mariotte,

$$u_1 = \frac{p_0 u_0}{p_1}$$

Substituting the expressions given, we obtain from (7):

$$u_0 = \frac{k \pi d^2}{2\psi(z)} \left(\frac{p_1 + P}{p_1}\right)^{\frac{k+1}{k}} \frac{p_1^2}{P p_0} vt \qquad (9)$$

This is the exact analytical general formula showing the relation between the volume of the air chamber, u_0, and the intensity of additional shock pressure, P_a*, which the air chamber will allow to pass through it, and which is transmitted through the pipe beyond the chamber.

This formula, owing to the complexity of the function $\psi(z)$, is inconvenient for practical use. But for the special case mentioned below it is possible to obtain, from this general and exact formula, a convenient and sufficiently approximate one. When the additional shock pressure, P_a, which is allowed to pass through the air chamber, is small,† the difference between the limits of integration

$$z_0 = \frac{p_1}{p_1 + P} \quad \text{and} \quad z_t = \frac{p_1 + P_a}{p_1 + P}$$

is also small, and we may write, approximately:

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* P is included in the designation ψ (z), in z_t.

† For instance, 0.7 atmosphere, as in Table XI, page 411.

$$\psi(z)=\int_{z_0}^{z_t}\frac{dz}{z^{\frac{k+1}{k}}(1-z)}=\frac{z_t-z_0}{z_0^{\frac{k+1}{k}}(1-z_0)};$$

or

$$\psi(z)=\frac{\frac{p_1+P_a}{p_1+P}-\frac{p_1}{p_1+P}}{\left(\frac{p_1}{p_1+P}\right)^{\frac{k+1}{k}}\left(1-\frac{p_1}{p_1+P}\right)}.$$

Simplifying, we have

$$\psi(z)=\left(\frac{p_1+P}{p_1}\right)^{\frac{k+1}{k}}\frac{P_a}{P} \qquad (10)$$

Substituting this approximate value of $\psi(z)$ in the formula (9), we obtain:

$$u_0=\frac{k\pi d^2}{2}vt\frac{p_1^2}{p_0 P_a} \qquad (11)$$

Here u_0 is the minimum volume which the air chamber must have, in order that the shock pressure, P_a, in that portion of the pipe lying between it and the origin, shall not exceed the allowed intensity.

But this formula (11) is sufficiently approximate only when the allowed P_a is small; that is to say, when the volume of the air chamber is great. With small air chambers, great shock pressures pass through them, and the exact formula (9) must be used.

For the direct use of formula (9), tables of the function ψ should be calculated; but Joukovsky used this formula for the determination of two limits, between which the true value of ψ is included.

According to formula (5), $z < 1$.

Therefore, by formula (8), where, as a matter of fact, $k = 1.4$;

$$\text{if } k = 1, \quad \psi_1 > \psi;$$
$$\text{if } k = 2, \quad \psi_2 < \psi.$$

The corresponding actual value of ψ lies between ψ_1 and ψ_2, or between

$$\left.\begin{aligned} \psi_1 &= \frac{1}{z_0} - \frac{1}{z_t} + \log\left(\frac{1}{z_0} - 1\right) - \log\left(\frac{1}{z_t} - 1\right); \\ \psi_2 &= 2\left(\frac{1}{y_0} - \frac{1}{y_t}\right) + \log\left[\frac{\frac{1}{y_0} - 1}{\frac{1}{y_0} + 1}\right] - \log\left[\frac{\frac{1}{y_t} - 1}{\frac{1}{y_t} + 1}\right]; \end{aligned}\right\} (12)$$

Here log = Napierian logarithm, and $y = \sqrt{z}$. Thus:

$$y_0 = \sqrt{z_0}, \quad \text{and } y_t = \sqrt{z_t}.$$

The volumes of the air chambers, in the first four experiments given in the table, were calculated by this method.

It must be observed that an air chamber, placed very close to the gate, and of half the volume required by the formula, will restrict the maximum shock pressure to the permissible pressure, P_s. This follows from what has been said above,* the water in this case entering the air chamber from one side only.

The following table gives the full data of the pressure curves obtained on October 9, 1897, and shows (in columns 4 and 5, and in columns 9 and 10) the extent to which the experiments sustained the formula:

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* See page 404.

TABLE XI.

