UNITED STATES
DEPARTMENT OF THE INTERIOR
BUREAU OF RECLAMATION

HYDRAULIC LABORATORY REPORT NO. HYD-3.2

Technical Memorandum No. 435

THE HYDRAULIC JUMP AND ITS TOP ROLL AND THE DISCHARGE OF SLUICE GATES
(Chapters III and IV)

by Kazimierz Woycicki

Translation from German by
I. B. HOSIG, Associate Engineer

Denver, Colorado
January 29, 1935

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MEMORANDUM TO CHIEF DESIGNING ENGINEER

Translation from German

SUBJECT: THE HYDRAULIC JUMP AND ITS TOP ROLL
AND THE DISCHARGE OF SLUICE GATES
(Chapters III and IV)

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TECHNICAL MEMORANDUM NO. 435

Denver, Colorado

January 23, 1934

(PRICE $3.00)

INTRODUCTORY REMARKS BY THE TRANSLATOR

This paper is on file in the Denver Public Library. It consists of the following principal parts:-

(1) A list of technical papers on the hydraulic jump and related hydraulic phenomena.

(2) Chapter I, Introductory, describing the relation of this paper to previous works on the jump and on sluice gate discharge formulae.

(3) Chapter II. A discussion of the theory of the jump, showing the derivation of the principal formulae connected with it.

(4) Chapter III. A description of the experimental equipment used in the work which the paper presents.

(5) Chapter IV. A description of the experiments made and the results deduced from them.

(6) A summary of the principal conclusions reached.

(7) An appendix of 7 tables giving detail measurements of a number of typical experiments of each class presented in the paper.

(8) A short biography of the author.

Chapters III and IV and the summary only have been translated. Chapter II was not translated principally because the theory is presented in this work much in the same form that it is presented in the "Hydraulics of Open Channels" by Bakmeteff.

The work appears valuable for the following features:-

(1) It follows the theory of conservation of momentum in the jump to its logical conclusion, namely that the momentum at every section of a jump is equal to the initial momentum and that from this relation

the height of the jump at any point can be computed. He presents experimental verification of this theorem. The important feature of this theorem is that the jump is divided into two parts, a main stream and a top roll cover. In the computation of the force-momentum equation the hydrostatic pressure is computed from the total depth of the section, but the momentum is computed from the velocity of the main stream only as the top roll has no downstream velocity effect.

(2) He shows the necessity for a given length of channel in order that a jump may develop and gives a formula for estimating the length of a jump.

(3) He covers the situation where a jump in front of a sluice gate is partially drowned by back water, developing a formula for the relation between the raise in tailwater and the raise in headwater in consequence of this drowning effect. This is the gap between free discharge from an orifice and submerged orifice discharge hitherto side stepped in American hydraulic literature, as far as the translator knows.

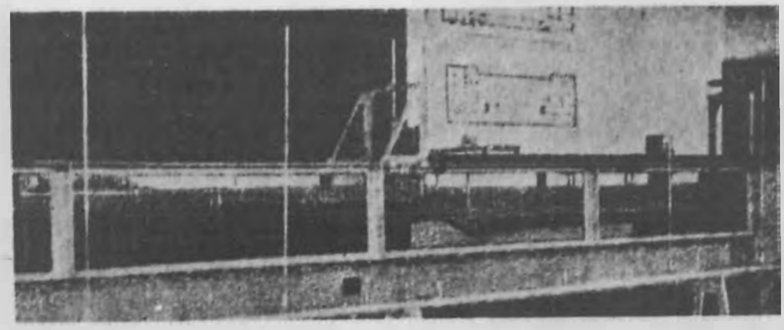

Fig. 10
General View of Flume

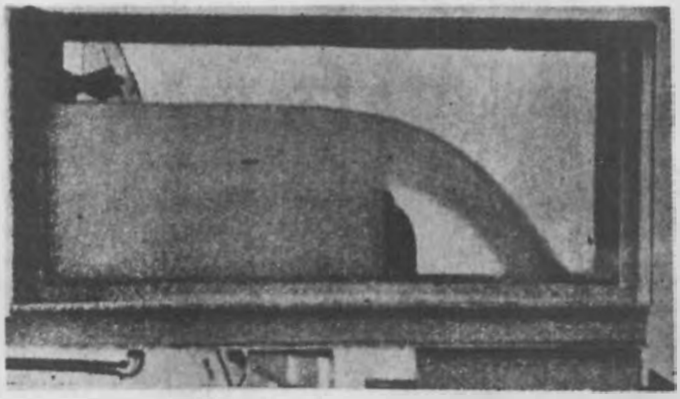

Fig. 11
Side View of Flume

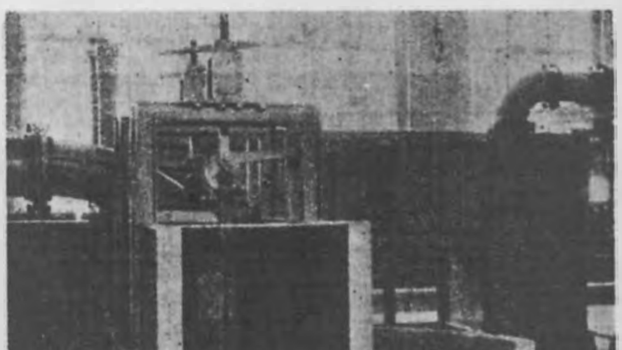

Fig. 12
End View of Flume

CHAPTER III

THE EXPERIMENTAL EQUIPMENT

1. Description of the Plant.

The experiments were conducted in the experimental flume of the new Hydraulic Laboratory of the "Eidgenossichen Technischen Hochschule" (Federal Institute of Technology) of Zurich, Switzerland. The flume is 7.0 meters [22' 11 5/8"] long, .30 meters [11 13/16"] wide, and .44 meters [1' 5 5/16"] high. The sides are made of glass. See Figure 10. The set-up is such that the bottom of the flume may be given any desired slope but the experiments herein described were all made with a horizontal bottom.

Water was admitted to the flume over a weir without side contractions. The weir served also to measure the flow. Its width was 300 mm. [11 13/16"] and it also had glass sides. See Fig. 11. The weir crest was 199 m.m. [7 13/16"] above the bottom of the canal and was aerated by means of two pipes 25 m.m. [63/64"] in diameter.

The head on the weir was measured with a hook gage set up in a transparent walled stilling well. The well connects with the flume through a pipe which enters the bottom 450 m.m. [17 23/32"] upstream from the weir. A cock in the pipe permits the reduction of its cross-sectional area so that surges in the stilling well may be damped.

The weir was carefully calibrated by the use of a tank. A special flume permitted of diverting the water suddenly into and from the tank. The runs were of about 1000 seconds duration. Time was measured with a stop watch. A quiet water surface above the weir

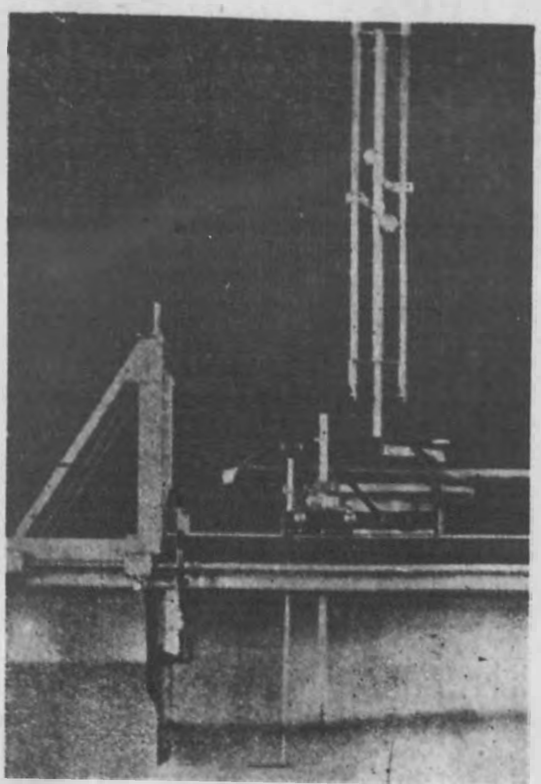

Fig. 13
Section Showing Coordinatograph and Pitot Tube Mounting

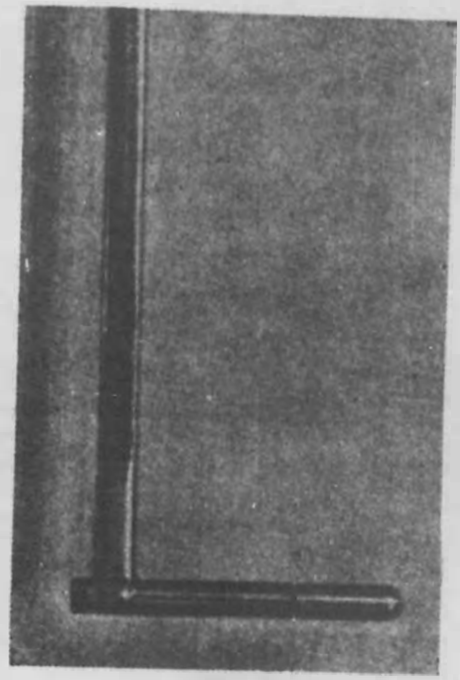

Fig. 14
Prandtl Pitot Tube

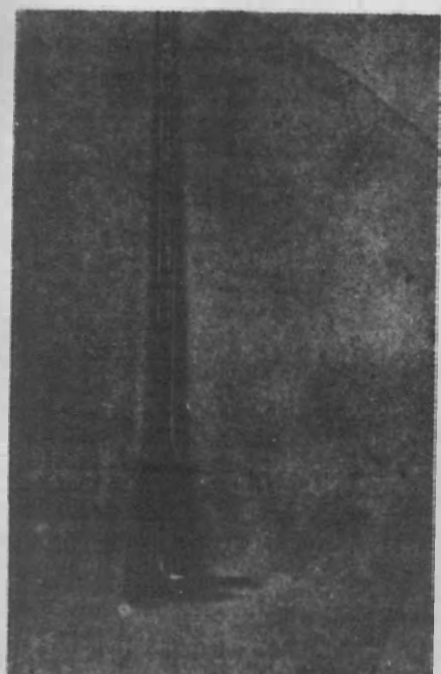

Fig. 15
Berlin Hydraulic and Marine Experiment Station Pitot Tube

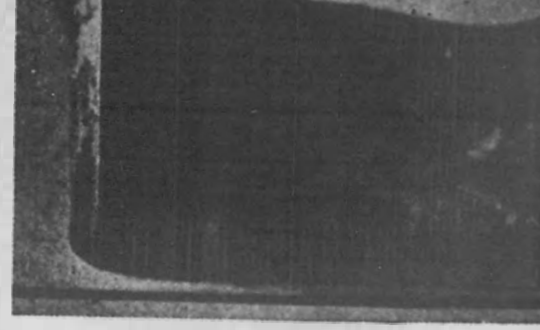

Fig. 38

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was obtained by setting up a series of baffle plates between the weir and the supply pipe.

For the experiments herein described a sluice gate was constructed about in the middle of the flume. The gate leaf was smooth and flat and without guides so that no side contractions were produced. The gate was fastened to the top of the flume and could be set at any desired height above the flume floor. The joints between the gate leaf and sides of the flume were stopped with tape strips which were cut off slightly above the top of the bottom edge of the gate so that they did not interfere with the stream flow. The small cracks below the tape strips were sealed with wax.

Water surface elevation could be measured with a coordinatograph which could cover any point of the flume and which was graduated 0.004"– to 0.1 m.m. Pipes for a pitot tube were also fastened to the coordinatograph carriage. Figures 10-13 show the entire set-up.

Most of the experiments were conducted with the plain flume bottom, the joints of which were made watertight and smooth with red lead. For experiments with a pool, wooden blocks which could be fastened to the flume bottom were used. These were made in sections so that a variable length of level pool bottom could be obtained by inserting flat pieces between the falling and rising pieces which gave the pool effect. See Figure 19.

2. Velocity Measurements by Means of Pitot Tubes.

To determine the velocities of the top rolls a pitot tube was used. It permitted measurements in very shallow water without observable disturbance of the flow.

M. Ricaud of Toulouse conducted some experiments with a pitot tube which gave velocity values too large for the discharge when computed according to the formula $V = \sqrt{2gH}$ (where H is the difference in height of the water columns of the manometer). In consequence a coefficient of less than unity has to be used with the above equation to obtain correct values. This coefficient is designated as **k**. Observations of the velocity in a pipe in which turbulence was produced by a screen gave a value of 0.90 for **k**.

The top roll is comparatively turbulent and the question arises whether serious error will be made if the pitot tube is used. To determine the proper value of the coefficient k in this case numerous experiments were made with various conditions of turbulent flow both rapid and tranquil. First the pitot tube of the form shown in Figure 14 was used. This form was shown to have a coefficient of 1.0 for non-turbulent flow by Prandtl. Following this the same flow was measured with a pitot tube of the form shown in Figure 15. This form was developed at the Berlin Hydraulic and Marine Experiment Station. In these measurements the former type was found to be more satisfactory but both types gave practically the same results.

The coefficient **k** was calculated from an equation, one side of which contained the discharge computed from the weir measurements and the other side of which contained the measured velocities.

TABLE I

Observations of Experiments for the Determination of the Coefficient "k" for Pitot Tubes

N	Nature of Flow	Depth of Stream	Distance from Gate	Velocity q Liters/sec. Actual	By Pitot Tube	Coefficient	Kind of Tube
1	Rapid	3,10	10	15,07	15,30	0,985	Prandtl
2	"	3,18	40	"		0,985	"
3	"	3,61	70	"	15,64	0,970	"
4	"	3,90	120	"	15,60	0,980	"
5	"	2,05	10	"	14,72	1,020	"
6	"	2,33	40	"	16,06	0,940	"
7	"	2,36	70	"	15,90	0,950	"
8	"	3,86	—	"	15,55	0,970	"
9	"	3,25	—	"	15,55	0,970	"
10	Tranquil	19,20	—	15,04	16,23	0,930	"
11	"	14,82	—	"	16,30	0,920	"
12	"	12,68	—	"	16,32	0,925	"
13	Tranquil below a top roll.	18,00	—	9,00	10,00	0,900	"
14	"	19,30	—	15,00	16,50	0,910	"
15	"	14,50	—	12,00	13,35	0,900	"
16	"	18,75	25	15,04	19,10	0,790	"
17	Below a rack	18,85	50	"	16,92	0,890	"
18	"	18,00	130	"	16,60	0,900	"
19	Rapid	3,28	10	"	15,30	0,985	Berlin Exp.Sta.
20	"	2,11	40	"	15,60	0,965	"
21	"	3,84	—	"	15,30	0,985	"
22	"	2,86	—	"	16,33	0,920	"
23	Tranquil	10,43	—	"	16,20	0,930	"
24	Below a rack	10,25	50	"	16,70	0,900	"

Rapid flow was obtained either by giving the flume a steep gradient or by the use of the sluice gate. In the case of tranquil flow, three series of measurements were taken: (1) in a normal flow in the flume; (2) in a normal flow in the flume immediately below a trash rack or screen; and (3) in the flow immediately downstream from a jump with a top roll. These measurements are shown in Table I.

The table also shows that for rapid flow produced by the use of the sluice gate the value of k varies between 0.94 - 1.00. It also shows that in the sections nearer the gate the value is the higher, in other words that the turbulence increases proceeding downstream. The more distant sections show a very nearly constant value of 0.96. In rapid flow produced by tilting the flume, k was found to be 0.97.

For tranquil flow the coefficient is practically constant at 0.92 to 0.93, although in sections immediately downstream from the top roll it drops to .90 - .91.

In the case of the flow through a trash rack or screen the turbulence immediately downstream from the obstruction was marked and the coefficient fell to 0.79. This value rose to 0.90 in sections further downstream from the rack and it was concluded that pitot tube observations cannot be depended on for observations made at points near such obstructions.

In the case of a jump with a top roll flow conditions are varied and the principal may be established that one value of the coefficient for the pitot tube observations will not give absolutely correct results. Because of the increase of turbulence in the direction of flow it is desirable to use a variable coefficient for the computation of velocities but because it was impossible to derive a relation between the value of the coefficient and distance of the observation below the gate, which was constant for the various experiments, this refinement had to be abandoned and all computations were made assuming a value of k = 0.95.

The coefficient actually varies between .90 and 1.00 and the errors resulting from the above assumption may then be ± 5.0% but this is not believed of sufficient moment to appreciably affect the principal conclusions reached.

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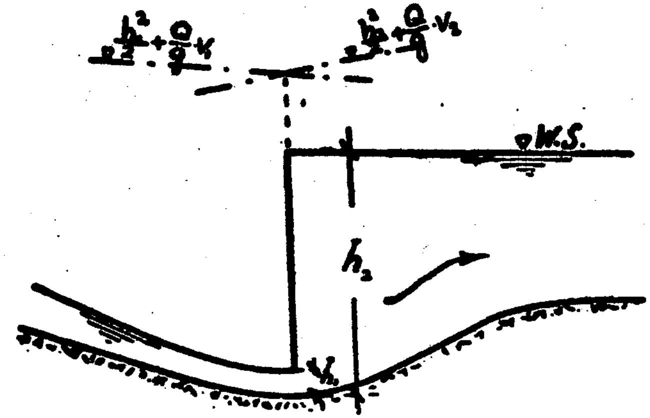


Fig. 16

Tailwater at a sluice gate with various depths of backwater

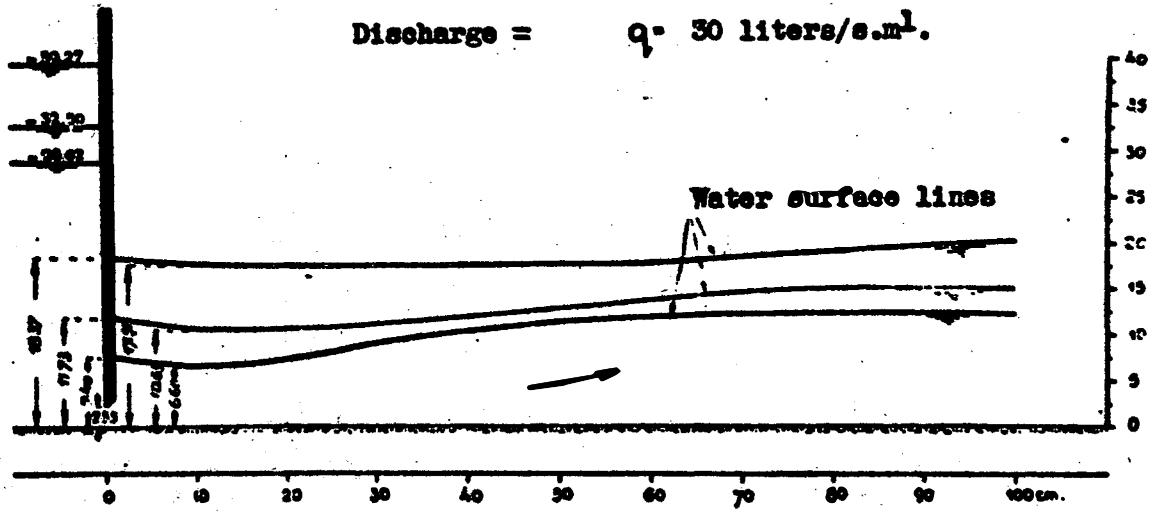
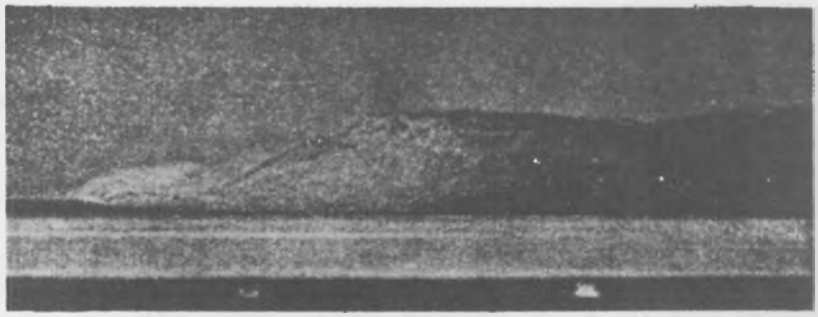


Fig. 39.

4.VI.30



$q = 10$ l./s m.'
$h_1 = 0.51$ cm.
$h_2 = 5.95$ cm.

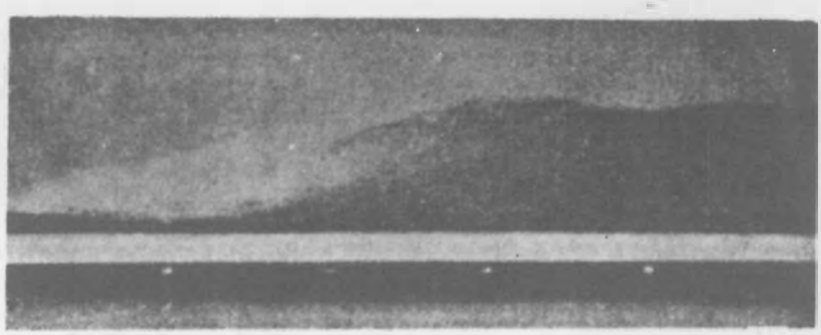
31.V.30

$q = 20$ l./s. m.'
$h_1 = 0.81$ cm.
$h_2 = 9.67$ cm.

5.VI.30

$q = 30$ l./s. m.'
$h_1 = 1.34$ cm.
$h_2 = 10.72$ cm.

2.VI.30



$q = 50$ l./s. m.'
$h_1 = 2.05$ cm.
$h_2 = 14.36$ cm.

Fig. 17
Photographs of Several Experimental Jumps

CHAPTER IV

EXPERIMENTS ON THE HYDRAULIC JUMP, TOP ROLL, AND DISCHARGE OF SLUICE GATES.

Part 1. The Length of the Hydraulic Jump.

The change from rapid to tranquil flow requires a definite length of channel the same as is required in a change from tranquil to rapid flow despite the sudden and violent nature of the phenomena. In many cases a very considerable length is necessary. It is important to observe the facts connected with this phenomena because hitherto, in work connected with the jump, it has been usual to consider it as a vertical rise at the point of intersection of the respective pressure-momentum curves for rapid flow and tranquil flow, plotted from the low and high stage sides of the jump as shown in Figure 16. In such consideration of the problem it is also necessary to determine the elevation of the lowest point in the bottom grade so that for the greatest discharge the depth of the tranquil flow and the rapid flow are in the agreement predicated by equation (6). See page 16

The numerous experiments made in connection with the preparation of this paper as well as those of several earlier investigators show that the jump does not occur in the form of a vertical rise at one point but that it requires a definite distance for its accomplishment. This length depends on the water depth before and after the jump and upon the height of the jump. The photographs of Figure 17 show several of many observed jumps. They show clearly that the length of the jump is several times its height.

TABLE II

Observations of Experiments Showing Relations of Water Depths to Length of Jump

No. of Exp.	Tranquil Flow Depth q l/s.[1] m.[1]	Depth of Stream h_2 cm.	Rapid Flow Depth h_1 cm.	$h_2 - h_1$ cm.	$h_2 : h_1$	Length of Jump l cm.	$\frac{l}{h_2 - h_1}$
1	120,0	21,27	5.12	16,15	4,16	125	7,72
2	,,	16,82	7,57	9,25	2,22	70	7,57
3	100,0	19,90	4,23	15,67	4,70	125	7,97
6	,,	14,52	6,82	7,70	2,13	60	7,79
7	80,0	17,38	3,40	13,98	5,11	110	7,87
10	,,	13,24	5,24	8,00	2,53	65	8,12
11	60,0	14,70	1,98	12,72	7,42	95	7,50
25	,,	9,26	4,10	5,16	2,26	40	7,77
26	40,0	11,39	2,10	9,29	5,42	70	7,53
28	,,	10,71	2,20	8,51	4,86	65	7,63
29	30,0	12,53	1,38	11,15	9,28	85	7,42
32	,,	9,45	2,22	7,23	4,26	55	7,61
33	20,0	11,78	1,20	10,58	9,82	75	7,09
40	,,	7,71	2,29	5,42	3,36	45	8,32
50	25,0	10,76	0,95	9,81	11,32	70	7,15
60	,,	5,60	2,76	2,84	2,03	23	8,10
61	20,0	9,67	0,81	8,86	11,95	62,5	7,07
64	,,	6,98	1,40	5,58	4,98	45,0	8,03
65	10,0	6,17	0,48	5,69	12,86	40,0	7,03
68	,,	4,91	0,74	4,17	6,62	32,5	7,80
69	5,0	4,51	0,23	4,28	19,60	30,0	7,02
71	,,	4,09	0,26	3,83	15,70	27,0	7,05

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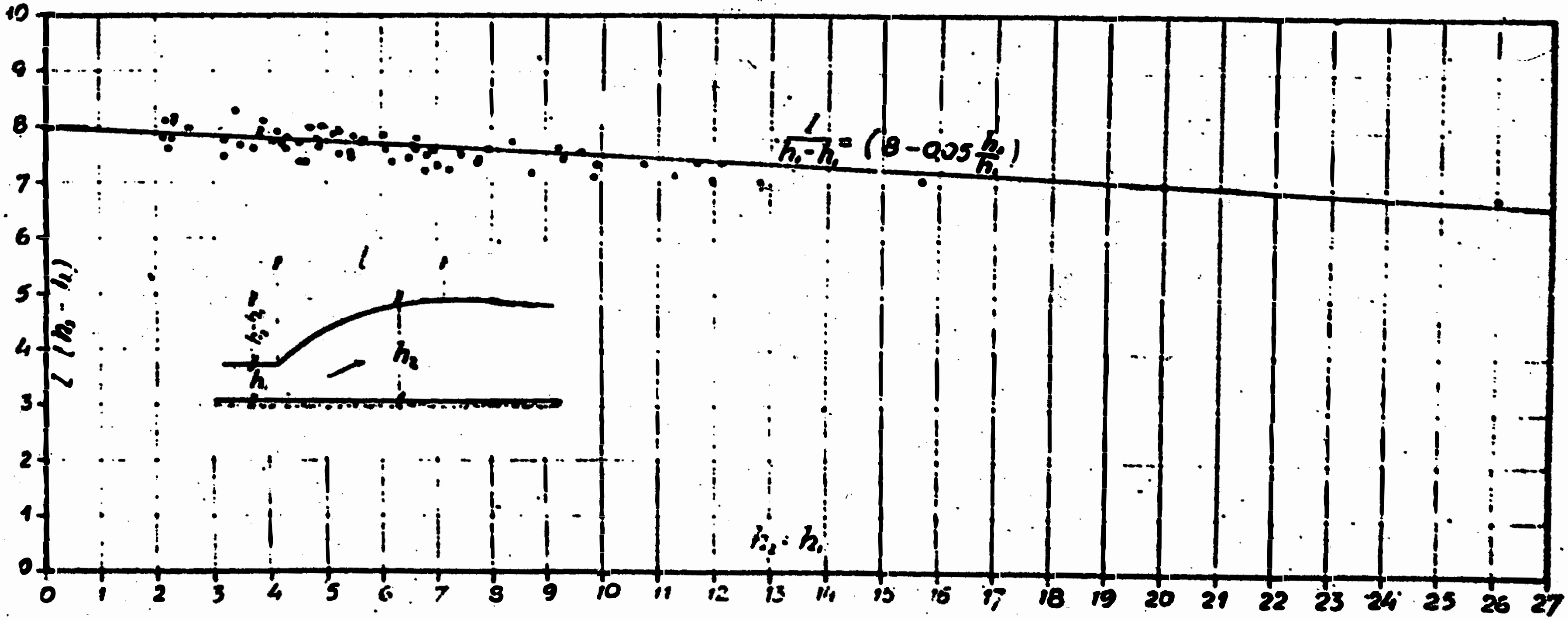
Fig. 18

Relation Between the Length and Height of the Hydraulic Jump



To demonstrate definite relations between the length and height of the jump a series of experiments was made varying the quantity and the water depths above and below the jump. In these experiments the water surface curve of the entire jump was measured with a pointed gage. Because of the fluctuations in the surface the maximum and minimum elevations at each point were determined. Results were plotted to a distorted scale to obtain close estimates of the lengths. Table II shows several series of measurements. The table also shows the ratios (1) of water depths below and above the jump $\frac{h_2}{h_1}$ and (2) of length of jump divided by height of jump $\frac{\ell}{h_2 - h_1}$. Plotting these ratios as abscissae and ordinates respectively, points are established for each jump as shown in Figure 18. An average line through these points is straight and has the following equation:

$$\ell = (8 - 0.05 \frac{h_2}{h_1}) \quad (h_2 - h_1) \text{ - - - - - - - - - - } \quad (10)$$

For known depths of flow before and after the jump this equation gives the length of the jump. If a jump is to occur at a given location, say in the pool below a drop, for instance; it is necessary that the horizontal distance between falling and rising bottom grades have a definite minimum length.

An arrangement such as is shown in Figure 16 will not produce a jump and the rapid flow stage will persist throughout its length. The jump will occur further downstream at the point where friction will have reduced the low stage velocity and increased the water depth so that a balance of the pressure-momentum quantities of rapid and tranquil flows exists. Such conditions may cause scour of the channel bottom and endanger structure foundations.

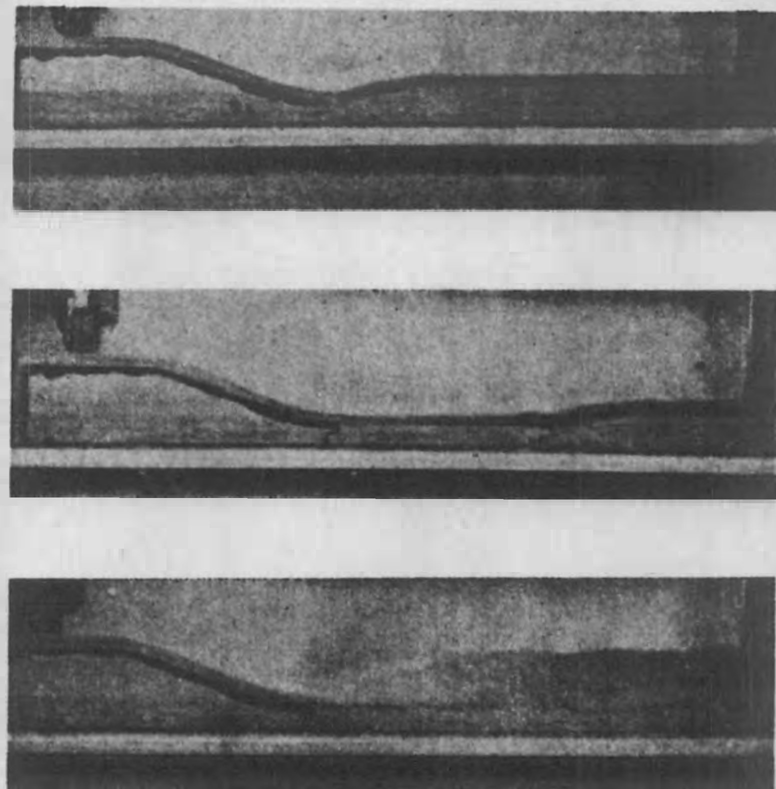
To demonstrate that the production of a jump requires a definite length a number of experiments were made with a model that had a dropped bottom grade; i.e., a pool. The model was so constructed that the pool could be lengthened as desired by inserting horizontal pieces between the falling and rising bottom grades.

Table III herewith

TABLE III

Observations of Jumps in Pools

N	liters/ sec./ m	Observed		Length of Horiz. Pieces Inserted in Pool Bot. cm.	Theor. h_2 cm.	$\frac{h_2}{h_1}$	$(h_2 - h_1)$	Length According to Equation (10) cm.
		h_1 cm.	h_2 cm.					
1	20	1.02	8.42	60	8.40	8.25	7.40	56
2	"	1.33	7.18	60	7.20	5.40	5.85	45
3	30	1.37	10.85	80	10.90	7.92	9.48	72
4	"	1.50	10.20	60	10.35	6.80	8.70	66
5	"	1.57	9.95	60	10.00	6.33	8.38	64
6	"	2.02	8.40	60	8.55	4.16	6.53	51
7	50	2.50	12.85	80	13.00	5.14	10.35	80
8	"	3.32	11.00	80	11.00	3.42	7.78	61
9	80	4.19	15.25	100	15.60	3.64	11.06	86
10	"	4.61	14.49	80	14.60	2.93	9.88	77
11	100	5.42	16.90	100	16.88	3.12	11.48	90
12	120	6.30	18.60	100	18.65	2.95	12.30	96

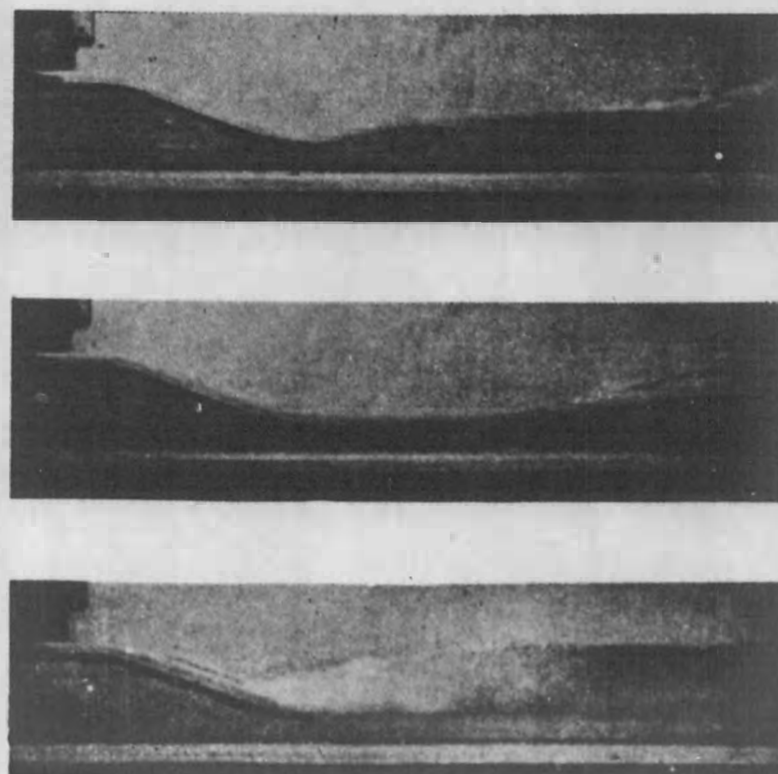


a — 21.VI.30

q = 30 l. s m.

h = 1.50 cm.

h_2 = 10.20 cm



b — 21.VI.30.

q = 30 l. s. m.

h_1 = 1.37 cm

h_2 = 10.85 cm

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Fig. 19
Effect of Various Lengths of Pool Bottom in the Formation of a Jump

Effect of increasing the pool length upon the formation of a jump.