INFLUENCE OF AIR CHAMBERS ON WATER HAMMER. RECORD OF EXPERIMENTS ON THE 2-INCH PIPE, OCTOBER 9, 1897.

No. of Experiment.	Original Velocity of Flow, v, in Feet per Second.	Duration, t, of a Round Trip of the Wave from the Air Chamber to the Nearest End of the Pipe and Back, in Seconds.	Maximum Shock Pressure, P, at House I, between the Gate and the Air Chamber.		Hydrostatic Pressure, p_0, in the Air Chamber before the Experiment.	Hydrodynamic Pressure, p_1, in the Air Chamber before Water Hammer.	Maximum Shock Pressure, P_a, in the Air Chamber.	Volume, u_0, of Air in Air Chamber in Cubic Inches. u_0		Ratio of Pressures, at House I, in the 2d and 1st Cycles.
	v	t	Observed.	Calculated $P = 4v$.	p_0	p_1	P_a	Observed.	By the Formula.	
1	2	3	4	5	6	7	8	9	10	11
1, 2, 3	4.4	0.50	17.3	17.6	5.4	2.7	14.6	60	55–69	1.3
4	3.7	0.50	14.8	14.8	5.3	2.5	13.4	40	41–66	1.5
5	3.9	0.50	15.7	15.6	5.4	3.1	0.7	548	523	1.1
6	1.8	0.50	7.1	7.2	5.4	4.6	0.7	548	532	1.1

The conclusions of Joukovsky are as follows:

"*Air chambers of adequate size protect that part of the water pipe between the air chamber and the origin. This adequate size is, in general, quite large.* If, for instance, in experiment No. 8, with the 6-inch pipe, Table IX, page 384, we should wish to reduce the water-hammer pressure from 29 atmospheres (as it was in that case) to 1 atmosphere, the formula,

$$u_o = \frac{k \pi d^2}{2} vt \frac{p_1^2}{p_o P_a}$$

(assuming $p_o = p_1 = 5.4$ atmospheres) would give 5.68 cubic feet as the necessary volume.

But the chief practical inconvenience of using air chambers is that *it is difficult to keep the adequate quantity of air in them.* The observations of October 9, 1897, have shown that the volume of air in the chambers, which at the beginning of the experiments was 60 and 40 cubic inches, was, at the end of the experiments, reduced to 50 and 37 cubic inches, respectively.

This circumstance necessitates the use of special machinery for keeping the air chambers supplied with air. *Hence the use of safety valves is preferable.**

* The practical experience with the Moscow waterworks is in accord with the views of Professor Joukovsky. N. P. Simin, chief engineer of these waterworks, has recently written as follows: "It is almost impossible to protect the pipe system from water hammer by the use of air chambers, because the air very quickly disappears from these devices, probably being entrained in the water and perhaps passing through the cast iron of the air chamber. The only places where it is possible to use air chambers successfully are the pumping plants, but here it is necessary to keep the air chambers artificially filled with air."

5. *Efficiency of Safety Valves.*

Experiments with safety valves were made with the 2-inch pipe which had been used for the experiments with the air chambers just described. Spring safety valves were located in about the same places where the air chambers had been set before. About a quarter of a second after the dropping of the weight (which closed the gate) the pressure wave arrived at the safety valve, and, opening it, threw out water from it in shape of a fountain. This lasted about a half second, or until the wave of reduced pressure reached the safety valve, when the valve closed itself. This opening and closing of the safety valve was repeated periodically several times because of the successive reflections of the pressure wave from the gate and from the open safety valve,* until the pressure was reduced to such an extent that it ceased to open the safety valve.

Diagrams of pressure were recorded in house I, near the gate, and in house III, beyond the safety valve. Samples of the diagrams obtained, corresponding to the velocity of flow, v = 3.81 feet, are given in Fig. 25. The first of these diagrams was obtained in house I. Its first rise shows a pressure of 15.3 atmospheres, which well corresponds with the theoretical 4 v = 15.2. The second of these diagrams was obtained in house III. Its first rise shows 3.1 atmospheres above the hydrostatic pressure, which accords with the elasticity of the spring of the safety valve. The table on page 415 gives the results of six observations, made October 9, 1897.

* For "reflected wave," see page 391.