Discharge = q=30 l/sm

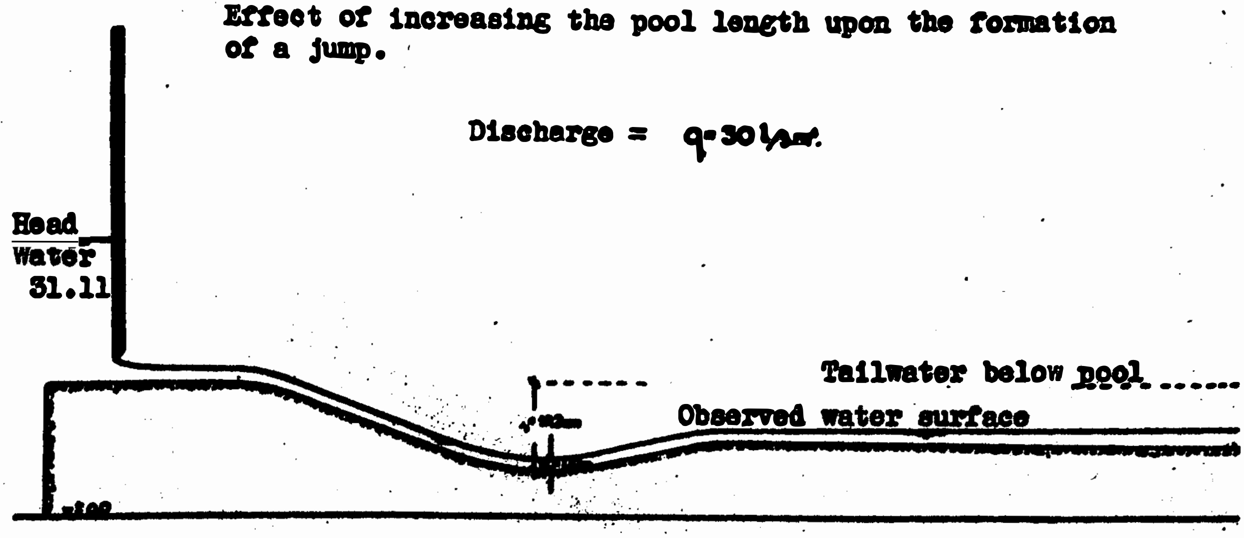

Fig. 20

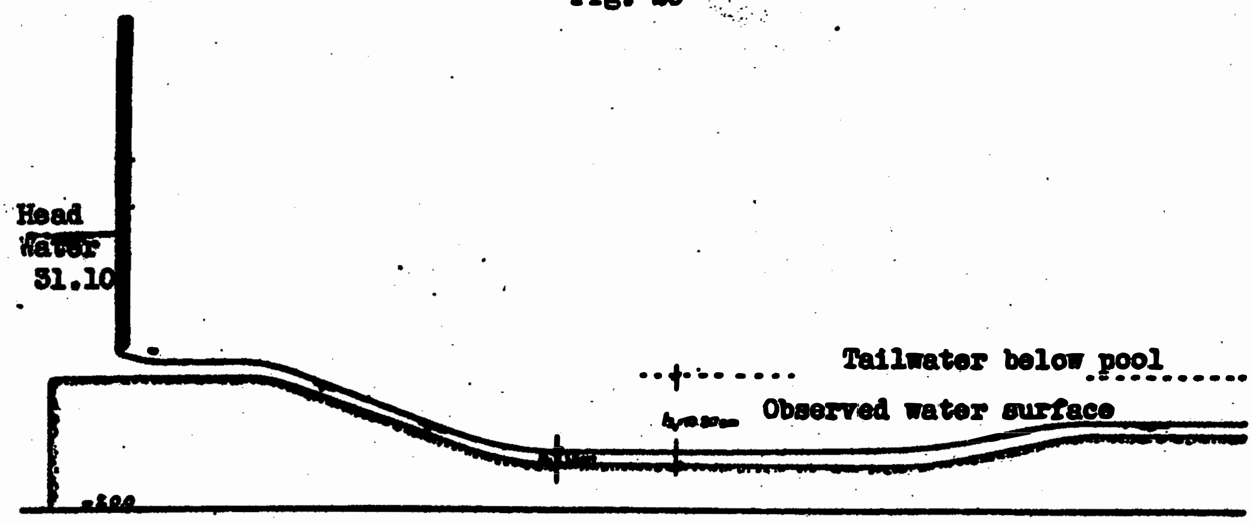

Fig. 21

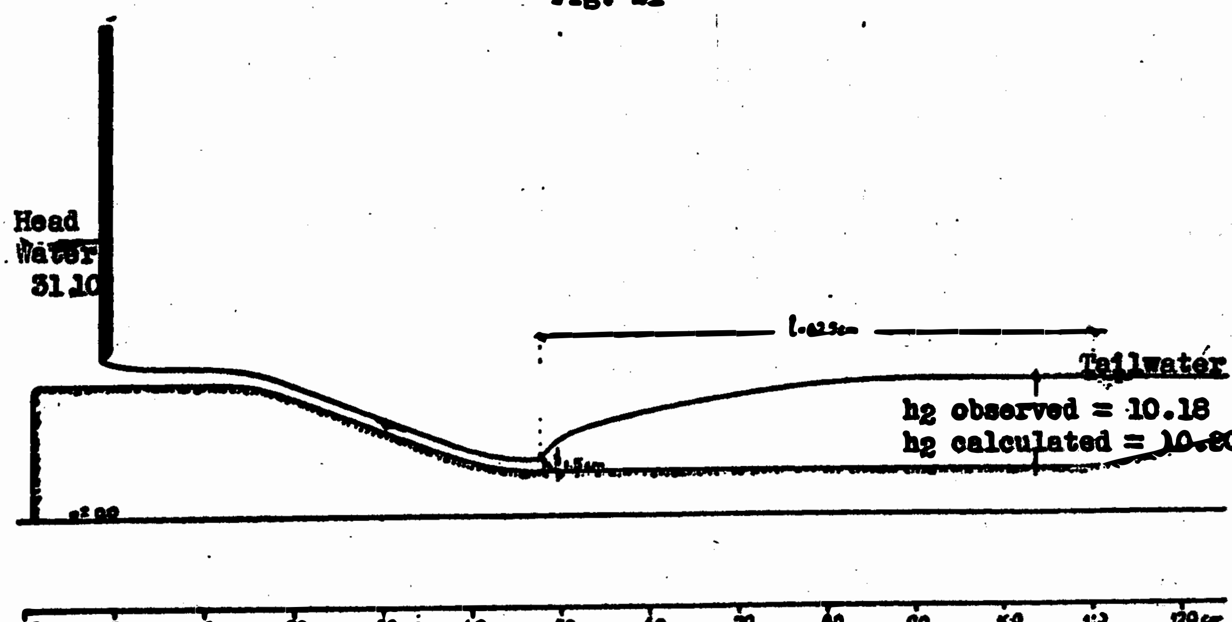

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Fig. 22

gives the results of 12 experiments proving that the length of the horizontal insertion must be that given by formula 10.

In these experiments the depth of the rapid flow stream was first measured. Then back water pressure was produced by means of a sluice gate at the lower end of the trough to the amount that calculations according to equation (6) indicated was necessary to produce a jump. If a sufficient length of horizontal insertion was in place in the pool bottom a jump did occur, but otherwise not. In experiments where the back water was first built up to the point where a top roll would occur under ordinary circumstances, and then was lowered slightly (but still not below the normal high stage), the top roll and jump would disappear in a definite length of time. The rapid flow water depth also increased and a jump was formed downstream from the pool on the higher bottom grade.

The photographs of Figure 19 show the water surface in the pool for various lengths of insertion but with constant discharge and water stages above and below. In the first case where the drop and rise of bottom grade were made without the insertion of a stretch of level pool bottom, as well as in the second case where the insertion was too short, no jump was observable, although conditions for a jump were propitious according to equation (6). In the third case the insertion was of sufficient length and a jump was formed. The water surfaces in the experiments shown in photographs, Figure 19a, as determined by hook gage measurements are shown to scale in Figures 20-22.

Value of hydrostatic pressure and momentum and their sum in terms of water depth for a discharge of 1 cu.m. per second per meter width of channel

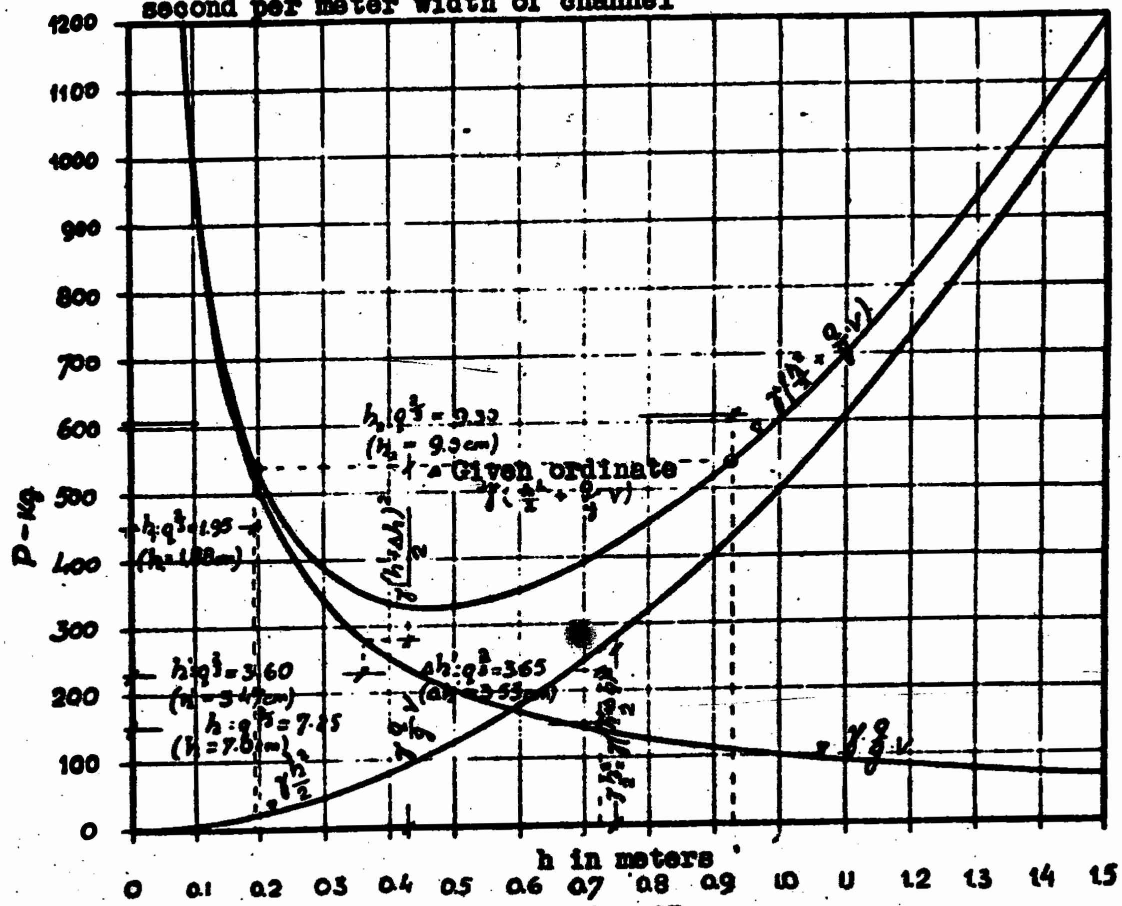

Fig. 23

In fixing the dimensions of a pool there are two alternatives. (1) The jump can be permitted to occur in the conventional manner; in this case proper tranquil flow water depth and sufficient length of level channel are necessary. (2) A back pressure top roll effect can be produced by lowering the pool. The first case will require works of greater volume, the other, extra foundation work. The proper solution will be determined by other considerations such as desirability or cost.

The length of the pool can be fixed by the use of equation (10). In the event that this length is considered impractical even after considering the greatest possible depth it may be necessary to resort to flow obstructions of some kind, such as the use of stone blocks or the dentated sill developed by Rehbock. Such a solution of a drop problem should be developed by experiments with models.

Part 2. Investigation of the Top Roll.

A. The Free Top Roll.

In consequence of the investigation of the hydraulic jump, especially from the viewpoint of the theory that at every cross-section of a stream the quantity, static pressure + $\frac{Qv}{g}$, is a constant, the conclusion is reached that the equation expressing this theory can be usefully applied to sections in a jump. To gain a better understanding of those relations the following explanation should be made concrete by solving the formulas for definite quantities. In figure 23 are given as curves the values (1) static pressure, (2) $\frac{Qv}{g}$, and