D = 2"; v = 3.81'

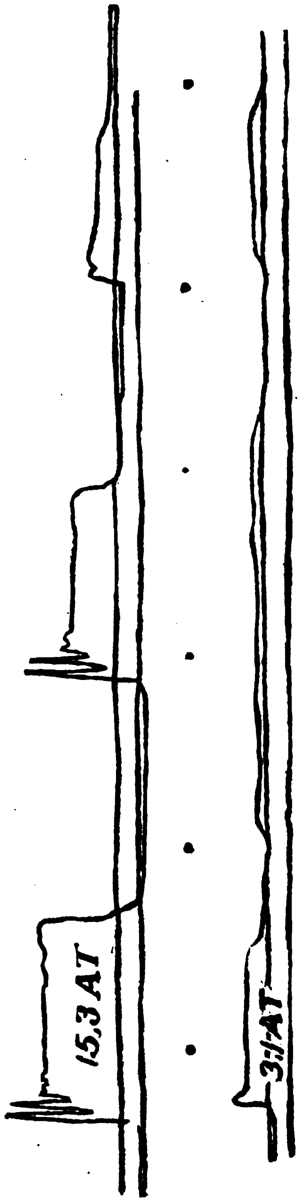

Fig. 25. Indicator Diagrams in the Case of a Safety Valve.

TABLE XII.

INFLUENCE OF SAFETY VALVES ON THE WATER HAMMER. RECORD OF EXPERIMENTS ON THE 2-INCH PIPE, OCTOBER 9, 1897.

No. of Experiment.	Velocity of Water, in Feet per Second, v.	Duration of Round Trip of the Wave from Safety Valve to the Gate, in Seconds.	P from Diagram in House I.	P by the Formula P = 4 v.	P from Diagram in House III.	Ratio between First and Second Rise, House I.
1	2	3	4	5	6	7
1	4.39	0.50	17.3	17.6	3.5	1.4
2	4 39	0.50	17.3	17.6	3.5	1.5
3	3.79	0.50	15.5	15.2	3.1	1.5
4	3.81	0.50	15.5	15.2	3.6	1.5
5	3.81	0.50	15.3	15.2	3.1	1.5
6	2 58	0.49	10.3	10.3	3.5	1.4

Beyond the safety valve (house III, column 6) the shock pressure is practically constant, depending upon and limited by the tension of the spring.

Column 7, which gives the relation of the height of the first to the height of the second rise of the curve obtained in house I, shows a rapid extinction of the vibration of the hammer pressure in the pipe. These experiments show that *the safety valve allows to pass through it only such a hammer pressure as corresponds to the elasticity of the spring of the safety valve,* and thus demonstrates the usefulness of the safety valve.

Mr. N. P. Simin writes as follows:

"About 18 years ago, when the waterworks in the city of Samara were built, I was much concerned with the question of protecting its pipe system from water hammer. These waterworks were designed for direct pressure fire service from hydrants under a pressure of 10 atmospheres. This was the first fire protection water supply in Russia, and many fears were expressed respecting the durability of its pipes. Some altogether refused to believe that such a water supply could work satisfactorily, and I was therefore especially careful in taking measures against water hammer. I decided to use safety valves. The Samara pipe system, which then had a length of 25 miles, was equipped with 16 safety valves of 2 inches diameter. This measure proved to be satisfactory, notwithstanding the frequent use of the fire hydrants of the waterworks in cases of fires and still more often in cases of false fire alarms. During the 16 years of the existence of the Samara waterworks there has not been recorded a single case of injury to the pipes or their joints. This can be explained only by the use of the safety valves."

As a result of the experiments of Joukovsky at the Alexeievskaia pumping plant, an ordinance was passed by the Imperial Water Commission of Moscow, placing safety valves at the beginnings of all service pipes of the waterworks. This ordinance is still in force, a specially devised type of 1¼-inch safety valve being used. The total number of safety valves at the Moscow waterworks is now over 1500.

6. *Water Hammer as a Means for Locating Accumulations of Air in Mains.*

Fig. 26 shows a pressure curve obtained in house I, on the 2-inch pipe. Curves of a similar shape were obtained in a series of experiments performed September 1, 1897. The aim of these experiments was to test the formula

$$P = \frac{v\lambda\gamma}{g}$$

where

P = additional pressure due to water hammer, in pounds per square inch.

v = velocity checked.