Form of the Top Roll of the Hydraulic Jump

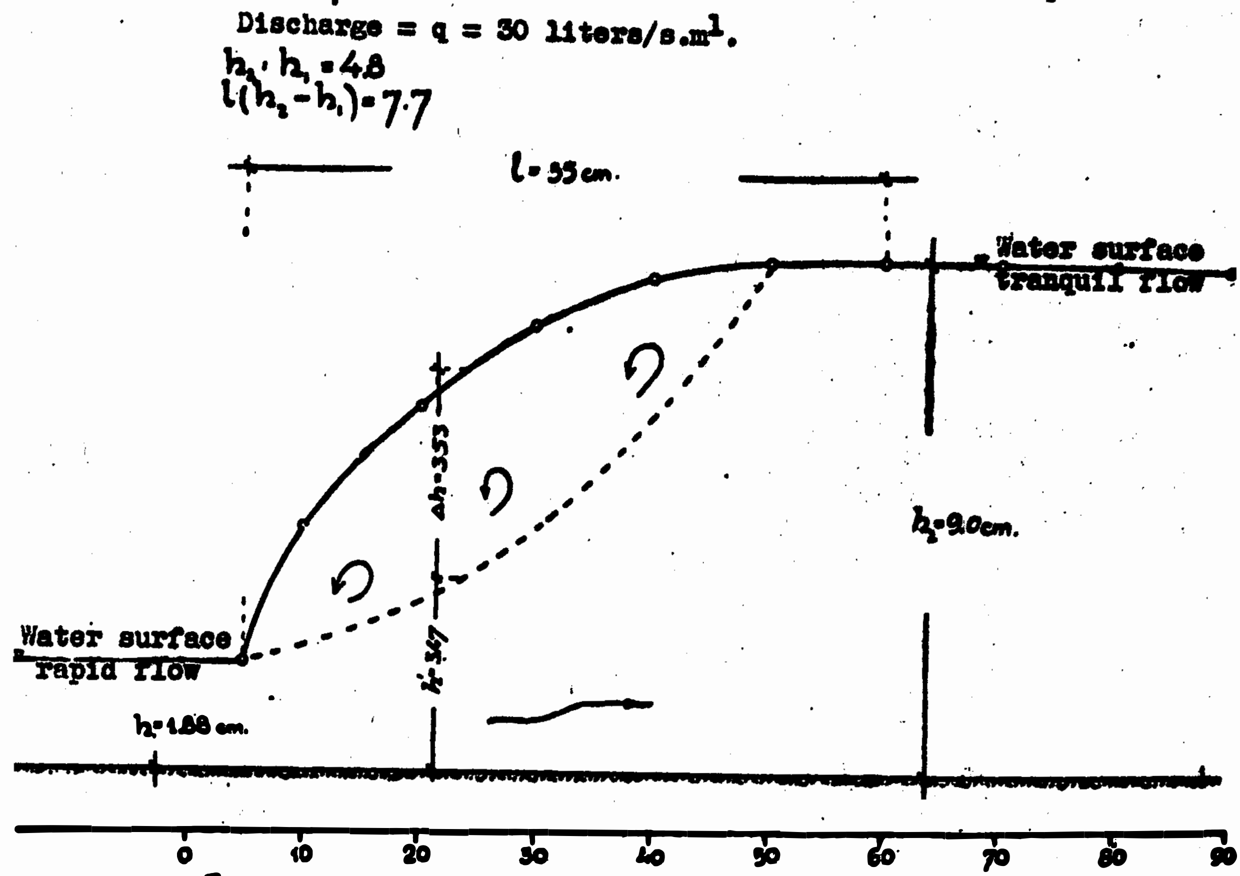

Fig. 24

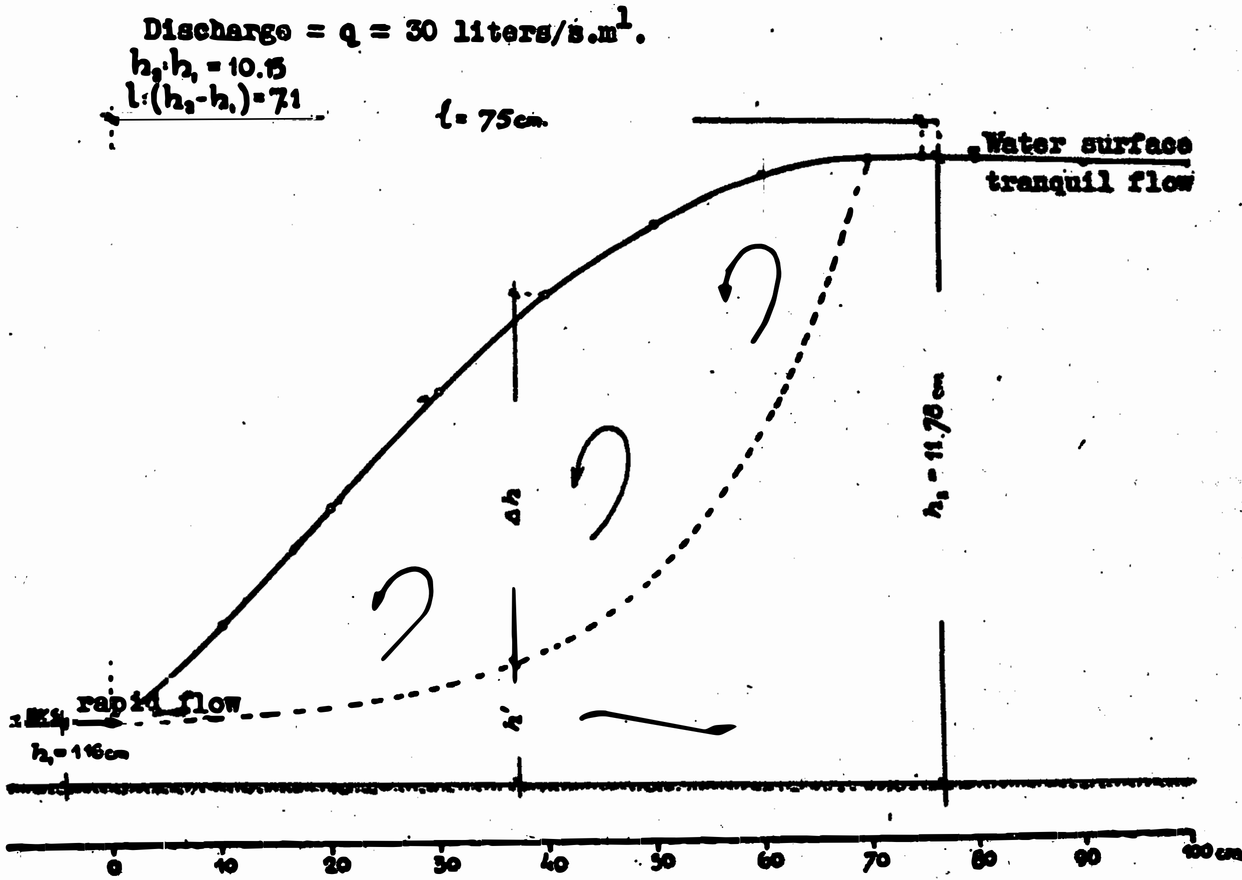

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Fig. 25

(3) their sum for various values of water depth h, for the quantity of 1 cubic meter per second over a width of 1 meter, or as usually expressed $1M^3/S.M.^1$. This sum is the so-called pressure-momentum curve (the literal translation of the German name of this curve, i.e., "Stutzkraftkurve", is "push-strength curve"). Figure 24 is the profile of a jump to which the quantities of Figure 23 can be applied by proper transformation. The expression of the pressure-momentum relation of a jump in an equation is as follows, where h_1 and h_2 are the water depths before and after the jump and γ is weight of liquid.

$$\gamma \cdot \frac{h_1^2}{2} + \gamma \cdot \frac{q}{g} \cdot v_1 = \gamma \cdot \frac{h_2^2}{2} + \gamma \cdot \frac{q}{g} \cdot v_2 \quad ---- \quad (4)$$

Consider a section downstream from h_1 with a main stream water depth h^1 which is slightly less than the critical depth. If the value of the pressure-momentum quantities for this point are taken from diagram Figure 23 obviously they will be less than at the value for h_1 and the equality of the pressure-momentum relations will be upset. If, however, the momentum part of the equation be picked from the momentum curve of Figure 23 for the height h^1 chosen and the hydrostatic pressure part be picked from the hydrostatic pressure curve so that the sum of the two is equal to the pressure-momentum quantity at h_1 it will be found that the depth at which this quantity occurs on the pressure curve is the depth that experiment indicates is the total water depth at section h^1. In other words to preserve the pressure-momentum relation the momentum value for a particular point is computed from the velocity of the main stream but the hydrostatic pressure is computed from the depth of the main stream + top roll. This presents a picture of continuity in the jump where hitherto the notion of discontinuity at the critical depth prevailed.

To make the demonstration concrete consider the jump shown in Figure 24.

h_1 = 1.88 cm.

h_2 = 9.00 cm.

q = 30 liters per second per meter width of channel.

The height of the energy gradient for depth h_1 equals 1.88 cm. plus the velocity head for a velocity $v = \frac{30}{1,000 \times .0188} = \frac{30}{18.8} = 1.60$ meters/second or 1.88 + 12.99 = 14.87 cm.; the critical depth then is 2/3 x 14.87 or 9.91 cm. and the depth 3.47 cm. is chosen. To use the chart, Figure 23, the depth must be converted to correspond to "h" on the chart. The corresponding h of the chart for the depth h_1 = 1.88 cm. is $\frac{.0188}{.03^{2/3}} = \frac{.0188}{.0965}$ = 0.195 m. and the pressure-momentum value is 540 Kg. The corresponding h of the chart for 3.47 cm. is .360 m. At this value the momentum is 280 Kg. To satisfy the pressure-momentum quantity of 540 Kg., 260 Kg. of pressure must be furnished by water depth. This quantity is found on the pressure curve at h = .725 m. The corresponding height of the quantity under discussion is .725 x .0965 = .0700 m. or 7.00 cm. This is the total water depth at the point chosen and the thickness of the top roll is 7.00 - 3.47 = 3.53 cm. It will be noticed that the momentum of the top roll is taken as zero. This is justifiable because the net longitudinal velocity of the top roll is zero, as shown in figures 31 and 32.

The top boundary of the main stream may be worked out by repeating this construction. See Figures 24 and 25.

The top roll is thus seen to be a necessary part of a jump by virtue of requirement of continuity in the pressure-momentum relation.

Consideration of the energy curve in Figure 2 leads to the same conclusion. If the curve is followed to the right from the abscissa corresponding to depth h_1 the critical depth or point of minimum energy is reached which of course would indicate a less amount of energy than that existing at h_2, the lower end of the jump. Since an accession of energy from outward source is precluded the energy must be in the jump and it is in fact in the top roll.

The water in the top roll obviously cannot remain at rest. It is carried forward along the contact line between it and the main stream by the dragging effect of the main stream. It is carried backward along the top by gravity. The total result is a swirling motion. There is considerable loss of energy in this mass of swirling water. It is not possible to tell how the total energy lost in the jump is divided between the top roll losses and internal friction losses in the main stream. It seems probable, however, that the latter is much the larger loss.

The just described form of top roll may be called a free top roll (the name adopted in this translation is "free jump") to destinguish it from other forms to be later described.

Experiments by J. Einwachter on jumps of small energy loss,

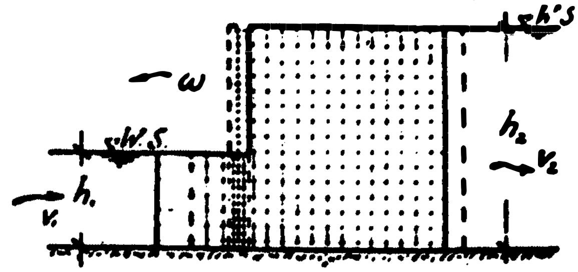

Fig. 26

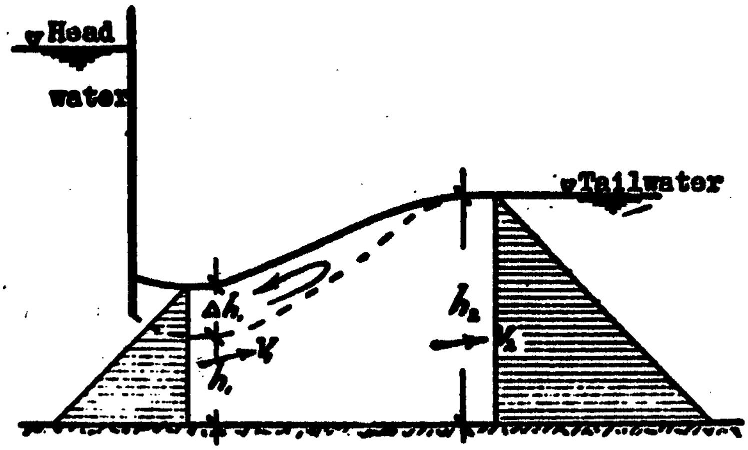

Fig. 27

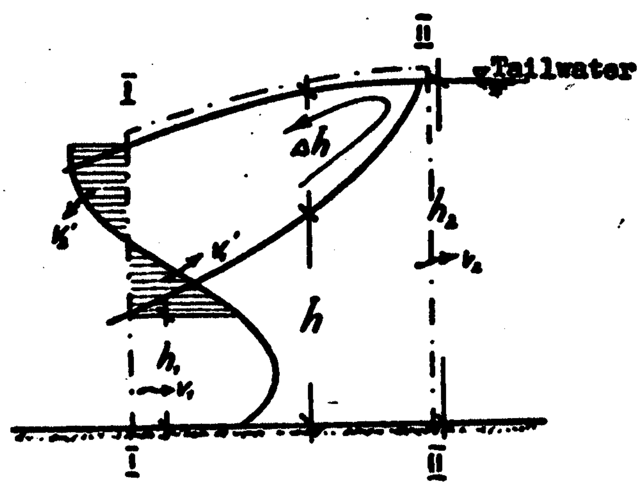

Fig. 28

and hence small jump, show that under such circumstances a small ground roll takes the place of the top roll. This appears to be the reason for the small energy loss.

B. The Moving Top Roll or Bore.

As previously stated, there are two conditions necessary for the formation of a jump: (1) the existence of the conjugate water depths implied by rapid and tranquil flow, (2) sufficient length of bottom grade at the proper elevation to support tranquil flow. The cases in which these conditions do not exist fall in two classes (1) Class 1 where $h_2 < -\frac{h_1}{2} + \sqrt{\frac{h_1^2}{4} + \frac{2v_1^2 h_1}{g}}$ in which case the change from rapid flow to tranquil flow cannot occur. (2) Class 2 where $h_2 > -\frac{h_1}{2} + \sqrt{\frac{h_1^2}{4} + \frac{2v_1^2 h_1}{g}}$. In this case the transformation of hydrostatic pressure means more than a change in velocity. Additionally the jump begins to travel upstream with such a velocity of translation that the pressure-momentum equation is satisfied. To investigate the case the equation must be set up in its general form. Following the nomenclature of Figure 26.

$$\frac{h_1^2}{2} - \frac{h_2^2}{2} = \frac{v_2^2 h_2}{g} - \frac{v_1^2 h_1}{g} + \frac{w(h_2 - h_1)v_2}{g} + \frac{w(v_2 - v_1)}{g}$$

(w = velocity of translation of the jump or bore)

reducing the above

$$\frac{h_1^2}{2} - \frac{h_2^2}{2} = \frac{v_2 h_2}{g}(v_2 - w) - \frac{v_1^2 h_1}{g} - \frac{h_1 v_1 w}{g} \quad \text{- - - - - - - - - - (11)}$$

but $v_1 h_1 = w(h_2 - h_1) + v_2 h_2$ - - - - - - - - - - - - - - - - - - - (12)

because Q above the jump must equal Q below

or $$h_2 = \frac{(v_1 + w)h_1}{w + v_2}$$

substituting the last expression in (11)

$$\frac{h_1^2}{2} - \frac{h_2^2}{2} = \frac{(v_1 + w)h_1}{g} \times (v_2 - v_1) \text{ -------- } (13)$$

Equation (12) may also be written

$$v_1h_1 - v_2h_1 = w(h_2 - h_1) + v_2(h_2 - h_1) \quad \text{or}$$

$$h_1(v_1 - v_2) = (h_2 - h_1)(w + v_2)$$

substituting this in equation (13)

$$\tfrac{1}{2}(h_1 + h_2)(h_1 - h_2) = \frac{v_1 + w}{g}(h_1 - h_2)(w - v_2) \quad \text{or}$$

$$w^2 + w(v_1 + v_2) + v_1v_2 - \frac{(h_1 + h_2)g}{2} = 0 \quad \text{or}$$

$$w = -\frac{v_1 + v_2}{2} + \sqrt{\frac{g(h_1 + h_2)}{2} + \frac{(v_1 - v_2)^2}{4}} \text{ ---- } (14)$$

This is the equation shown by Ph. Forchheimer (Wassorschwall und Wasserfunk Leipzig und Wien 1924) for the velocity of the bore. It is also the equation of the translation of a jump. If w is made equal to 0, then

$$0 = \frac{v_1 + v_2}{2} + \sqrt{\frac{g(h_1 + h_2)}{2} + \frac{(v_1 - v_2)^2}{4}}$$

$$v_1^2 + 2v_1v_2 + v_2^2 = 2g(h_1 + h_2) + v_1^2 - 2v_1v_2 + v_2^2 \quad \text{or}$$

$\frac{2v_1v_2}{g} = h_1 + h_2$, but since in this case $v_2 = \frac{v_1h_1}{h_2}$

$$h_2 + h_1 = \frac{2v_1}{g} \times \frac{v_1h_1}{h_2} = \frac{2v_1^2h_1}{gh_2} \quad \text{and}$$

$h_2^2 + h_1h_2 = \frac{2v_1^2h_1}{g}$ which is equation ------------ (5)

This equation was previously deduced to show the relation between rapid flow and tranquil flow in the ordinary jump. It is a modification of the ordinary jump formula of Bresse, namely

$$h_2 = -\frac{h_1}{2} + \sqrt{\frac{h_1^2}{4} + \frac{2v_1^2 h_1}{g}} \qquad (6)$$

When the bore or moving jump reaches the sluice gate or weir no further translatory motion is possible and a new phenomena appears. The water over the top of the main stream because of the viscosity and turbulence along the border zone between it and the main stream receives energy which keeps it in constant motion. This motion is in the form of a big swirl with a horizontal axis and it is here called a back pressure top roll to distinguish it from the ordinary top roll. Under this top roll the main stream changes from rapid to tranquil flow passing through critical depth and of course it looses energy. A part of the energy is lost by internal friction in the jump and the remainder as friction in the contact zone between the main stream and the top roll. These losses are transformations of kinetic energy to heat energy. The water in the top roll despite its swirling motion is constantly and rapidly changing as was demonstrated by the use of dyes.

C. The Back Pressure Top Roll.

The water surface of the top roll must satisfy the conditions enumerated in the following explanation.

The existence of a back pressure top roll is conditioned upon the existence of a tailwater depth in excess of that which a free jump can sustain for the same conditions of quantity and gate opening or h_2, the tailwater, must be greater than h_2 in the formula

$$h_2 = -\frac{h_1}{2} + \sqrt{\frac{h_1^2}{4} + \frac{2v_1^2 h_1}{g}} \quad \text{- - - - - - - -} \quad (6)$$

The pressure-momentum equation is applicable to all cases of the hydraulic jump. In the case of the moving jump or bore just discussed, the variable relations between the hydrostatic pressure and the momentum of the moving waves were satisfied by the translation of the jump. In the case of the back pressure top roll they are satisfied by an increase in water depths. As shown in Figure 27 the pressure-momentum equation must be as follows:-

$$\frac{\gamma(h_1 + \Delta h_1)^2}{2} + \gamma \frac{q}{g} v_1 = \gamma \frac{h_2^2}{2} + \gamma \frac{q}{g} v_2 \quad \text{- - - -} \quad (15)$$

(In this paper the discussion is confined to rectangular channels and the relations are written for a longitudinal section one unit wide thus eliminating b, the width, from the equations)

In the back pressure top roll the thickness of the top roll increases as the tail water depth h_2 increases. The water surface becomes more and more quiet the greater the increase in thickness of the top roll. The water currents in the jump show regular flow and even though the water in the top roll is constantly being renewed, the direction and velocities of the stream lines in the various sections of the jump remain tolerably constant. For this reason an exact study of the pressure-momentum relations requires consideration also of the momentum of the top roll (this is in contra-distinction to the consideration of the ordinary free jump in which the momentum of the top roll is neglected). For a reach between two sections such as shown in Figure 28 the integration of the pressure and momentum gives the following values to equation (15)

$$Y \frac{(h_1 + \Delta h_1)^2}{2} + Y \frac{q}{g} v_1 + Y \frac{q'}{g} (v_1^1 - v_2^1) = Y \frac{h_2^2}{2} + Y \frac{q}{g} v_2 \quad --- (16)$$

In this equation q', v_1^1, and v_2^1 are the quantity and mean velocities of the top [redacted] roll.

The solution of this equation for Δh_1 for any chosen section and depth of the main stream will give the raise in the water surface

$$\Delta h_1 = -h_1 + \sqrt{h_2^2 - \frac{2q}{g}(v_1 - v_2) - \frac{2q'}{g}(v_1^1 - v_2^1)} \quad -- (17)$$

In many places, for instance in the vicinity of the gate, and also at the points of greater thickness of the top roll, the value of the momentum in the roll is of small value and the expression $\frac{2q'}{g}(v_1^1 - v_2^1)$ may be omitted from the above formula. Two swirls with vertical axes and located right next to the gate are always observable and here the component of the momentum in the direction of the main stream flow is zero.

To test equation (17) experiments were conducted with various conditions for the flow thru the gate, and with various tail water depths. Velocities were measured at various sections of the top roll with a pitot tube. From these observations it was always possible to determine the depth of the main stream and also the velocities and discharge of the top roll. The water surface at each cross-section

TABLE IV

Observations of Experiments Showing Comparison of Observed and Calculated Depths of Back Pressure Top Rolls

No. of Exp.	Observed				Calculated	Differences (1)—(2)	
	q l.s.^{1} m.^{1}	Distance from Gate cm.	h_2 cm	(1) $h_1 + \Delta h$	(2) $h_1 + \Delta h$	cm.	%
1	120	10,0	25,79	20,86	21,38	— 0,52	— 2,43
2	•	30,0	25,79	21,29	21,80	— 0,61	— 2,80
3	•	50,0	25,79	22,45	23,43	— 0,68	— 2,94
4	1.0	20,0	27,22	23,40	23,29	+ 0,11	+ 0,47
5	•	40,0	27,22	23,83	23,89	— 0,06	— 0,03
6	•	30,0	22,56	17,69	17,50	+ 0,19	+ 1,08
7	80	30,0	23,90	18,33	17,69	+ 0,64	+ 3,62
8	•	40,0	23,90	17,35	17,61	— 0,26	- 1,48
10	60	25,0	19,30	13,90	14,50	— 0,60	— 4,13
11	•	40,0	19,46	14,60	15,70	— 1,10	— 7,01
13	40	10,0	19,46	15,50	15,40	+ 0,10	+ 0,65
18	•	10,0	14,49	8,00	8,07	— 0,07	— 0,89
22	36	25,0	17,00	13,50	13,80	— 0,30	— 2,17
29	•	25,0	13,80	8,80	9,10	— 0,30	— 3,29
32	30	40,0	18,00	14,70	15,00	— 0,30	— 2,00
33	•	20,0	14,40	9,50	9,24	+ 0,26	+ 2,81
34	25	20,0	17,40	14,70	14,40	+ 0,30	+ 2,08
30	•	80,0	17,40	17,30	16,95	+ 0,45	+ 2,66
40	•	15,0	12,80	8,70	8,20	+ 0,50	+ 6,10
46	20	25,0	16,80	14,70	14,80	— 0,10	— 0,67
47	10	25,0	14,80	13,70	14,30	— 0,90	— 4,19
49	•	15,0	8,30	6,40	6,40	± 0,00	+ 0,00

Distribution of velocities in the longitudinal section of a jump.

Discharge = q = 40 liters/s.ml.

0.5 0 1.0 2.0 m/s
Scale of velocities

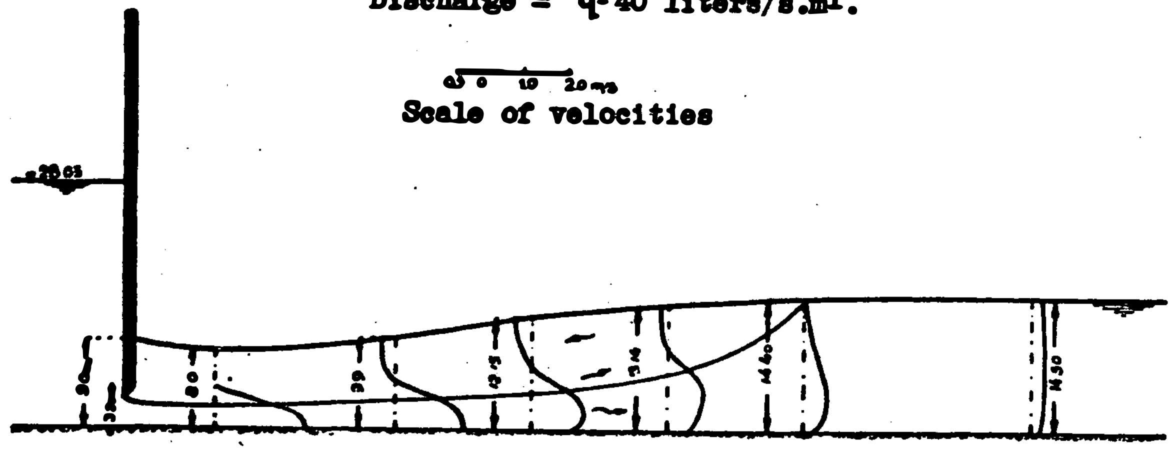

Fig. 29

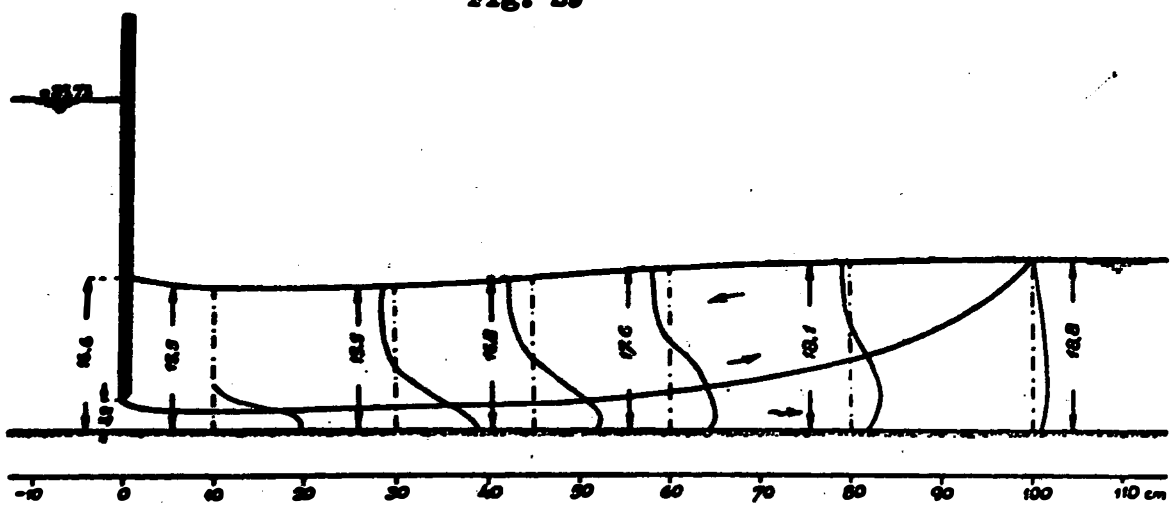

Fig. 30

Velocities in the Top Roll

Discharge = $q = 36\ l/s\,m'$
Distance below sluice gate = 25 cm.
Tailwater depth $h_s = 17.0$ cm.

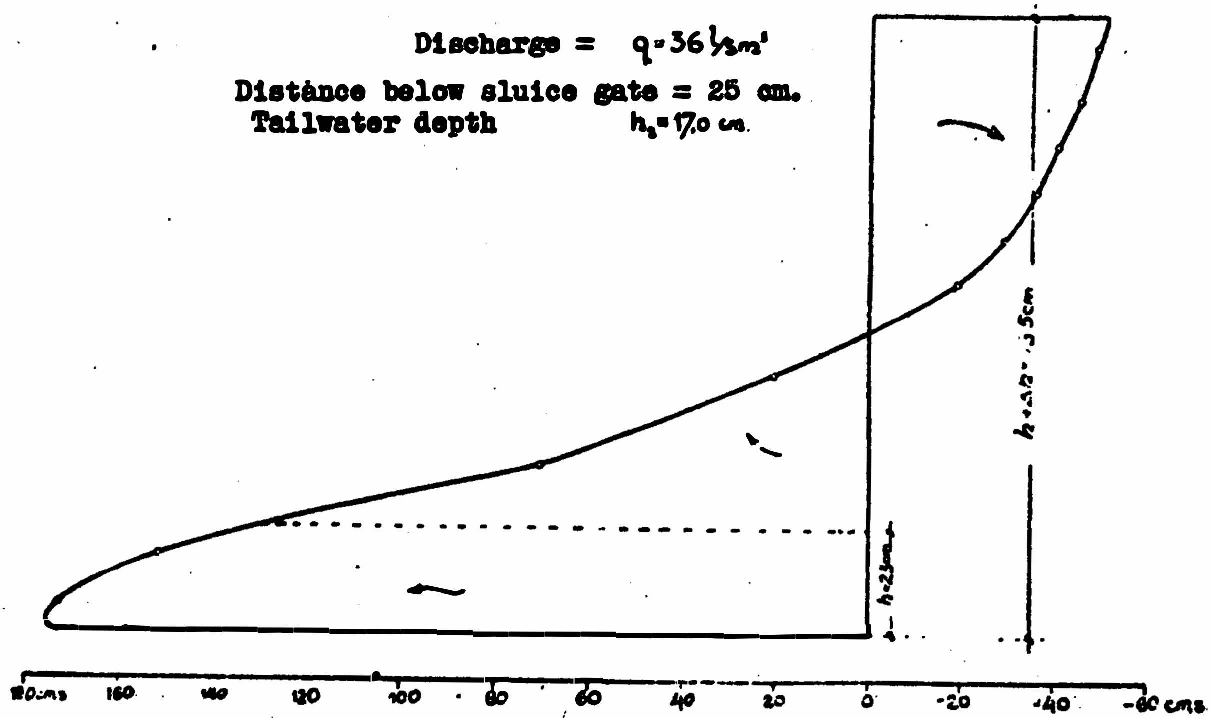

Fig. 31

Discharge = $q = 36\ l/s\,m'$
Distance below sluice gate = 40 cm.
Tailwater depth $h_s = 17.0$ cm

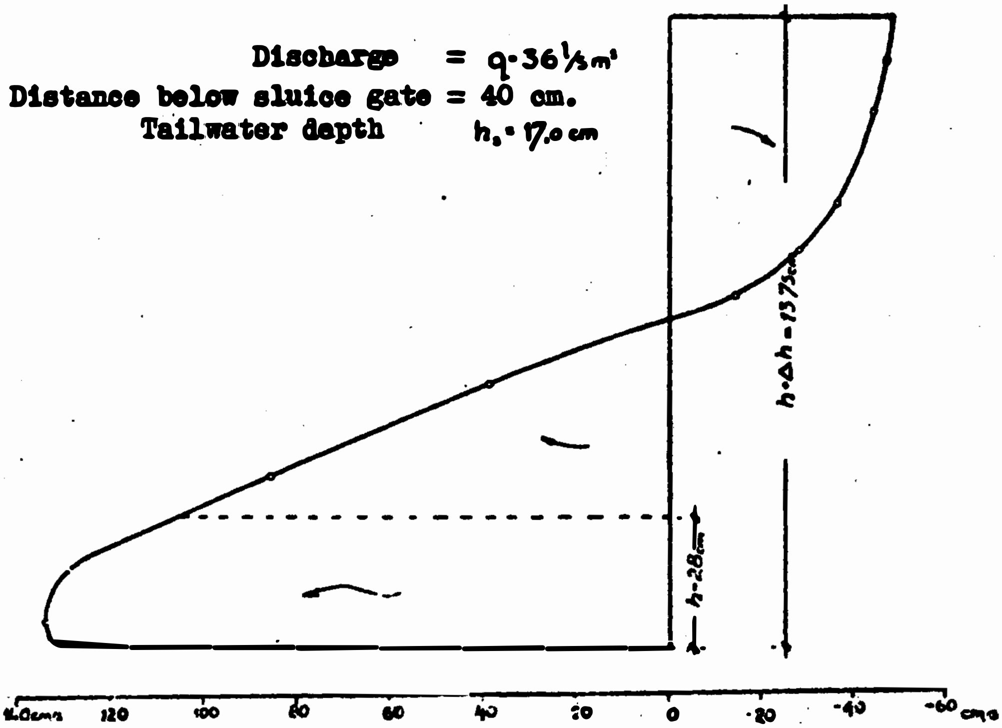

Fig. 32.

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Velocities in the Top Roll

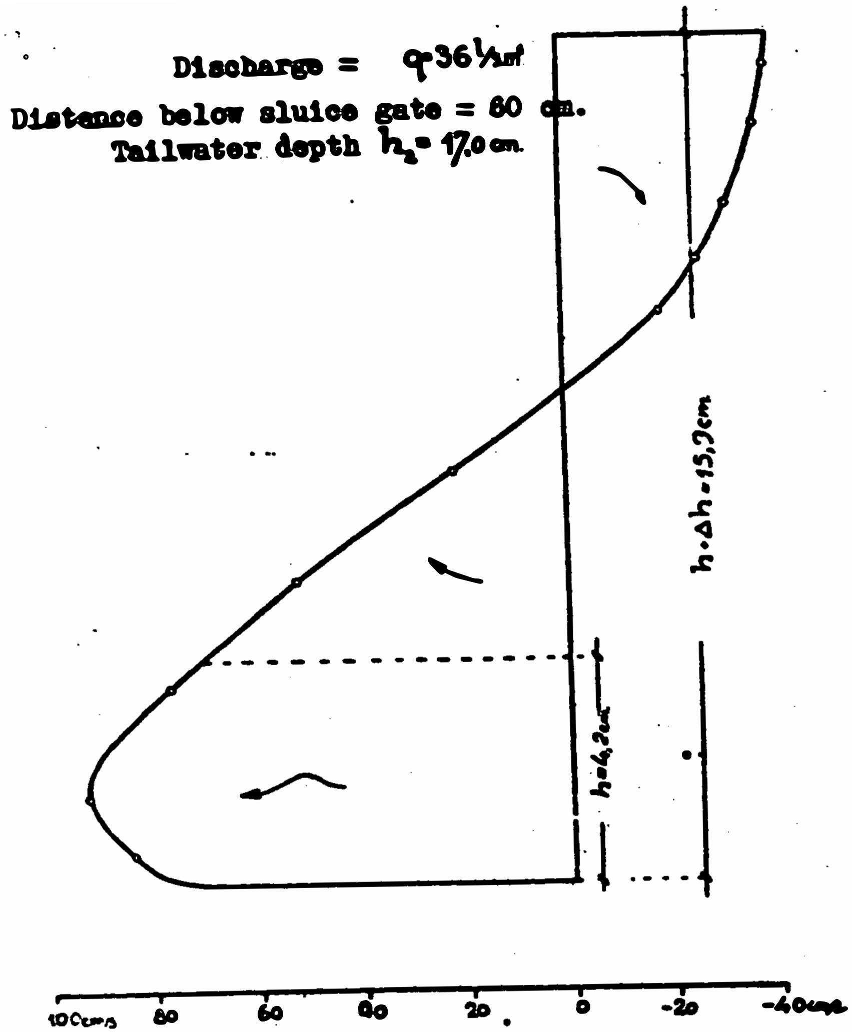


Fig. 33

Velocities in the Top Roll

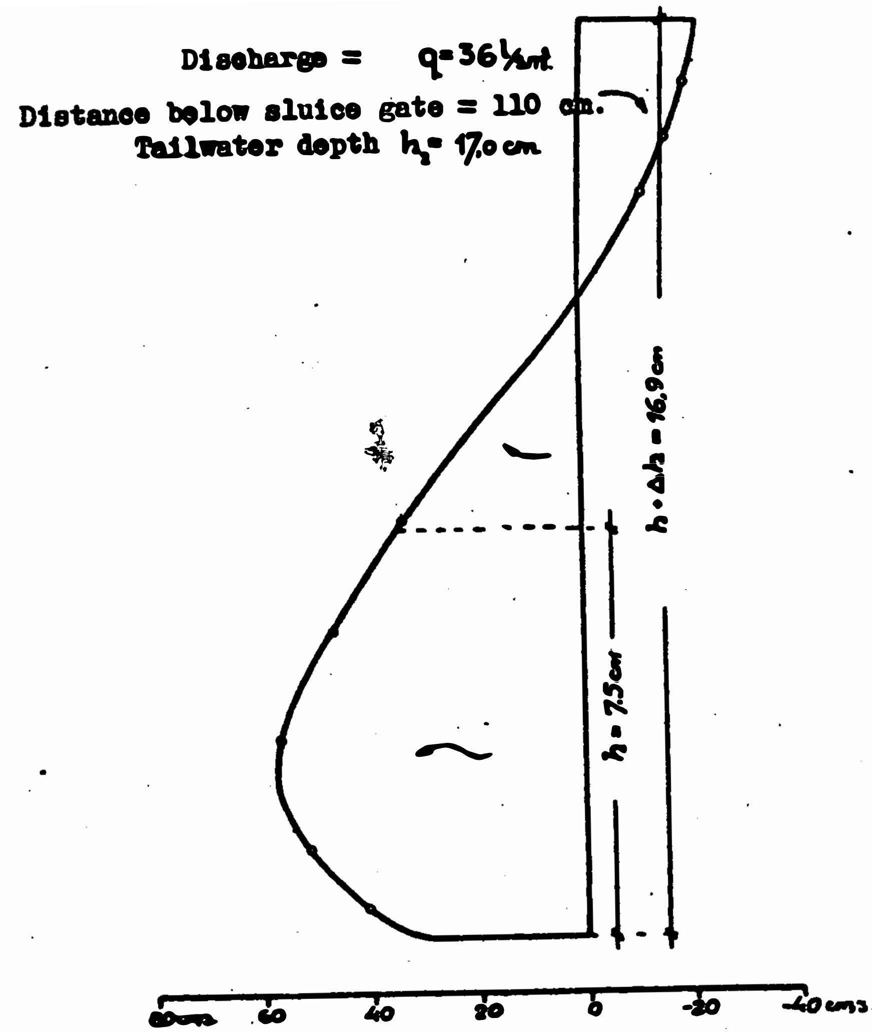
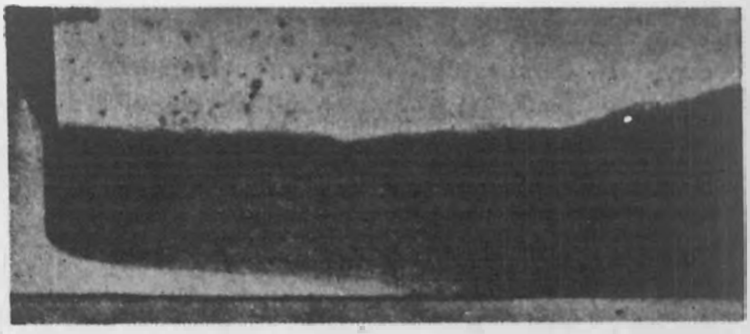


Fig. 34

a



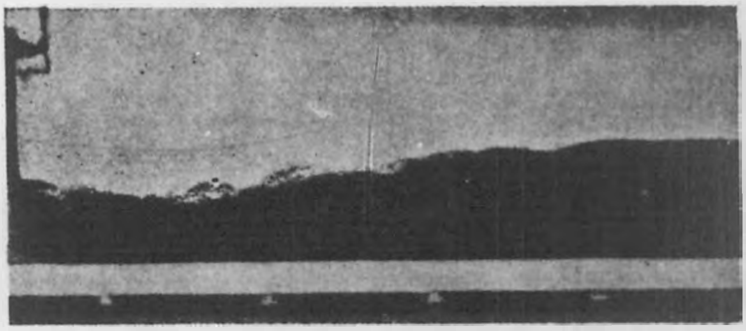
6.VI30.