λ = speed of progagation of pressure wave.
γ = density of water.
g = acceleration of gravity.

The diagrams obtained (Fig. 26) were at first regarded as failures, owing to the sharp depressions shown at a, b and c; but these depressions attracted the attention of Professor Joukovsky by the regularity with which they appeared at the same point on each of the diagrams; and, upon inspection of the system of pipes upon which

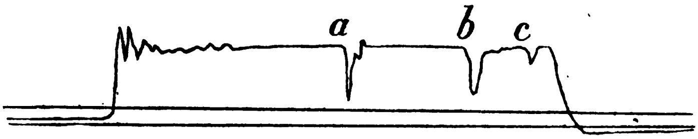

FIG. 26. INDICATOR DIAGRAM IN THE CASE OF AIR POCKETS.

the experiments were performed, it was found that, at distances of 1357 feet, 2066 feet and 2351 feet, respectively, from the gate (where the manometer tubes had been attached to the 2-inch pipe), air pockets had formed in the pipe line.

The distances on the diagram from the first increase of pressure to each of the 3 depressions, a, b and c, Fig. 26, expressed in time units, were found to be in all cases double the distances of the air pockets, respectively, from the point where the diagram was taken. This enables us readily to locate an air pocket by means of the diagram.

The width of the depression, indicated in time units, is roughly proportional to the volume of the air pocket.

When an air pocket is located nearer to the gate than to the origin, a second depression, due to the reflection, by the gate, of the effect of the same air pocket, appears in the pipe at a distance, from the beginning of the diagram, double that of the first depression.

The table on page 418 gives the results of 7 experiments made September 1, 1897:

TABLE XIII.

DETERMINATION OF LOCATION OF AIR POCKETS. RECORD OF EXPERIMENTS ON THE 2-INCH PIPE, SEPTEMBER 1, 1897.

No. of Experiment.	Velocity, in Feet per Second, v	Distances on the Diagram, in Seconds, of the Depressions, from the Point Corresponding to the First Increase of Pressure. A	B	C
1	2	3	4	5
1	4.42	0.64	1.00	1.15
2	4.42	0.65	1.00	1.13
3	4.37	0.64	1.00	1.14
4	4.34	0.65	1.00	1.14
5	3.29	0.64	1.00	1.14
6	3.17	0.63	0.96	1.13
7	3.18	0.65	0.99	1.15
Average.	...	0.64	0.99	1.14
Corresponding distances of air pocket, calculated ($\lambda = 4200$ feet per second) . . Feet		1344	2079	2394
Actual . . . . . Feet		1357	2066	2351
Discrepancy . . Feet		13	13	43

A very slight water hammer is sufficient for this purpose. Hence the use of this method need involve no risk to the mains.

7. *Water Hammer as a Means of Locating Leaks.*

A number of small holes were made in the 2-inch pipe, and provided with means for closing them. With one of these holes opened, the gate was opened and the flow measured. The gate was then suddenly closed, and pressure curves were obtained in the 3 houses (although but one curve from house I really sufficed for location of the hole).

Fig. 27 represents such a curve obtained with a velocity of 4.2 feet per second and with a hole open 949 feet from the gate. The depression at *a* marks the location of the hole.

The diagram gives 0.44 second* as the time interval between the first rise of the curve and the depression, a. Taking $\lambda =$ 4200 feet per second, we have, for the calculated distance of the hole from the gate, $\frac{0.44\,\lambda}{2} = \frac{0.44 \times 4200}{2} = 924$ feet. This is only 25 feet less than the actual distance, and it will be seen that this method furnishes a convenient means for the location of leaks. Here, as in the location of air pockets, a very slight water hammer suffices for the purpose.

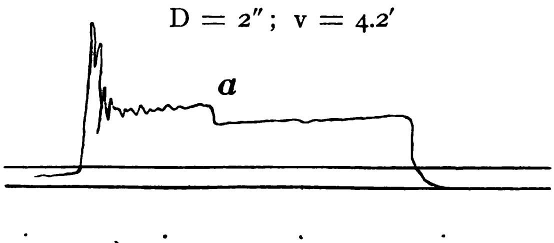

Fig. 27. Indicator Diagram in the Case of a Leak.

Table XIV, page 420, gives the results of the observations made September 25, 1897, upon 2-inch pipe line 2494 feet long.