$q = 50$ l./s. m[1]

$h_1 = 2.82$ cm.

$h_2 = 17.55$ cm.

b



6 VI.30.

$q = 30$ l./s. m[1]

$h_1 = 1.36$ cm.

$h_2 = 13.58$ cm.

c

7 VI.30.

$q = 40$ l./s. m.[1]

$h_1 = 2.15$ cm.

$h_2 = 17.60$ cm.

-1398

Fig. 35

Back Pressure Top Roll Below a Sluice Gate.

was taken to be the mean of the highest and lowest position of the surface as observed with a pointed gage. Table IV shows the observation of several tests. A comparison is shown between observed and calculated values of $h = h_1 + \Delta h_1$. The differences have maximum values of +6.1 and -7.8% and are attributed to errors in measurements.

Figures 29 and 30 depict in profile the division of main stream and top roll and the variations in velocity at a number of sections for two experiments with the same discharges but different head and tail waters. The sections nearest the gate show the highest velocities at the bottom, and show a rapid decrease of velocities as the distance from the bottom increases. Further downstream the highest velocities occur above the bottom. In these figures the retarding effect of the top roll can be observed. The cause of the decrease in velocities is attributable to the border zone between the main stream and the top roll. The main stream imparts a part of its energy to the top roll and this is converted into heat by the internal friction and turbulence of the roll. The change in style of flow and the passage of the main stream through the critical depth always occur near the end of the top roll. The velocity can be measured in only the lower portion of the first section below the gate as the swirls above mentioned make measurement nearer the surface impossible.

To show up the variations in velocity in more detail, Figures 31-34 were prepared giving the details of several observations to a large scale.

The photographs, page 35, show the position and division of the flow into main stream and top roll immediately below the gate. This showing was made possible by the use of dyes. Because of the intermingling at the border zone the sharpness of the division line decreases with the distance from the gate so that at the lower end of the top roll it is lost and the whole mass is one color. Figure 35b shows this especially well.

The experiments show that back pressure top roll cuts down the discharge for a given gate opening and also that the head water must be raised in order to secure the same discharge with a given gate opening as compared with conditions when free flow prevails. This head increase ΔH, (see Figure 36a) is equal to the depth of the top roll at the point where the water surface in a free discharge condition (i.e., at the vena contracta) is a minimum.

The assumption of exact correspondence of the minimum point in the top roll surface and the vena contracta of the free discharge jet disregard the small variations in the contraction which arises in consequence of the ΔH increase in the head water.

The experiments confirm the applicability of the theory of a constant value for the combined pressure-momentum expression for the back pressure top roll. Considering the case at the border line between free discharge and back pressure top roll it is possible to set up two equations which show the connection between tail water and head water increases. In this process certain restatements and omissions are necessary, the principal of which is the omission of the effect of the momentum in the top roll at the gate as previously

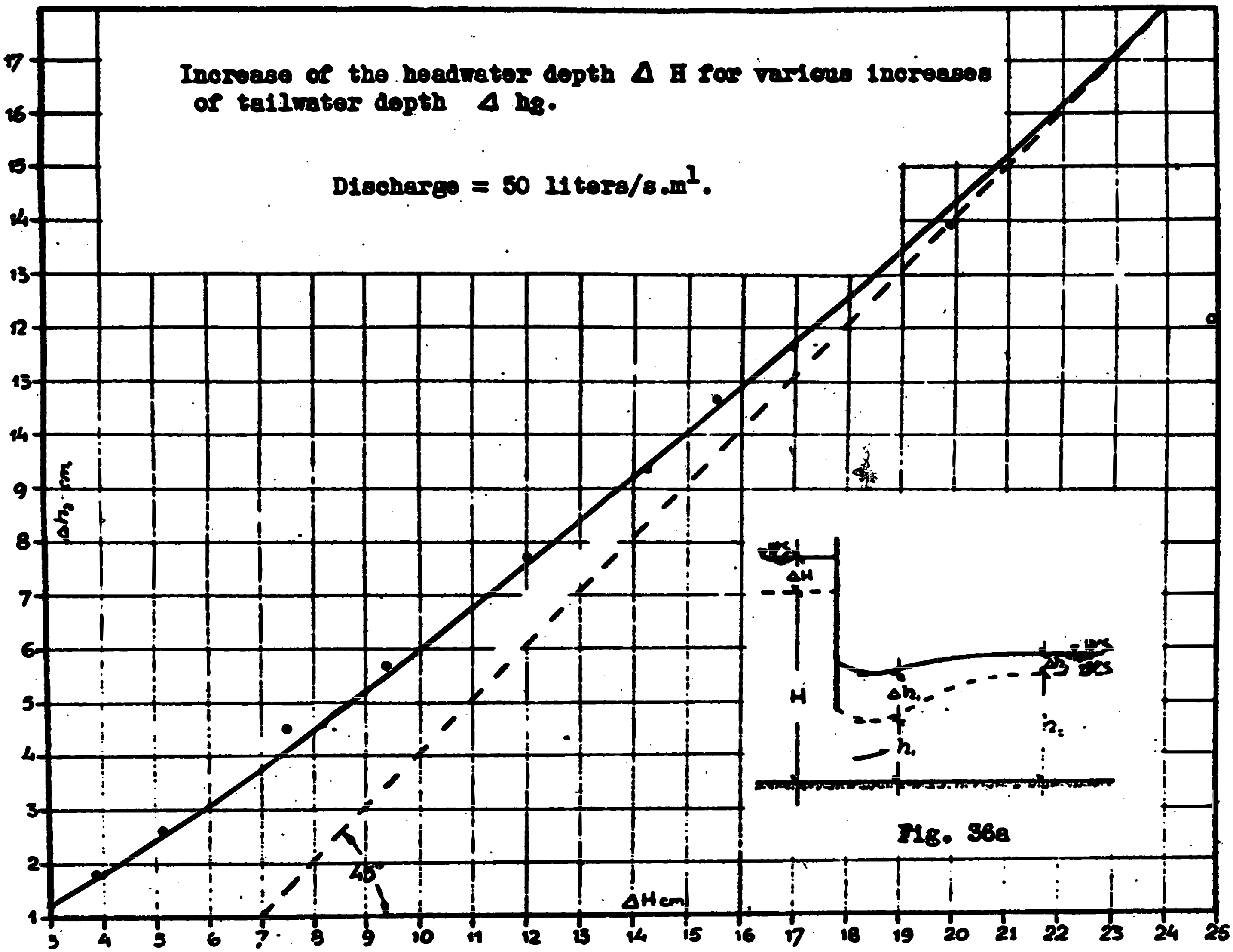

Increase of the headwater depth Δ H for various increases of tailwater depth Δ hg.
Discharge = 50 liters/s.m[1].
ΔH cm
45°
H
Fig. 36a

Fig. 36

mentioned. See Figure 36a for the meaning of symbols.

$$Y \frac{h_1^2}{2} + Y \frac{q}{g} v_1 = Y \frac{h_2^2}{2} + Y \frac{q}{g} v_2 \text{ - - - equation (6) free discharge}$$

$$Y \frac{(h_1 + \Delta h_1)^2}{2} + Y \frac{q}{g} v_1 = Y \frac{(h_2 + \Delta h_2)^2}{2} + Y \frac{q^2}{g(h_2 + \Delta h_2)} \text{ ---- equa-}$$

tion (16) modified

subtracting

$$\frac{(h_1 + \Delta h_1)^2}{2} - \frac{1}{2} h_1^2 = \frac{(h_2 + \Delta h_2)^2}{2} - \frac{h_2^2}{2} - \frac{q^2}{g} \left(\frac{1}{h_2} - \frac{1}{h_2 + \Delta h_2} \right)$$

Solving for $\Delta h_1 \cong \Delta H$

$$\Delta H \cong \Delta h_1 = -h_1 + \sqrt{h_1^2 + \Delta h_2 (2h_2 + \Delta h_2) - \frac{2q^2}{g} \left(\frac{1}{h_2} - \frac{1}{h_2 - \Delta h_2} \right)} \text{ ---(18)}$$

Figure 36 shows the computed and measured relations between head and tail waters for the discharge of 50 liters/second per meter width of channel and initial h_2 of 11.80 cm. The experimental data is given in Table V. The variation between computed and observed values falls between the limits of +3.3% and -7.8%. The experiment was made by first taking measurements from a free flowing jump and then drowning the jump by various amounts.

From these observed relations a formula was deduced which is applicable to practical cases of flow from sluice gates with back pressure top rolls. (See part 3).

D. Length of the Back Pressure Top Roll.

In the course of the experiments on back pressure top rolls the length of the top roll was also studied. This length is considered

TABLE VI

Observations of Experiments to Determine the Length of the Back Pressure Top Roll

No. of Exp.	q $l\,s.^{-1}\,m^{-1}$	h_2 cm.	h_1 cm.	$h_2 - h_1$ cm.	$h_2 : h_1$	Length of Top Roll l cm.	$h_2 - q_1$
1	120	8.05	21.89	13.84	2.72	85	6.13
3	″	7.57	28.12	20.55	3.71	120	5.83
4	100	6.82	26.27	19.45	3.85	115	5.92
7	″	6.05	26.40	20.35	4.36	145	5.65
8	80	3.70	19.90	16.20	5.38	87.5	5.40
10	″	5.24	25.82	19.59	4.92	115	5.87
11	50	3.00	25.72	22.72	8.56	125	5.31
18	″	3.00	14.30	11.30	4.80	65	5.70
19	40	2.10	19.46	17.36	9.28	100	5.77
21	″	2.10	12.85	10.75	6.11	50	5.50
23	30	1.85	24.90	23.05	13.45	130	5.65
26	″	1.85	10.76	8.91	5.82	47.5	5.33
30	25	1.53	25.61	24.08	16.70	123	5.10
32	″	1.53	20.79	19.26	13.60	105	5.46
39	20	1.40	27.40	26.00	19.65	145	5.17
45	″	1.40	8.36	6.96	5.97	40	5.74
46	10	0.51	14.61	14.10	28.70	69	4.82
47	″	0.51	12.61	12.10	24.80	55	4.55
48	″	0.51	12.15	11.64	23.80	60	5.15
53	″	0.51	7.16	6.65	14.05	37.5	5.03

Curve showing relation between height and length of jump with back pressure top roll.

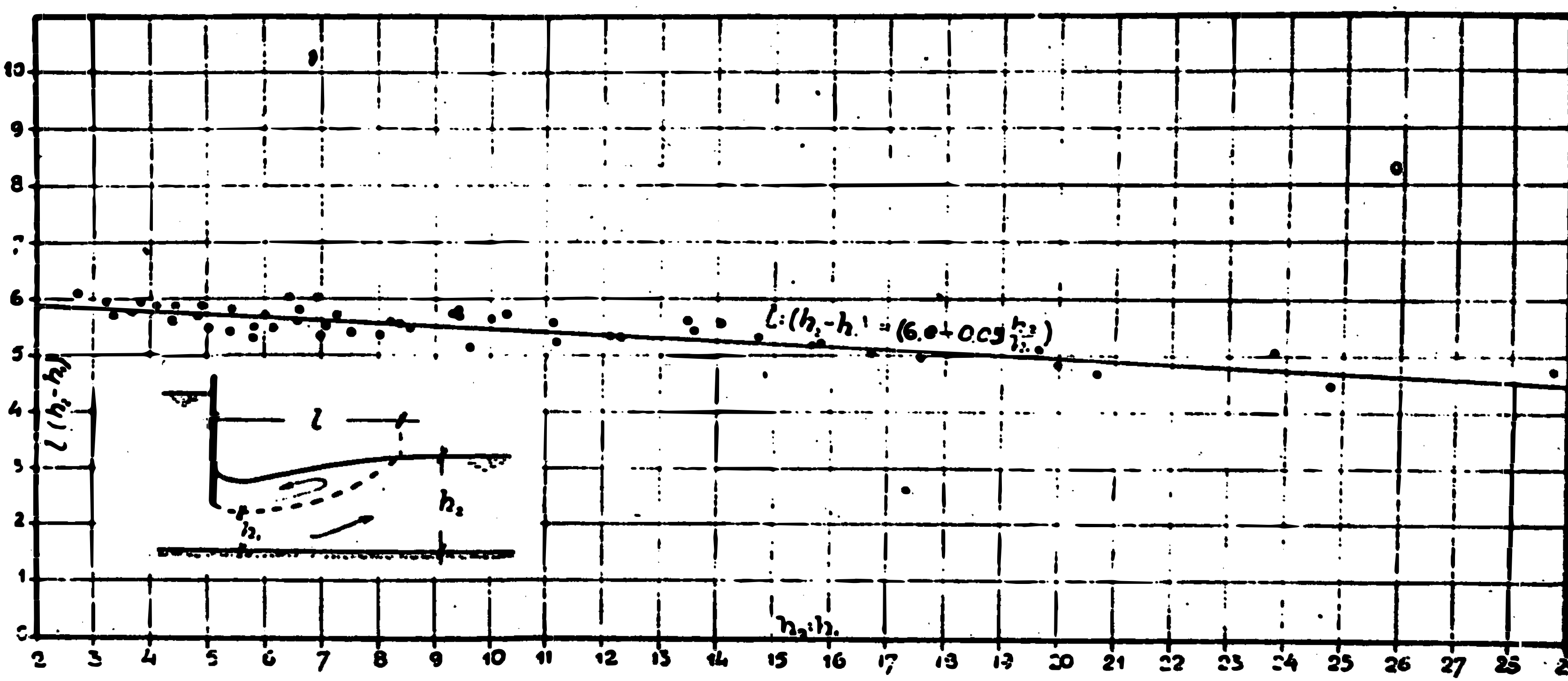


Fig. 37

to be the reach from the gate to the point where the backward movement of water ceased as determined by the use of dyes. Table VI shows various values for this length. The points shown in Figure 37 were derived from the relations $\frac{h_2}{h_1}$ and $\frac{\ell}{(h_2 - h_1)}$ in the same manner that they were set up in the discussion of the length of the free jump. The straight line through these points shows the same slight downward slope in the direction of increasing values of $\frac{h_2}{h_1}$ that was observed in the case of the free jump. The equation of this line is

$$\ell = \left(6 - 0.05\frac{h_2}{h_1}\right)(h_2 - h_1) \quad ----- \quad (19)$$

The measurements of the length of the swirling zone of the jump, and their expression in terms similar to those used for the entire length of the jump give points which also lie close to a line. These phenomena are very similar in the two cases of free jump and back pressure top rolls. The length of the jump is always larger than the over-lying top roll both in the case of the free jump and the back pressure top roll jump. The length of the jump is taken as the length of the reach from the point where h_1 is measured (vena contracta), the point where the water begins to rise, to the point where h_2 is measured, the point where the water surface reaches its greatest height and begins to fall in the direction of the stream flow.

Part 3. Experiments on the Flow From a Sluice Gate.

A. Water Surface Below the Gate.

In the case of flow under a sluice where a back pressure top roll occurs it was observed that the tail water had a maximum

Fig. 38 on same plate with Figs. 13, 14 & 15

Fig. 39 on same plate with Fig. 16

depression at a certain distance from the gate. At the gate the water surface is always somewhat higher. Examination of photographs, Figures 35 and 38, show this plainly.

Figure 39 shows the measured water surface for three different circumstances of tail water. Such a form of water surface occurs in all cases of drowned outflow. The rise of the water evidences itself by the contraction of the outflowing stream. At the point of greatest contraction, where $Y \frac{q}{g} v$ has its greatest value, there is also found the lowest point in the water surface of the top roll. The contraction of the outflow requires a certain length. Right at the gate the contraction is small hence the expression $Y \frac{q}{g} v$ of the momentum-pressure equation has not attained its maximum value. Assuming that the total value of the pressure-momentum equation is constant it is therefore necessary that the $\frac{Y(h + \Delta h)^2}{2}$ part of the equation be larger to make up the deficiency or that the water depth at the gate be greater than at the vena contracta.

B. Discharge Formula.

The contraction of the water surface at the vena contracta is of great importance in the practical application of formula to flow through sluices with back pressure flow conditions.

Consider the usual flow formula

$$Q = \mu b a \sqrt{2gh} \quad \text{------------} \quad (20)$$

Assuming that $h = H_0 - h_2$, Figure 40, serious differences between computed and observed discharge occur in certain cases. The computed

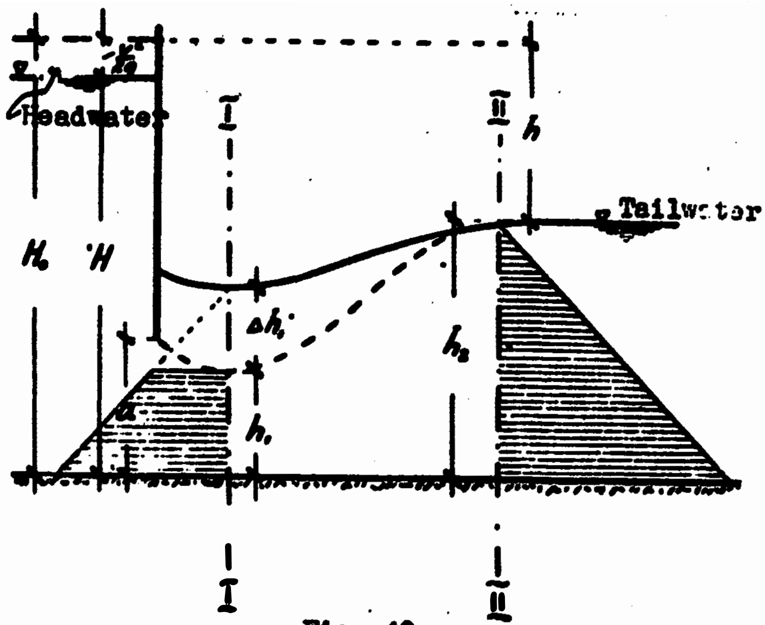


Fig. 40

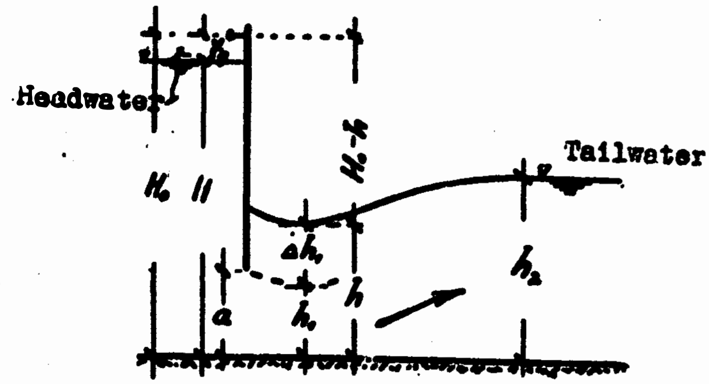


Fig. 41

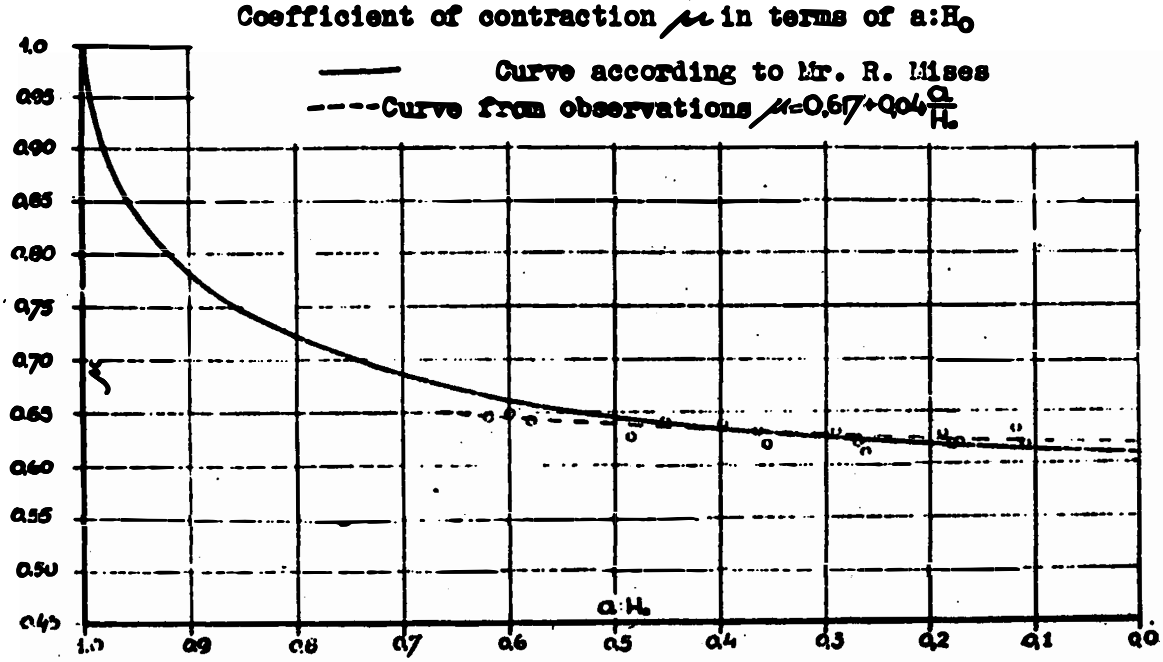


Fig. 42

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value is always smaller than the actual value. Koch called attention to this situation. In his book he gives the method of calculating the flow of drowned sluices based upon the pressure-momentum principle but he considers the pressure for the section below the gate to be applied only to the depth of the main stream flow at the vena contracta as shown in Figure 40. To make correct use of the momentum-pressure theory for the section between the gate and the end of the top roll the pressure must be considered as applied to the entire height of the water as previously pointed out.

In the sections directly below the gate it is permissible to neglect the momentum of the top roll as previously explained, but it is not permissible to neglect the influence of the rise in the water surface of the top roll in calculations of the hydrostatic pressure. Such neglect has produced serious error. The described experiments prove this point in toto. The depth at the vena contracta must be taken into consideration in the calculation of the discharges of sluice gates with drowned outflow. The governing pressure head is $H_o - h$, see Figure 41.

By combining equations

$$Q = bh_1 \sqrt{2g(H_o - h)} \qquad \text{---------- (21)}$$

and

$$\frac{(h_1 + \Delta h_1)^2}{\mu^2} + \frac{Q}{gb} v_1 = \frac{h_2^2}{2} + \frac{Q}{gb} v_2 \qquad \text{----------- (15)}$$

where $h_1 = \mu a$ and μ = contraction coefficient it is possible to deduce the formula for the case under consideration by changing,

$$(h_1 + \Delta h_1)^2 = h_2^2 - \frac{2q}{g}(v_1 - v_2) = h_2^2 - \frac{2q^2}{g}\left(\frac{h_2 - h_1}{h_2 h_1}\right)$$

or $$h_1 + \Delta h_1 = h = \sqrt{h_2^2 - \frac{2q^2}{g}\frac{(h_2 - h_1)}{h_2 h_1}} \quad \text{- - - - - - - - - -} \quad (22)$$

By eliminating q an expression is obtained for h which can be substituted in formula (21) so as to make this a correct equation for the discharge.

$$h_1^2\, 2g\,(H_o - h) = q^2$$

$$h^2 = h_2^2 - \frac{2}{g}\frac{h_2 - h_1}{h_2 h_1} \times h_1^2 \frac{2}{g}(H_o - h)$$

$$h = \frac{2h_1}{h_2} \times (h_2 - h_1) + \sqrt{\left(\frac{2h_1}{h_2}(h_2 - h_1)\right)^2 + h_2^2 - 2H_o\left(\frac{2h_1}{h_2}(h_2 - h_1)\right)} \quad \text{- - - - - - - -} \quad (23)$$

Let $t = \frac{2h_1}{h_2}(h_2 - h_1)$

then $$h = t + \sqrt{t^2 + h_2^2 - 2H_o t} \quad \text{- - - - - - - - - - - - - - -} \quad (24)$$

and the discharge formula will be

$$Q = \mu ab\sqrt{2g\left(H_o - t - \sqrt{t^2 + h_2^2 - 2H_o t}\right)} \quad \text{or}$$

$$= b\mu a\sqrt{2g(H_o - h)} \quad \text{- - - - - - - - - - - - - - - - -} \quad (25)$$

As the tail water lowers, the thickness of the top roll decreases and at the moment when the depth at the vena contracta, h_1, and depth of the tail water h_2 satisfy equation (6) a free jump begins. At this moment Δh in equation (15) becomes zero, the discharge under the gate becomes independent of the tail water depth. Also Δh of Figure 41 becomes equal to zero and h becomes equal to h_1.

Should the tail water drop still more the jump will move downstream and finally should the values of h_2 reach h_k, the critical depth, the jump would disappear.

C. Contraction of the Jet With a Sharp Gate Edge.

For given depths of head and tail water and a given gate opening the form of the water surface of the pressure top roll in the vicinity of the gate depends upon the contraction of the main stream. With a sharp gate edge or other cause of increased contraction the water surface of the top roll is depressed further than in the case of a rounded gate edge.

To test the correctness of the formulas above deduced, the values of μ, the coefficient of contraction, were determined by experiment.

R. Mises deduced theoretically the value of this coefficient for various cases of discharge from a vessel. His work is based upon the "potential" theory and is given in a book entitled "Calculation of Orifice and Weir Coefficients". By assuming that gravity effects on the jet may be neglected and by assuming the insertion of a flat plate in the middle of the orifice it may be reasoned that the discharge is very similar to that under a sluice gate as in this case.

Because the sluice gate of the experiments was sharp edged the value used for the coefficient of contraction was that calculated by Mises for the similar character of orifice edge.

Figure 42 shows a curve giving the theoretical value of the coefficient for various values of the ratio of the gate opening "a" and the head on the gate sill H_o. In the same figure and with dashed lines is shown a similar curve based upon the experiments of this work. The observed values from which the curve was constructed are shown as

TABLE V

Observations of Experiments Showing the Relation Between an Increase in Tailwater Depth Δh_2 and the Corresponding Increase of Headwater Depth ΔH.

No. of Exp.	Observed q l. s.[1] m.[1]	h_1 cm.	h_2 cm.	Δh_2 cm.	H cm.	(1) ΔH cm.	Calc'd (2) ΔH cm.	Difference (1) — (2) cm.	Difference (1) — (2) %
1	120	6,05	19,20		26,61				
			23,20	4,00	33,91	7,30	7,36	— 0,06	— 0,82
2	..	7,67	16,10		19,81				
			26,80	10,70	33,56	13,75	13,43	+ 0,32	+ 2,39
3	..	7,76	15,05		19,81				
			21,89	5,94	27,72	7,91	7,56	+ 0,25	+ 3,31
4	100	4,65	18,60		28,24				
			20,92	2,32	33,69	5,44	5,41	+ 0,03	+ 0,06
5	..	5,34	16,90		23,16				
			22,97	6,07	33,13	9,98	9,91	+ 0,07	+ 0,07
6	..	6,76	14,25		18,01				
			24,30	10,15	30,66	12,65	12,65	± 0,00	± 0,00
7	80	4,03	15,02		20,35				
			20,35	4,43	32,97	8,62	8,58	+ 0,04	+ 0,05
8	..	4,58	14,41		20,18				
			22,75	8,34	32,88	12,70	12,69	+ 0,01	+ 0,01
10	50	3,00	11,90		17,53				
			25,72	13,92	37,51	19,98	19,53	+ 0,45	+ 2,31
13	..	..	21,13	9,33	31,83	14,30	14,24	+ 0,06	+ 0,48
18	..	..	13,59	1,79	21,32	3,70	3,93	— 0,19	— 4,77
23	30	1,76	9,40		16,44				
			24,90	15,50	37,93	21,49	21,11	+ 0,38	+ 1,80
25	..	..	15,73	6,33	27,04	10,60	10,37	+ 0,23	+ 2,22
27	..	..	11,16	1,79	20,86	4,42	4,28	+ 0,14	+ 3,26
32	25	1,45	8,65		16,45				
			23,96	15,31	37,53	21,08	21,13	— 0,05	— 0,24
37	..	..	16,14	7,49	28,27	11,82	12,00	— 0,18	— 1,50
40	..	..	10,23	1,58	20,47	4,02	4,00	+ 0,02	+ 0,50
41	20	1,32	7,05		13,19				
			27,58	20,53	38,71	25,52	25,22	+ 0,30	+ 1,19
44	..	..	13,29	6,24	23,00	9,81	9,78	+ 0,03	+ 0,31
47	..	..	8,45	1,40	16,40	3,21	3,32	— 0,11	— 3,31
48	30	1,36	10,85		27,50				
			13,58	2,73	33,69	6,19	6,71	— 0,52	— 7,75

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TABLE VII

Observations of Experiments on the Discharge of Sluice Gates

No. of Exp.	Actual q l./s.¹ m.¹	Observed H_0	Observed a	Observed h_2	Calculated According to Formula (25) q_1 l./s¹ m.¹	Differences $q - q_1$ l./s.¹ m¹	Differences $q - q_1$ %
2	20	34.42	2.22	24.78	19.70	−0.30	−1.50
4	20	38.23	2.58	31.16	19.60	−0.40	−2.00
6	20	20.38	2.58	13.90	20.25	+0.25	+1.25
13	40	34.52	2.92	15.50	40.25	+0.25	+0.62
15	40	27.65	3.83	15.13	40.40	+0.40	+1.00
16	40	35.46	3.53	21.53	39.70	−0.30	−0.75
24	60	20.13	5.37	17.86	61.80	+1.80	+3.00
28	60	22.58	6.125	15.47	60.90	+0.90	+1.50
30	60	19.25	7.00	15.30	59.10	−0.90	−1.50
35	80	32.07	6.55	20.35	81.75	+1.75	+2.19
36	80	29.85	6.55	18.72	78.50	−1.50	−1.88
39	80	25.64	7.44	17.82	79.80	−0.20	−0.25
44	100	33.68	7.55	20.92	103.90	+3.90	+3.90
46	100	33.13	8.62	22.97	99.70	−0.30	−0.30
48	100	30.11	10.76	24.30	99.50	−0.50	−0.50
50	120	33.91	9.725	23.20	124.50	+4.50	+3.75
52	120	33.56	12.18	26.80	119.60	−0.40	−0.33
53	120	27.72	12.18	21.80	121.70	+1.70	+1.42

TABLE VIII

Comparison of Calculated and Observed Discharges of Sluice Operating Under Pressure

Actual Discharge q_0 l. s. m.[1]	Observed Values H_0 cm.	a cm.	h_2 cm.	Values Computed According To Equation (20) Q	Depart. from q_0 l. s. m.'		Equation (25) Q	Depart. from q_0 l. s. m.'	
20	11.09	2.58	7.38	13.55	6.45	12.98	20.10	+0.10	+0.50
40	29.00	2.92	12.75	32.00	8.00	20.00	40.30	+0.30	+0.75
50	21.32	4.80	13.59	31.40	18.60	—37.20	49.40	—0.60	-1.20
60	34.19	4.60	18.27	49.50	10.50	17.50	60.50	+0.50	+0.83
80	19.44	9.30	16.00	49.00	31.00	—38.70	78.25	—1.75	—2.19
100	30.66	10.76	24.30	75.50	24.50	—24.50	99.50	—0.50	—0.50
120	27.72	12.18	21.89	83.00	37.00	—30.85	121.70	+1.70	+1.42
120	33.91	9.725	23.20	87.60	32.40	—27.00	124.50	+4.50	+3.75

points. The difference between the two curves is not very great. The upper limit of experimental values is $\frac{a}{H_o} = .62$ or $\frac{a}{H} = .67$. This limit was set by the experimental equipment as the point past which the gate could not be opened and still maintain a free jump. It might have been possible to over-step this limit by tilting the apparatus, but this would also have had an effect upon the head water. This was further than it was desired to carry the experiments.

The following equation fits the experimental results:

$$\mu = 0.617 + 0.04 \frac{a}{H_o}$$

(This value is not in accord with other experiments and cannot be substantiated by the experiments, Table V. They give a value of $\mu = .70$, a value also given by King for orifices with contraction suppressed on the sides and bottom).

D. Results of Experiments.

The experiments show close agreement between measured discharges and those computed according to formula (25). The biggest discrepancy was +3.90%. The use of the old formula (20) gave values consistently smaller than those measured, the biggest difference was 38.7%. Table VII gives detail values for a number of experiments and compares the actual values observed with values computed according to formula (25), Table VIII similarly compares experiments and values of Q computed both according to formula (20) and formula (25).

E. Diagrams for the Computation of Discharges under Sluice Gates.

In the calculation of the discharge of sluice gates it is desirable to know (1) the character of the discharge for particular circumstances and (2) the discharge. Usually the head water elevation is given and it is desired to know the items in question assuming various values of the gate opening or the tail water. It is laborious to work out cases using formula (25) and to lighten this work three diagrams were prepared for sharp edged gates using the values shown in Figure 42 as the Mises values of the coefficient of contraction.

To make the diagrams universally applicable, quantities were expressed in terms of ratios instead of giving absolute values. In diagram II these values are $\frac{H}{q^{2/3}}$, $\frac{a}{q^{2/3}}$, and $\frac{h_2}{q^{2/3}}$. In diagrams III and IV they are $\frac{q}{H^{3/2}}$, $\frac{h_2}{H}$, and $\frac{a}{H}$. In all cases H is the height of head water above the gate sill, h_2 is the depth of tail water, "a", the gate opening and q the discharge. (Unfortunately this did not work out as the author intended as he uses for q the discharge in cubic meters per second per meter width of opening and this is not made a pure number by multiplication or division by linear dimensions such as H, h_2 and a.) Notched lines near the borders of the diagrams indicate the limits of experimentally verified relations.