Joukovsky assumed in his calculations $\lambda = 4200$, but he notes that, according to the duration of the double run of the wave through the length of the pipe, the quantity $\lambda = 4200$, the assumed value, was correct only at the beginning of the water hammer. Afterward (when the air had been disengaged from the water) λ increased to 4333 feet, making the calculated distance 1.03 times greater. Thus, instead of 924 feet, we have 952 feet, or 3 feet greater than the actual distance.

* The points under the curve in Fig. 27 indicate half-second intervals.

TABLE XIV.

DETERMINATION OF LOCATION OF LEAKS. RECORD OF EXPERIMENTS ON THE 2-INCH PIPE, SEPTEMBER 25, 1897.

No. of Experiment.	No. of Hole.	Velocity, in Feet per Second. v	Duration of Round Trip of Wave Through the Whole Pipe, in Seconds.	Diagram I.			Diagram II.			Diagram III.		
				Duration of Round Trip,* in Seconds.	Distance, in *sagens*,† from Gate to Hole.		Duration of Round Trip,* in Seconds.	Distance, in *sagens*,† from Gate to Hole.		Duration of Round Trip,* in Seconds.	Distance, in *sagens*,† from Gate to Hole.	
					Calculated.	Actual.		Calculated.	Actual.		Calcu-lated.	Actual.
1	2	3	4	5	6	7	8	9	10	11	12	13
1	3	3.92	1.18	0.20	60	56.5	. . . .	. . . .	. . . .	. . .	. . .	. . .
2	4 (a)	3.86	1.16	0.25	75	76.9	0.25	75	76.9	. . .	. . .	. . .
3	4 (a)	4.18	1.15	0.26	78	76.9	0.25	75	76.9	. . .	. . .	. . .
4	6	3.61	1.15	0.46	138	135.6	0.44	132	135.6	. . .	. . .	. . .
5	6	3.60	1.15	0.44	132	135.6	0.47	141	135.6	. . .	. . .	. . .
							Distance from House II.					
6	7 (a)	3.87	1.13	0.66	198	193.9	0.25	75	76.2	. . .	. . .	. . .
7	7 (a)	4.42	1.15	0.66	198	193.9	. . . .	. . . .	. . . .	. . .	. . .	. . .
8	8 (a)	3.87	1.14	0.86	258	250.6	0.46	138	133.13	Distance from House III.		
9	8 (a)	4.42	1.14	0.82	246	250.6	. . . .	. . . .	. . . .	0.064	19.2	20.2
10	10	4.13	1.15	1.02	306	315.0	0.62	186	197.9	0.270	8.10	80.9

*From gate to hole and back. †1 sagen = 7 feet.

V. SUMMARY AND CONCLUSIONS.

1. *The shock pressure is transmitted through the pipe with a constant velocity, which seems to be independent of the intensity of the shock. This velocity depends upon the elasticity of the material of the pipes and upon the ratio of the thickness of their walls to their diameter.* Ordinarily, in cast-iron pipes the ratio of thickness to diameter decreases somewhat with increase of the diameter; hence the velocity of the pressure wave is a little less, in pipes of large diameters, than in pipes of smaller diameters. *For pipes of diameters from 2 to 6 inches this velocity is about 4200 feet per second; for 24-inch pipes it is about 3290 feet per second.*

The speed of propagation of the pressure wave remains the same, whether the shock is caused by arresting the flow of a column of water moving in a pipe, or by suddenly changing the pressure in the column of water (flowing or standing) in any part and by any other means.

2. *The shock pressure is transmitted along the pipe with a constant intensity. The shock pressure is proportional to the destroyed velocity of flow and to the speed of propagation of the pressure wave. For ordinary cast-iron pipes, of diameters from 2 to 6 inches, the increase of pressure, for every foot per second of extinguished velocity of flow, is about 4 atmospheres, and, for a 24-inch pipe, about 3 atmospheres.*

3. The phenomenon of periodical vibration of the shock pressure is completely explained by the reflection of the pressure wave from the ends of the pipe, *i. e.*, from the gate and from the origin.

4. If the water column continues flowing, such flow exerts no noticeable influence upon the shock pressure. In a pipe from which water is flowing, the pressure wave is reflected from the open end of the pipe, in the same way as from a reservoir with constant pressure.