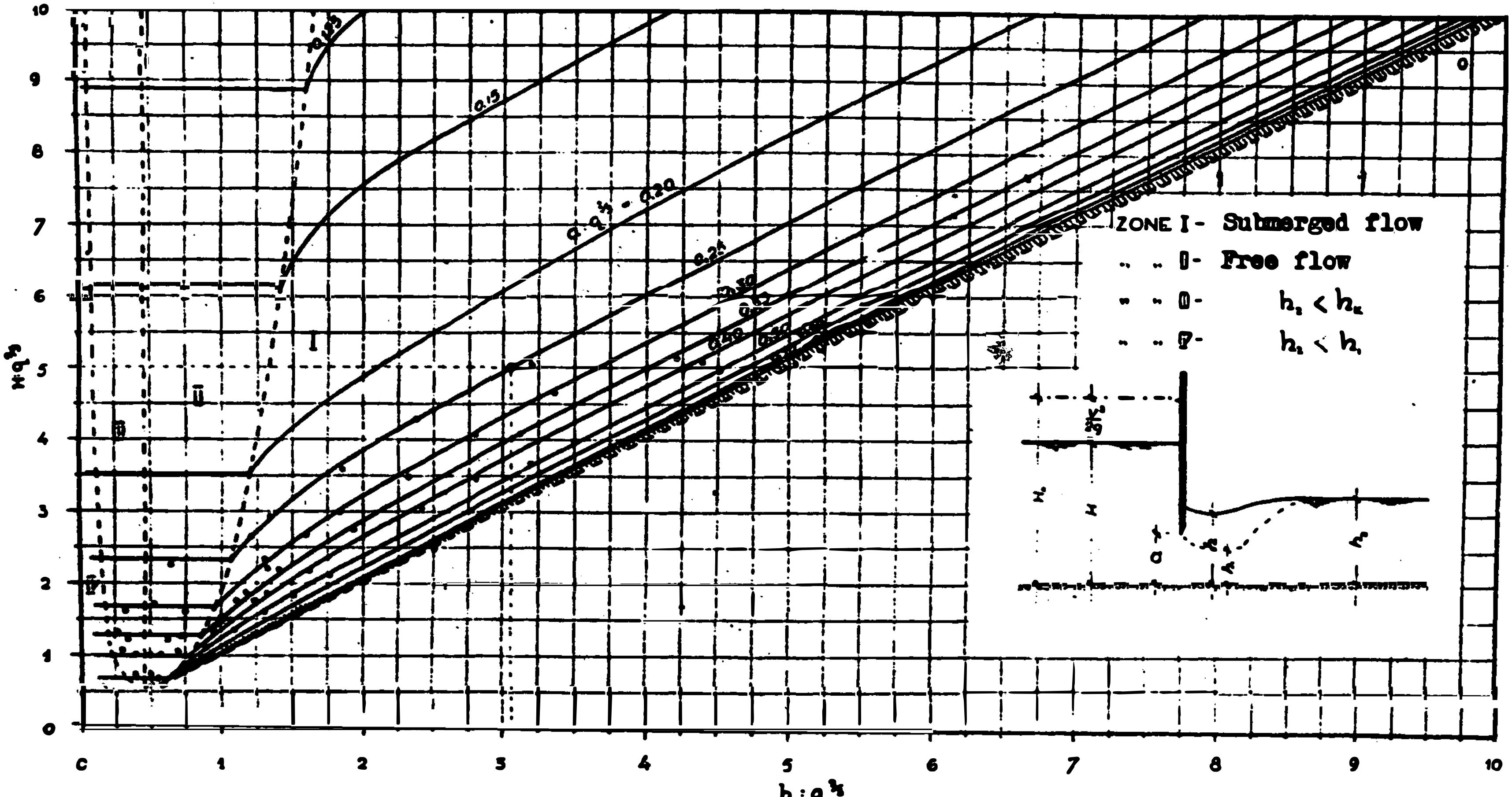
To make the use of the diagrams easier the values of H_o, height of the energy gradient above the gate sill, in equation (25) were replaced, after appropriate correction, by the values of H, the height of the head water above the gate sill.

The diagrams are divided into 4 zones:

Zone 1: Discharge where the top roll causes back pressure on the

DIAGRAM II

Headwater H as a Function of the Gate Opening (a) and the Tailwater Depth (h_2) for Various Discharges (q) per Meter Width of Channel



Discharge of Sluice Gates

gate opening, the case where formula (25) is applicable. Zones II, III, and IV free discharge, where the tail water depth is to be taken at the value applicable to the vena contracta, i.e., h_1.

Zone II, $h_2 > h_k$ covers the case where the jump occurs at some distance from the gate.

Zone III, $h_2 < h_k$ covers the case where the jump does not occur because conditions permit of keeping up a velocity in excess of critical.

Zone IV, covers the case where $h_2 < h_k$ and the velocity continues to accelerate after leaving the vena contracta.

The border line between Zones I and II is that where conditions are such that a jump below the gate is just obvious; the line between Zones II and III is that where the critical depth is just reached; the line between Zones III and IV is that where $h_2 = h_1$.

Diagram II shows the head water H as functions of the gate opening "a", and the tail water h_2 for given values of q. By its use H can be determined for chosen values of q, a and h_2.

For instance:

given q = 10.00 cubic meters per sec. per meter width of gate

h_2 = 14.20 m.

a = 1.16 m. what is H?

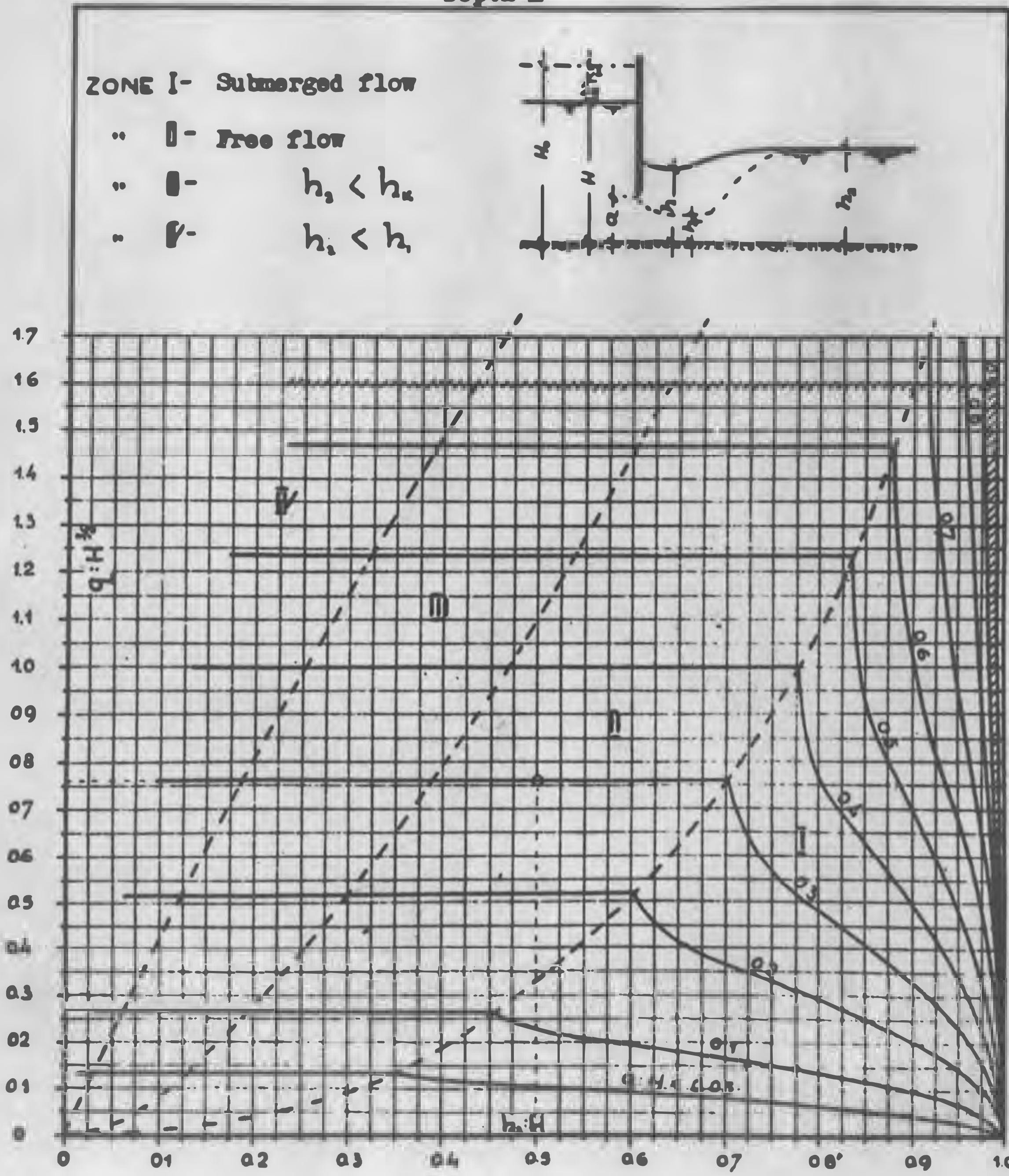
$$q^{2/3} = 4.642 \text{ and hence } \frac{h_2}{q^{2/3}} = 3.06$$

$$\frac{a}{q^{2/3}} = 0.25$$

The point of abscissa value 3.06 lying on the curve $\frac{a}{q^{2/3}} = 0.25$ falls

DIAGRAM III

Discharge per Meter Width of Channel as a Function of Tailwater Depth h_2 and Gate Opening (a) for Given Values of Headwater Depth H



Discharge of Sluice Gates

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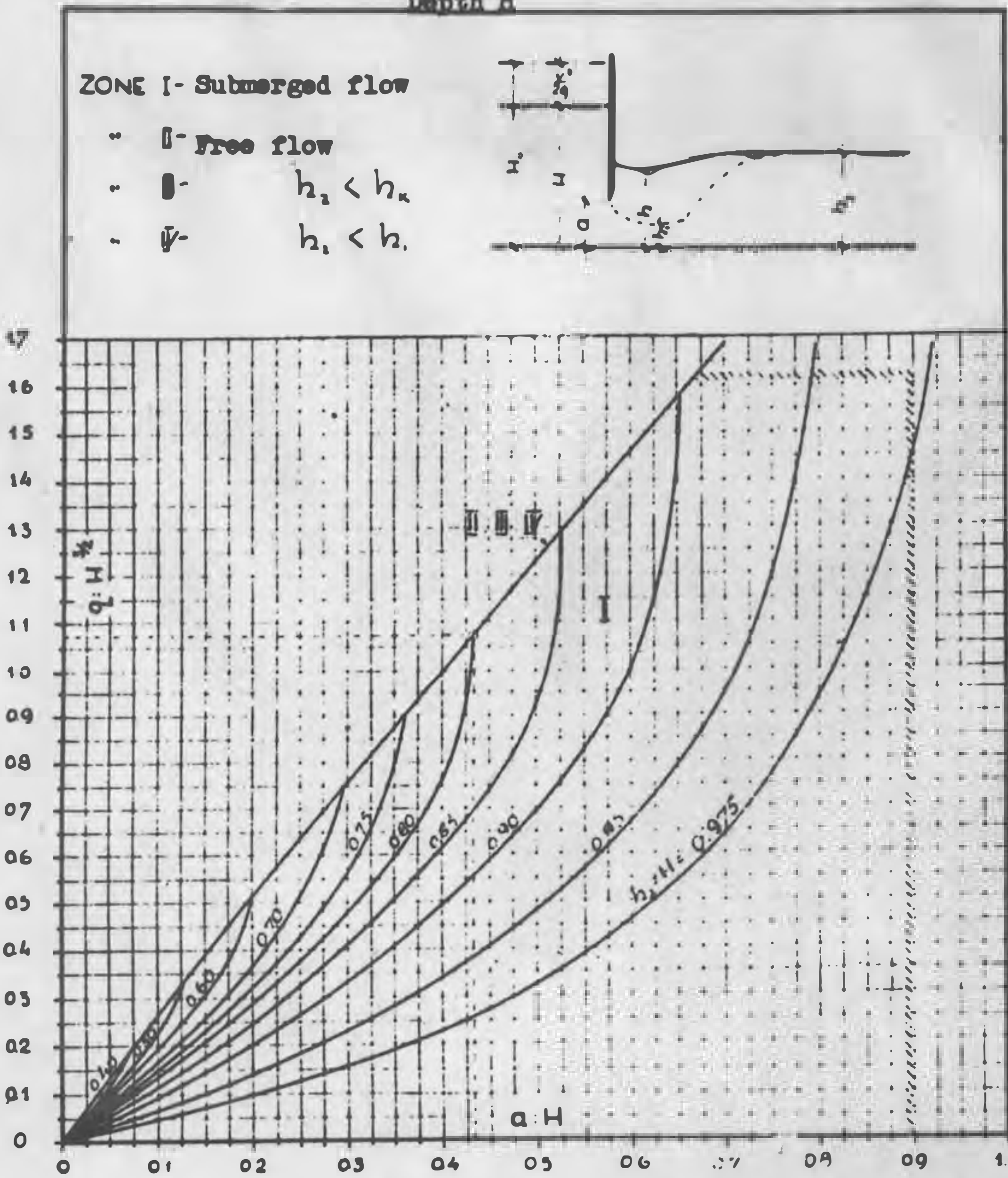
DIAGRAM IV

Discharge per Meter Width of Channel as a Function of Tailwater Depth h2 and Gate Opening (a) for Given Values of Headwater Depth H



Discharge of Sluice Gates

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in Zone I. This means that the flow will be under back pressure top roll conditions. The point above described also has as ordinate the value 5.00 or $\frac{H}{q^{2/3}}$ = 5.00 and hence H = 5.00 x 4.642 = 23.21 meters.

Diagram III gives discharge q as functions of gate opening (a) and tail water depth h_2 for given values of H and is convenient to obtain q for fixed values of a and h_2 and chosen values of H. It shows clearly in Zone I how a decrease in the depth of the tail water brings about an increase in discharge for a given gate opening.

For instance given H = 10.00 meters

a = 3.00 meters

h_2 = 5.00 meters required q

$$H^{3/2} = 31.623; \quad \frac{h_2}{H} = \frac{5.0}{10.0} = 0.50; \quad \frac{a}{H} = \frac{3.00}{10.00} = .30$$

The point on the curve $\frac{a}{H}$ = .30 which has the abscissa of .50 has the ordinate of 0.762 which is the value of $\frac{q}{H^{3/2}}$, q therefore equals 0.762 x 31.623 = 24.10 cubic meters per sec. per meter width of gate. This point falls in zone II which indicates that a free flowing discharge with a jump at some distance from the gate is to be expected.

Diagram IV gives the discharge q as functions of the tail water depth h_2 and gate opening a for given values of the head water H similar to Diagram III but is arranged to show the effect of varying gate openings upon the discharge when the tail water is fixed.

For instance given

$$H = 10.00 \text{ meters}$$

$$a = 4.32 \text{ meters}$$

$$h_2 = 8.0 \text{ meters, sought } q.$$

$$H^{3/2} = 31.623; \frac{h_2}{H} = 0.80, \frac{a}{H} = 4.32$$

The point on the curve $\frac{h_2}{H} = 0.80$ with abscissa of 0.432 has the ordinate of $\frac{q}{H^{3/2}} = 1.075$ so that $q = 1.075 \times 31.623 = 34.00\ m^3$/sec./m. The point falls on the border line between zones I and II. This indicates that a free flowing discharge with a jump immediately below the gate is to be expected.

CONCLUSION

The experiments demonstrate the applicability of the momentum theory to all cases of water flow where there is a transition from rapid to tranquil flow. In consequence of this theorem the depth of water above and below such transitions can be computed from the following formula

$$h_2 = -\frac{h_1}{2} + \sqrt{\frac{h_1^2}{4} + \frac{2v_1^2h_1}{g}} \quad \text{- - - - - - - - -} \quad (6)$$

If h_2 is greater than the right hand of the above equation the jump disappears or travels downstream. If h_2 is smaller than the right hand side of the equation then the jump travels upstream until it reaches the sluice gate or weir and a back pressure top roll results.

A jump cannot occur without the formation of a top roll. The energy loss in a jump is connected with the top roll and the entire length of the jump has a cover of turbulent water.

The loss of energy in a jump is given by the formula:

$$\Delta E = \gamma q \frac{(h_2 - h_1)}{4h_1h_2} \text{ kg per second per meter width of channel.}$$

It is not possible to make a division of the loss between losses in the main stream of the jump and the losses in the top roll.

The dimensions of a jump can be fairly well determined. The depth by formula (6) and the length by the following formula:

$$l = \left[(h_2 - h_1) \quad (8.0 - 0.05\frac{h_2}{h})\right]$$

The discharge of a sluice gate operating under back pressure can be computed by the use of the following formula:

$$Q = \mu ba\sqrt{2g(H_o - h)}$$

whore: H_o is the onergy hoad of the hoad water above the gate sill; μ is the coefficient of contraction of the corrosponding froo jot; h_2 is the tail water dopth and h is computed according to the following formula:

$$h = \frac{2h_1}{h_2}(h_2-h_1) + \sqrt{\left[\frac{2h_1}{h_2}(h_2-h_1)\right]^2 + h_2^2 - 2H_o\left[\frac{2h_1}{h_2}(h_2-h_1)\right]}$$